Reading assignments don’t have to be turned in.

1. **Reading assignment.** Read chapters 7, 8, 9, 10 and 11 of Dym’s book.

2. **Reading assignment.** In the next few lectures we will cover chapters 4 and 6 in Dym’s book.

3. For any $m \times n$ real matrix $A$ show that
   
   A. $\|A\|_2 \leq \|A\|_F \leq \sqrt{n}\|A\|_2$ without using the SVD,
   
   B. $\max_{i,j} |a_{i,j}| \leq \|A\|_2 \leq \sqrt{mn} \max_{i,j} |a_{i,j}|$,
   
   C. $\frac{1}{\sqrt{n}}\|A\|_\infty \leq \|A\|_2 \leq \sqrt{m}\|A\|_\infty$,
   
   D. $\frac{1}{\sqrt{m}}\|A\|_1 \leq \|A\|_2 \leq \sqrt{n}\|A\|_1$.

4. Establish the following statements:
   
   A. A triangular orthogonal matrix is diagonal.
   
   B. Permutation matrices are orthogonal.
   
   C. The product of two orthogonal matrices is orthogonal.

5. Show that if $Q_1 + iQ_2$ is unitary then the real matrix
   
   $$Z = \begin{pmatrix} Q_1 & -Q_2 \\ Q_2 & Q_1 \end{pmatrix},$$
   
   is orthogonal.

6. Show that any matrix in $\mathcal{R}^{m \times n}$ is the limit of a sequence of full rank matrices.

7. Show that if $A$ has full column rank then $\|A(A^T A)^{-1} A^T\|_2 = 1$.

8. Let $X$ and $Y$ and be two real full column-rank matrices such that $Y^T Y = X^T X$. We know from a previous home assignment that there exist real matrices $A$ such that $AX = Y$ and $Y^T A = X^T$. 
A. Show that $\|Y\|_2 = \|X\|_2$.

B. Show that for any such matrix $A$, $\|A\|_2 \geq 1$.

C. Find an $A$ such that $\|A\|_2 = 1$.

9. A. Show that the set of skew-symmetric $n \times n$ real matrices is the orthogonal complement of the set of symmetric $n \times n$ real matrices in the vector space $\mathbb{R}^{n \times n}$ using the standard inner-product.

B. Let $A$ be a real square matrix. Show that if $x^T Ax = 0$ for all vectors $x$, then $A$ is skew-symmetric.

10. Find a matrix $P$ that is not an orthogonal projector that satisfies the condition that $x - Px$ is perpendicular to $Px$ for all $x$. **Hint:** Rotate by $\theta$ and scale by $1/(\cos \theta)$.

11. Show that for any matrix $M$

$$\|M\|_2 = \max_{\|A\|_F = 1} \|MA\|_F,$$

where $A$ is allowed to be any (finite) matrix such that $MA$ is well-defined. **Hint:** Show that $\|MA\|_F \leq \|M\|_2 \|A\|_F$. 