Home Work 5

Two pages.
Due on November 6, 2008

Reading assignments don't have to be turned in.

1. **Reading assignment.** Read chapters 7, 8, 9, 10 and 11 of Dym's book.
2. **Reading assignment.** In the next few lectures we will cover chapters 4 and 6 in Dym's book.
3. For any $m \times n$ real matrix A show that

   A. $\|A\|_2 \leq \|A\|_F \leq \sqrt{n}\|A\|_2$ without using the SVD,

   B. $\max\limits_{i,j} |a_{i,j}| \leq \|A\|_2 \leq \sqrt{mn} \max\limits_{i,j} |a_{i,j}|$,

   C. $\frac{1}{\sqrt{n}}\|A\|_\infty \leq \|A\|_2 \leq \sqrt{m}\|A\|_\infty$,

   D. $\frac{1}{\sqrt{m}}\|A\|_1 \leq \|A\|_2 \leq \sqrt{n}\|A\|_1$.
4. Establish the following statments:

   A. A triangular orthogonal matrix is diagonal.

   B. Permutation matrices are orthogonal.

   C. The product of two orthogonal matrices is orthogonal.
5. Show that if $Q_1 + iQ_2$ is unitary then the real matrix

   $$Z = \begin{pmatrix} Q_1 & -Q_2 \\ Q_2 & Q_1 \end{pmatrix},$$

   is orthogonal.
6. Show that any matrix in $\mathcal{R}^{m\times n}$ is the limit of a sequence of full rank matrices.
7. Show that if A has full column rank then $\|A(A^TA)^{-1}A^T\|_2 = 1$.
8. Let X and Y and be two real full column-rank matrices such that $Y^TY = X^TX$. We know from a previous home assignment that there exist real matrices A such that $AX = Y$ and $Y^TA = X^T$.

A. Show that $\|Y\|_2 = \|X\|_2$.

B. Show that for any such matrix A, $\|A\|_2 \geq 1$.

C. Find an A such that $\|A\|_2 = 1$.

9. A. Show that the set of skew-symmetric $n \times n$ real matrices is the **orthogonal complement** of the set of symmetric $n \times n$ real matrices in the vector space $\mathbb{R}^{n \times n}$ using the standard inner-product.

   B. Let A be a real square matrix. Show that if $x^T Ax = 0$ for all vectors x, then A is skew-symmetric.

10. Find a matrix P that is *not* an orthogonal projector that satisfies the condition that $x - Px$ is perpendicular to Px for all x. *Hint:* Rotate by θ and scale by $1/(\cos\theta)$.

11. Show that for any matrix M

$$\|M\|_2 = \max_{\|A\|_F = 1} \|MA\|_F,$$

where A is allowed to be any (finite) matrix such that MA is well-defined. *Hint:* Show that $\|MA\|_F \leq \|M\|_2 \|A\|_F$.