Reading assignments don’t have to be turned in.

1. **Reading assignment.** Finish reading chapters 4, 5 and 6 in Dym’s book.

2. Find the shortest distance between two infinite straight lines in $\mathbb{R}^3$. Note, the lines do not necessarily pass through the origin. Provide the simplest expression possible.

3. A matrix $A$ is said to be **normal** if $A^H A = AA^H$. Show that if a triangular matrix is normal, then it is diagonal.

4. Show that if $P$ is an orthogonal projection, then $Q = I - 2P$ is orthogonal.

5. What are the singular values of an orthogonal projection?

6. Find the singular values of the matrix

   $$ C = \begin{pmatrix} X \\ I \end{pmatrix} $$

   in terms of the singular values of the matrix $X$.

7. Find explicit expressions for the singular values of the $2 \times 2$ real matrix

   $$ A = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}.$$ 

8. Find the singular values of the matrix

   $$ B = \begin{pmatrix} I & X \\ 0 & I \end{pmatrix} $$

   in terms of the singular values of the matrix $X$.

9. Prove the following statement of the **CS Decomposition** (thin version). Consider the matrix

   $$ Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}, \quad Q_1 \in \mathbb{R}^{m_1 \times n}, \ Q_2 \in \mathbb{R}^{m_2 \times n}, $$

   where $m_1 \geq n$ and $m_2 \geq n$. If the columns of $Q$ are orthonormal, then there exist orthogonal matrices $U_1 \in \mathbb{R}^{m_1 \times m_1}, \ U_2 \in \mathbb{R}^{m_2 \times m_2}, \text{ and } V_1 \in \mathbb{R}^{n \times n}$ such that

   $$ \begin{pmatrix} U_1 & 0 \\ 0 & U_2 \end{pmatrix}^T \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} V_1 = \begin{pmatrix} C \\ S \end{pmatrix} $$
where

\[ C = \text{diag}(\cos(\theta_1), \ldots, \cos(\theta_n)), \]
\[ S = \text{diag}(\sin(\theta_1), \ldots, \sin(\theta_n)), \]

and

\[ 0 \leq \theta_1 \leq \theta_2 \leq \cdots \leq \theta_n \leq \frac{\pi}{2}. \]