Home Work 6

Two pages.

Due on November 13, 2008

Reading assignments don't have to be turned in.

1. **Reading assignment.** Finish reading chapters 4, 5 and 6 in Dym's book.
2. Find the shortest distance between two infinite straight lines in $\mathcal{R}^3$. Note, the lines do not necessarily pass through the origin. Provide the simplest expression possible.
3. A matrix A is said to be **normal** if $A^H A = AA^H$. Show that if a triangular matrix is normal, then it is diagonal.
4. Show that if P is an orthogonal projection, then $Q = I - 2P$ is orthogonal.
5. What are the singular values of an orthogonal projection?
6. Find the singular values of the matrix
$$C = \begin{pmatrix} X \\ I \end{pmatrix}$$
in terms of the singular values of the matrix X.
7. Find explicit expressions for the singular values of the 2×2 real matrix
$$A = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}.$$
8. Find the singular values of the matrix
$$B = \begin{pmatrix} I & X \\ 0 & I \end{pmatrix}$$
in terms of the singular values of the matrix X.
9. Prove the following statement of the **CS Decomposition** (thin version). Consider the matrix
$$Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \qquad Q_1 \in \mathcal{R}^{m_1 \times n}, Q_2 \in \mathcal{R}^{m_2 \times n},$$
where $m_1 \geq n$ and $m_2 \geq n$. If the columns of Q are orthonormal, then there exist orthogonal matrices $U_1 \in \mathcal{R}^{m_1 \times m_1}$, $U_2 \in \mathcal{R}^{m_2 \times m_2}$, and $V_1 \in \mathcal{R}^{n \times n}$ such that
$$\begin{pmatrix} U_1 & 0 \\ 0 & U_2 \end{pmatrix}^T \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} V_1 = \begin{pmatrix} C \\ S \end{pmatrix}$$

where

$$\begin{aligned} C &= \operatorname{diag}(\cos(\theta_1), \ldots, \cos(\theta_n)), \\ S &= \operatorname{diag}(\sin(\theta_1), \ldots, \sin(\theta_n)), \end{aligned}$$

and

$$0 \leq \theta_1 \leq \theta_2 \leq \cdots \leq \theta_n \leq \frac{\pi}{2}.$$