Home Work 7

Two pages

Due on November 20, 2008

Reading assignments don't have to be turned in.

1. **Reading assignment.** Finish reading chapters 4, 5 and 6 in Dym's book.

2. **Reading assignment.** Read the proof of the Jordan decomposition theorem in the class notes. Make sure you can supply the missing details.

3. Verify that if X diagonalizes
$$\begin{pmatrix} 1+\epsilon & 1 \\ 0 & 1-\epsilon \end{pmatrix}, \qquad 0 < \epsilon < 0.5,$$
then $\|X\|_1 \|X^{-1}\|_1 = \kappa_1(X) \geq 1/(2\epsilon)$.

4. Show that there is no non-singular X such that
$$X \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} X^{-1}$$
is diagonal.

5. Let $L_{A,B}$ be the linear operator
$$L_{A,B}(X) = AXB^H,$$
where A, B and X are matrices of suitable sizes. Find the eigenvalues of $L_{A,B}$ in terms of the eigenvalues of A and B.

6. Let $L_{A,B}$ be the linear operator
$$L_{A,B}(X) = AX + XB^H,$$
where A, B and X are matrices of suitable sizes. Find the eigenvalues of $L_{A,B}$ in terms of the eigenvalues of A and B.

7. Show that the eigenvalues of AB are the same as the eigenvalues of BA except for some eigenvalues at 0. This is true even if A and B are not square matrices. *Hint*: This is a bit tricky. A famous short proof is available in the text book.