

ECE215A/Materials206A Winter 2008
 Prof. Brown/ECE Dept/UCSB
 HW#1 Solutions

(1) Inspecting the Table 2.2 in Kittel, and other sources (see for example, <http://www.physicsofmatter.com/Edition2/Download/Tables/pdf/Tab07.pdf>):

(a) lowest compressibility, diamond form of C, $\kappa = 2.26 \times 10^{-12} \text{ m}^2/\text{n}$; (a) highest compressibility Cs , $\kappa = \sim 500 \times 10^{-12} \text{ m}^2/\text{n}$ (but barely a solid at 300 K, $T_{\text{melt}} = 301.6 \text{ K}$!)

(b) lowest expansivity W , $\alpha = 4.5 \times 10^{-6} \text{ K}^{-1}$; (b) highest expansivity Cd and Zn , $\alpha \approx 30 \times 10^{-6} \text{ K}^{-1}$

(from above Website, Table 7.7: α is linear expansivity; volume expansivity, $\beta = 3 \alpha$)

(c) lowest thermal conductivity S, $K = 0.15 \text{ W/K-m}$; (c) highest thermal conductivity diamond form of C, $K = 700\text{-}1700 \text{ W/K-m}$

(d) lowest electrical conductivity S, $\sigma = 5 \times 10^{-16} \text{ S/m}$ (d) highest electrical conductivity Ag, $\sigma = 6.2 \times 10^7 \text{ S/m}$

(2) The ideal gas and block of copper, each of volume 0.5 m^3 @ 300K and 1 atm. Pressure is increased to 5 atm isothermally.

(a) Work done in each process is given by $-\int_{V_1}^{V_2} P dV$ where V_1 is the volume at $P = 1 \text{ atm}$ and

V_2 is the volume at $P = 5 \text{ atm}$. It is helpful to draw a P - V diagram as shown in the figure below. In general, P decreases with increasing V for a gas or solid, but it decreases much slower for a gas because it is so much more compressible. Therefore, the change in volume ΔV and work done is greater for the gas. Graphically, the work done is the area under the P - V curve, as shown below.

(b) • For the ideal gas, $PV = nRT$

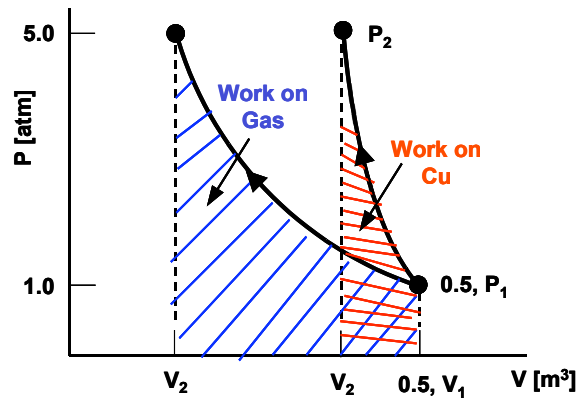
$$n = \# \text{ mole}, \quad R = \text{gas constant} = (8.31 \times 10^3 \text{ J / mole} - \text{K})$$

$$\text{So, } V = \frac{nRT}{P} \text{ and } dV = \frac{-nRTdP}{P^2} \text{ and } -\int_{V_1}^{V_2} P dV = \int_{P_1}^{P_2} \frac{nRT P dP}{P^2} = nRT \ln P \Big|_{P_1}^{P_2} = nRT \ln \left(\frac{P_2}{P_1} \right)$$

But all ideal gases have the same molal density, $v = 22.41 \text{ m}^3/\text{kilomole}$

$$\text{So, } n = \frac{0.5 \text{ m}^3}{22.41} = 0.0223 \text{ kmole} \text{ and } -\int P dV = (0.0223)(8.31 \times 10^3)(300) \ln(5/1) = 8.95 \times 10^4 \text{ J} .$$

• For copper, we know compressibility $\kappa = -\frac{1}{V} \frac{\partial V}{\partial P} = 0.7 \times 10^{-6} \text{ atm}^{-1}$. As in most solids, we can assume this κ is constant over small pressure changes (we will see how small when we study elasticity theory). So we can integrate,



The P-V description for Problem 2

$$-\kappa dP = \frac{dV}{V} \text{ or } \ln \frac{V_2}{V_1} = -\kappa (P_2 - P_1) \text{ or } V_2 = V_1 \exp[-\kappa(P_2 - P_1)]$$

$$\text{So, } \int_{V_1}^{V_2} -PdV = \int_{P_1}^{P_2} P\kappa V dP = \int_{P_1}^{P_2} P\kappa V_1 \exp[-\kappa(P - P_1)] dP$$

For $\kappa = 0.7 \times 10^{-6} / \text{atm}$, $P_1 = 1 \text{ atm}$, $P_2 = 5 \text{ atm}$ the term $\exp[-\kappa(P - P_1)]$ varies slowly

around unity, so that $\int_{P_1}^{P_2} P\kappa V_1 \exp[-\kappa(P - P_1)] dP \approx \int_{P_1}^{P_2} P\kappa V_1 dP = \kappa V_1 \frac{P^2}{2}$

$$= (0.7 \times 10^{-6})(0.5 \text{ m}^3) \left(\frac{1}{2}\right) (5^2 - 1^2) = 4.2 \times 10^{-6} \text{ m}^3 \cdot \text{Atm} = 0.42 \text{ J (using } 1 \text{ Atm} \approx 10^5 \text{ N/m}^2)$$

- (c) • volume change for ideal gas

$$|V_2 - V_1| = nRT \left(\frac{1}{P_1} - \frac{1}{P_2} \right) = (0.0223)(8.31 \times 10^3)(300) \left(1 - \frac{1}{5} \right) \left(\frac{\text{Atm}}{10^5 \text{ N/m}^2} \right) = 0.44 \text{ m}^3$$

- volume change for copper $|V_2 - V_1| = V_1 - V_1 \exp[-K(P_2 - P_1)]$

$$K(P_2 - P_1) \ll 1, \exp[-K(P_2 - P_1)] \approx 1 - K(P_2 - P_1) \text{ and}$$

$$|V_2 - V_1| \approx V_1 K(P_2 - P_1) = 0.5(0.7 \times 10^{-6})(5 - 1) = 1.4 \times 10^{-6} \text{ m}^3$$

- (3) (a) since the volume is fixed and the temperature is rising, we expect the pressure must rise. To calculate this, we take advantage of the fact that V is an exact differential in T and P. So $dV = \partial V / \partial T|_P dT + \partial V / \partial P|_T dP = 0$ because of constant volume constraint. So we get

$$dP = \frac{-\partial V / \partial T|_P}{\partial V / \partial P|_T} dT = (\beta / \kappa) dT. \text{ And if } \beta \text{ and } \kappa \text{ are assumed constant with respect to}$$

temperature (not a bad approximation, particularly over a 100 K excursion in temperature around 300 K), then by integration $\Delta P = (\beta / \kappa) \Delta T$. So for the copper, we get (in MKSA units) $\Delta P = (55 \times 10^{-6} / 0.7 \times 10^{-11}) * 100 = 7.9 \times 10^8 \text{ N/m}^2 \approx 7.9 \times 10^3 \text{ Atm}$. For the Si we get $\Delta P = (8.4 \times 10^{-6} / 1.0 \times 10^{-11}) * 100 = 8.4 \times 10^7 \text{ N/m}^2 \approx 840 \text{ Atm}$

HW#1 Solutions

- (b) Because of the constant volume, no work is done and the change in energy of the cubes is all by heat, making the heat a total differential. This allows us to obtain the heat input by integrating the specific heat capacity C'_V or $\Delta Q = M C'_V \Delta T$ where M is the mass of each cube. The densities of Cu and Si are (from Kittel's material Table 1.4) $8.93 \times 10^3 \text{ kg/m}^3$ and $2.33 \times 10^3 \text{ kg/m}^3$, respectively. So we find $\Delta Q = 346 \text{ J}$ for Cu and $\Delta Q = 163 \text{ J}$ for Si.
- (c) To keep the hydrostatic pressure constant with the rising temperature we consider the exact differential $dP = \partial P/\partial T|_V dT + \partial P/\partial V|_T dV = 0 = (\beta/\kappa) dT + (-1/\kappa)dV$, so that $\Delta V = \beta \Delta T = V_0 \beta \Delta T = 5.5 \times 10^{-3} \text{ cm}^3$ for the Cu, and $8.4 \times 10^{-4} \text{ cm}^3$ for the Si.
- (4) (a) From basic circuit theory and electrostatics, the electrical potential energy of a capacitor is $U_E = (1/2)CV^2$ where $C = \epsilon_r \epsilon_0 A/d$, A is the area of the plates and d is their separation. Also from electrostatics we know for parallel plates the internal electric field is $E = V/d$. So the potential energy can be restated as $U_E = (1/2)\epsilon_r \epsilon_0 A d E^2 = (1/2)\epsilon_r \epsilon_0 V E^2$, and the energy density is $U_E' = U_E/V = (1/2)\epsilon_r \epsilon_0 E^2$, which has a max value of $4.4 \times 10^4 \text{ J/m}^3 = 0.044 \text{ J/cm}^3$.
- (b) In the notes we defined $(\chi_E)_{ij} = (\epsilon_0)^{-1} \partial \mathbf{P}_i / \partial \mathbf{E}_j$ which is generally a 2nd rank tensor. But we know that glass is an "LIH" (linear, isotropic, homogeneous) dielectric so this reduces to a scalar relationship, $\mathbf{P}_E = \epsilon_0 \chi_E \mathbf{E}$, where from electrostatics $\chi_E = \epsilon_r - 1$. So for glass, we get $(\mathbf{P}_E)_{\max} = \epsilon_0 (4-1) E_{\max} = 1.33 \times 10^{-3} \text{ Cb/m}^2 = 1.33 \times 10^{-7} \text{ Cb/cm}^2$.

- (5) (a) From electrostatics we know the potential energy of a neutral (i.e., equal + and - charge) dipole in an electric field is $U = -\mathbf{p} \cdot \mathbf{E}$ where \mathbf{p} is the dipole moment that, by convention, points between the - and + charges and has magnitude $q \cdot d$ where q is the dipole charge and d is the charge separation $= a/(2)^{1/2}$ according to the Figure below. So for the \mathbf{E} oriented as shown between the - and + charges, the potential energy of the four possible interstitial sites as enumerated is

$$U_1 = -\mathbf{p}_1 \cdot \mathbf{E} = 0 \text{ (since } \mathbf{p}_1 \text{ and } \mathbf{E} \text{ are perpendicular);} \quad U_2 = -\mathbf{p}_2 \cdot \mathbf{E} = -eaE/(2)^{1/2}$$

$$U_3 = -\mathbf{p}_3 \cdot \mathbf{E} = eaE/(2)^{1/2}; \quad U_4 = -\mathbf{p}_4 \cdot \mathbf{E} = 0 \text{ (since } \mathbf{p}_4 \text{ and } \mathbf{E} \text{ are perpendicular)}$$

By assuming M-B Statistics, we can thus write probability of occupancy of the i th dipole

$$\text{level } P_i = C e^{-U_i/k_B T} \quad \text{where } C = \left(\sum_{i=1}^4 e^{-U_i/k_B T} \right)^{-1} \text{ is the normalization constant.}$$

$$\text{Exponential factors: Dipole (1): } e^{-U_1/k_B T} = e^{-0} = 1; \quad \text{Dipole (2): } e^{-U_2/k_B T} = e^{-\left(\frac{eaE}{\sqrt{2}}\right)/k_B T}$$

$$\text{Dipole (3): } e^{-U_3/k_B T} = e^{-\left(\frac{eaE}{\sqrt{2}}\right)/k_B T}; \quad \text{Dipole (4): } e^{-U_4/k_B T} = e^{-0} = 1$$

$$C = \{2 + \exp[eaE/(2^{1/2}k_B T)] + \exp[-eaE/(2^{1/2}k_B T)]\}^{-1}$$

- (b) Substitution of $a = 3 \times 10^{-10} \text{ m}$, $E = 1 \times 10^8 \text{ V/m}$, and $T = 300 \text{ K}$ yields $eaE/(2^{1/2}k_B T) = 0.821$

so $C = (2 + 2.27 + 0.44)^{-1} = 0.2123$, and we get for the probabilities:

$$P_1 = 0.212 \quad P_2 = 0.482 \quad P_3 = 0.094 \quad P_4 = 0.212$$

{Note: $P_1 + P_2 + P_3 + P_4 = 1.0$ as expected for any probability calculation}