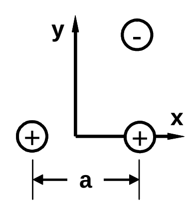
Homework 2

1. (Good exercise in continuous-state probability theory and statistical mechanics). In class we discussed the Maxwell-Boltzmann pdf, describing the probability of a particle having a velocity **v** when it is fully distinguishable from all other particles, has kinetic energy only, and this kinetic energy is given from Newtonian mechanics as $U_K = m|\mathbf{v}|^2/2$: $P(\mathbf{v})d\mathbf{v} = (m/2\pi k_B T)^{3/2} \exp(-mv^2/2k_B T)d\mathbf{v}$ where $v = |\mathbf{v}|$. [Note: this is very important in the development of kinetic theory and the more general Boltzman transport theory of charge carriers in semiconductors].
   (a) Derive the mean velocity $<v>$ using basic probability theory.
   (b) Derive the "most likely" velocity v_{max} using calculus [i.e., where $P(\mathbf{v})d\mathbf{v}$ reaches its max].
   (c) Derive the variance of the velocity from the definition $(\Delta\mathbf{v})^2 = <(\mathbf{v} - <\mathbf{v}>)^2>$, and from this calculate the rms deviation, $v_{rms} = [(\Delta\mathbf{v})^2]^{1/2}$
   (d) By integrating the $P(\mathbf{v})d\mathbf{v}$ over elevation and angle in spherical coordinates, find the function M(v)dv – the Maxwellian *distribution* of velocity (not a pdf anymore because we have integrated partially over the independent variable space). Now plot dM(v)/dv using your favorite graphical tool (e.g., Excel) for the fundamental particle of choice in electrical engineering – the electron – and use this plot to contrast the three characteristic velocities - the mean from (a), the most likely from (b), and the rms value from (c).

2. (Good exercise in discrete-state probability theory and statistical mechanics) One example given in class was for the Boltzman pdf applied to a subsystem consisting of a single particle having a "ladder" of energy states u_i with equal spacing. This is the basis for a much more important problem in statistical mechanics in which the Boltzmann subsystem is now a population of particles that can all occupy a "ladder" of single-particle states. This describes elegantly the statistical behavior of quantized lattice vibrations (i.e., phonons) in solids [not to mention quantized electromagnetic waves (i.e., photons) in free space].
   (a) Starting with the Boltzman pdf, derive the mean number of particles in each energy state assuming the ladder can be written as $u_i = i \cdot u_0$ where u_0 is a constant.
   (b) Now derive the variance or mean-square fluctuations, $<(\Delta n_i)^2>$. [clue: start with the Boltzman distribution and utilize the general result for random variables, $<(\Delta n_i)^2> = <(n - <n_i>)^2> = <(n_i)^2> - <n_i>^2$].

3. (Good exercise in covalent bonding). In class we stated that the ionized hydrogen molecule, H_2^+ is a good model for the electrostatic aspect of covalent bonding since it has only one electron which is not subject to Pauli exclusion. Referring to the figure at right, find the loci in two dimensions describing the possible position of the electron for which the net electrostatic force on each proton in the x direction is exactly zero. Comment on what you expect the relative forces on the protons to be when the electron is located at other positions away from these loci (clue: there are two such loci).



4. (A good exercise in matrix inversion) It is often easier to measure the compliance coefficients than the stiffness coefficients because of the relative ease of applying a known uniaxial pressure and measuring the displacement. Suppose we have a solid whose compliance matrix is given as follows [in units of 10^{-11} m^2/N]

$$\begin{matrix} 0.7664 & -0.2130 & -0.2130 & 0 & 0 & 0 \\ -0.2130 & 0.7664 & -0.2130 & 0 & 0 & 0 \\ -0.2130 & -0.2130 & 0.7664 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.2563 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.2563 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.2563 \end{matrix}$$

(a) What is the form of the stiffness matrix ?
(b) What are the stiffness coefficients (accurate to 3 decimal points) ?
(clue: this problem is very easy with a matrix math tool, such as MATLAB; if you don't have access to such a tool, just inspect the solution)

5. (Young's modulus and Poisson's ratio). An isotropic solid is subject to tension along the x axis (refer to Fig. 21 in Chap. 3 of Kittel)
   (a) Find expressions for the stiffness coefficients in terms of Young's modulus and Poisson's ratio.
   (b) (Optional) Now find the Lame coefficients λ_L and μ_L in terms of Young's modulus and Poisson's ratio

6. (Elasticity of Important Isotropic Solids). It is well known that purified aluminum – the metal of choice in silicon VLSI - has a Young's modulus of 73 GPa and a Poisson ratio of 0.33. Another very important solid in VLSI is amorphous SiO_2 (glass). Although the glasses vary between types slightly, good "ballpark" numbers are: Young's modulus = 85 GPa, Poisson ratio = 0.25. To first order, both of these solids can be assumed isotropic.
   (a) From these values, calculate the Lame coefficients λ_L and μ_L.
   (b) From the Lame coefficients, calculate the stiffness coefficients: C_{11}, C_{12}, and C_{44}.

7. (Speed of Sound in an Isotropic Solid)
   (a) Find a formula for the velocity of a longitudinal acoustic wave in an isotropic solid along the **x**, **y**, or **z** directions in terms of the stiffness and Lame coefficients.
   (b) Same as (a) but for a shear acoustic wave.
   (c) Now work out the formula for the velocity along the **x + y + z** direction referring to Kittel Chapter 3, Problem 9.
   (d) Evaluate the longitudinal and shear velocities in isotropic aluminum and glass using the data from 5 and 6 above.

8. (Speed of Sound in an Anisotropic Solid) A solid having *cubic* symmetry in an x, y, z cartesian system is subjected to tension along the **x** axis.
   (a) Find expressions in terms of the elastic stiffness coefficients for Young's modulus and Poisson's ratio as defined in Fig. 21 of Kittel, Chapter 3.
   (b) Evaluate Young's Modulus and Poisson's ratio for cubic Si and NaCl (assuming both to be *crystalline*) in MKS units using the stiffness coefficients in Kittel Chap. 3 Table 12.
   (c) Now evaluate the velocity of a longitudinal acoustic wave along the **x** direction and along the **x + y + z** directions for cubic Si and NaCl. Comment about any differences in contrast to 7(a) and (c) above.