Homework 2

1. Diamagnetic susceptibility of hydrogen atoms.

   The wave function of the hydrogen atom in its ground state (1s) is $\psi = \left(\pi a_0^3\right)^{-1/2} \exp\left(-r/a_0\right)$,

   where $a_0 = \hbar^2/me^2 = 0.529\times10^{-8} cm$. The charge density is $\rho\left(x,y,z\right) = -e\left|\psi\right|^2$, according to

   the statistical interpretation of the wave function. Show that for this state $<r^2>= 3a_o^2$, and calculate the magnetic susceptibility of a sample of atomic hydrogen containing 1 mole ($6.023\text{x}10^{23}$ atoms).

2. Heat capacity of magnetic systems.

(a) Consider a two-level system of unpaired spings, each having an energy splitting $k_B\Delta$ between upper and lower states. Show using Boltzmann statistics that the heat capacity per system is

$$C = \left(\frac{\partial U}{\partial T}\right)_\Delta = k_B \frac{\left(\Delta/T\right)^2 e^{\Delta/T}}{\left(1+e^{\Delta/T}\right)^2}.$$

(b) Plot the above heat capacity vs the parameter Δ/T using your favorite graphical tool. Then by graphical or numerical techniques, determine the value of Δ/T that produces the maximum C.

3. Langevin Model of Paramagnetism

Suppose a material is composed entirely of one molecular species having magnetic moment $\vec{m}_0$.

Assume that the magnetic dipolar potential energy is $U_P = -\vec{m}_0 \cdot \vec{B}_{local}$ and the magnetic moment is randomly oriented (with respect to θ and ϕ in spherical coordinates) in the absence of a B field.

(a) A magnetic induction $\vec{B}_{local}$ is applied along the z axis. If the dipole can be considered as a statistically-independent subsystem so that the Boltzmann pdf applies, calculate its mean value.

(b) If there is a density of n such dipoles in the solid, calculate the mean magnetization per unit volume and the approximate value of this at high temperatures. Also calculate the high-temperature limit of the (para)magnetic susceptibility.

4. Curie-Weiss-Heisenberg Model of Ferromagnetism

(a) Suppose the $\vec{B}_{local} = \vec{B}_{in} + \gamma\mu_0\vec{M}$, where according to the simple (Lorentz) theory $\gamma = 1/3$, but according to the Heisenberg γ can be much larger in (spin) ferromagnets, ~1000. Derive an (implicit) equation that relates B_{local} to n, m_0, T, and γ, assuming B_{local} and B_{in} are both pointed along the z axis.

(b) Now solve for the spontaneous magnetization, i.e., the value of M that creates a non-zero solution for B_{local} when B_{in} goes to zero. Plot the solution vs the parameter $\beta = m_0B_{in}/k_BT$ assuming n, γ , and m_0 stay constant. Make sure you go all the way to where M reaches zero, which defines the Curie temperature T_C

(c) Now suppose $m_0 = \mu_B$, the Bohr magneton. How big must γ be to achieve a ferromagnet with T_C = 300 K ?