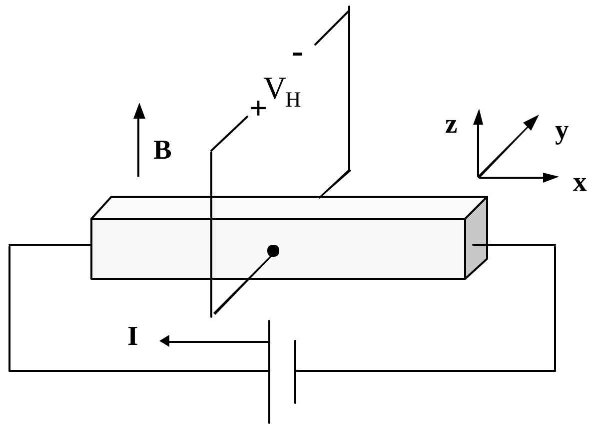
Homework #3 (Kinetic Theory)

1. (Hall effect): The semiconductor parallelapiped of dimensions L_x, L_y, and L_z (see figure below) has a carrier density ρ_c and is subject to a bias voltage V_x and current I_x along the x axis and a uniform magnetic field B_0 along the z axis. Under these conditions, a voltage V_y is induced along the y axis, although no current flows. Assuming uniform fields along the x and y axes and one carrier type, determine the following:

a) Starting with the kinetic equations of motion for a carrier of mass m_c and relaxation time τ, derive an expression for V_y in terms of V_x , B_0, τ, and m_c.

b) Re-derive an expression for V_y in terms of I_x , B_0, and ρ_c.

c) Suppose in an experiment we have $L_x = 1$ cm, $L_y = L_z = 1$ mm, $B_0 = 0.1$ T, $V_x = 1.0$ V and $V_y = +5$ mV, find the carrier charge polarity and mobility (MKSA units)



2. (Magnetoconductivity): a). Use the kinetic theory to show that in the presence of a magnetic field B, the static current density can be written in matrix form as

$$\begin{pmatrix} J_X \\ J_Y \\ J_Z \end{pmatrix} = \frac{s_0}{1+(w_C t)^2} \begin{pmatrix} 1 & -w_c t & 0 \\ w_c t & 1 & 0 \\ 0 & 0 & 1+(w_C t)^2 \end{pmatrix} \begin{pmatrix} E_X \\ E_Y \\ E_Z \end{pmatrix}$$

b). In the high magnetic field limit of $\omega_C\tau >> 1$, show that $\sigma_{XY} = ne/B = -\sigma_{XY}$ where σ_{XY} is called the Hall conductivity

3. (Joule heating):

Consider a metal at uniform temperature in a static electric field **E**. An electron experiences a collision, and then, after a time t, a second collision. In the Drude model energy is not conserved in collisions, since the mean speed of an electron emerging from a collision does not depend on the energy acquired from the field since the preceding collision.

(a) Show that the average energy lost to the ions in the second of two collisions separated by a time t is $(eEt)^2/2m$. (The average is over all directions in which the electron emerged from the first collision).

(b) Show that the average energy loss to the ions per collision is $(eE\tau)^2/2m$ using the fact that the probability of collision between t and t + dt = P(t,dt) = dt $e^{-t/\tau} / \tau$. Hence, show that the average loss per cubic centimeter per second is $(ne^2t/m)E^2 = sE^2$. Deduce that the power loss in a wire of length L and cross section A is I^2R, where I is the current flowing and R is the resistance of the wire.