Homework#4 (Transport Theory cont)

1. (Phonon thermal conductivity problem) In our coverage of lattice waves and phonons, we learned that the atomic lattice almost always contributes significantly to the heat capacity of solids. Consider a 3-dim lattice containing N atoms per primitive cell volume Vc. Assume that all three acoustic lattice waves have the same velocity of sound , v.

   a) Evaluate the high-temp specific heat capacity C' using the 3D Dulong-Petit expression, $3nk_B$ in MKS units assuming N = 2 and V_c = 45 Ang^3.

   b) Estimate the thermal conductivity at 300 K using the expression $\kappa = C'\upsilon\frac{X}{3}$ where v is the speed of sound, v = 7000 m/s, and X is the mean-free path (assume scattering time for phonons of τ = 1 ps).

2. (Electron thermal conductivity problem). Using kinetic theory we derived an expression for thermal conductivity, $\kappa = (C_V)v^2\tau/3$ where C_V is the specific heat capacity per unit volume, v is the mean particle velocity, and τ is the collision time. It is often true that this τ is very close to the collision time in the Drude expression for electrical conductivity, $\sigma = nq^2\tau/m$, where n is the electron density, q is the electron charge, and m is its mass.

(a) Using the known values of σ and n for pure aluminum at room temperature (find online or in almost any solid state book, such as Kittel), evaluate τ.

(b) Use the Sommerfeld-Fermi expression to evaluate the electronic heat capacity.

(c) Use (a) and (b) estimate the thermal conductivity for aluminum at 295 K.

Find an accepted value of the room temp thermal conductivity of Al from any credible source. Compare to the estimated value and comment on any discrepancy.

2. For Maxwell Boltzmann statistics evaluate the mean relaxation time assuming that $\tau(U) = AU^{-s}$, where U is the particle energy, and A and s are constants. The result should be expressed in terms of A, s, k_B, T, and the gamma function.

3. Show that if τ is independent of v, the distribution function $f(\vec{v}) = A\exp[-m(\vec{v}-\vec{v}_0)^2/(2k_BT)];\vec{v}_0 = (e\tau/m)\vec{E}_0$ is an approximate solution of the Boltzmann transport equation if $v_0 \ll (k_BT/m)^{1/2}$.

4. Show that if the mean-free-path L is independent of energy, then the electrical conductivity of a Maxwell-Boltzmann-distributed free-electron gas may be written $\sigma = (4/3)ne^2L/(2\pi mk_BT)^{1/2}$.

[clues for Problems 3 & 5: Gamma function and rules: $\Gamma(z) = \int_0^\infty e^{-x}x^{z-1}dx; \Gamma(z+1) = z\Gamma(z); \Gamma(0.5) = \sqrt{\pi}$]