Homework#5

Problem (1). Cyclotron resonance for spheroidal bands. Consider the energy surface

$$U(k) = \hbar^2\left(\frac{k_X^2 + k_Y^2}{2m_T} + \frac{k_z^2}{2m_L}\right)$$

where m_T and m_L are the transverse and longitudinal mass parameters, respectively. That is, the surface of constant U is a spheroid. Using the semiclassical equations of motion with $p = \hbar k$ and $v_g = (\hbar)^{-1}(\partial U_n / \partial k)$ and seeking an oscillatory solution with frequency ω for the crystal wave vector k of a given electron, show that $\omega = \omega_C = qB/(m_L m_T)^{1/2}$ when the static magnetic field lies in the x-y plane.

Problem (2). Semiclassical transport theory: Hall effect with two carrier types.
Assume we have a solid with two partially-occupied bands about band extrema at k = 0. The solid is contained in a parallelpiped sample with an electric field applied along the x axis and current density Jx along this axis, and the magnetic field B0 applied along the z axis with no current flowing along either the z or y axes. Suppose the higher-energy (conduction) band has $U_e = \hbar^2 k^2 / 2m_e$ and the lower (valence) band has $U_h = \hbar^2 k^2 / 2m_h$. Using the semiclassical equations with k-independent relaxation times τ_e and τ_h, show that the Hall coefficient (R_H) is given by

$$R_H = \frac{1}{q}\frac{p - nb^2}{(p + nb)^2}$$

where $b = \mu_e/\mu_h$. (Clues: neglect terms of order B^2; in the presence of the longitudinal bias field; find the transverse electric field by requiring the transverse current to vanish).

Problem 3: Electron-ionized-impurity scattering via the screened Coulomb potential

(a) Assume that a solid contains a density of singly-ionized impurities that are screened by the background free-electron concentration such that the electronic potential energy is given by

$$V(r) = \frac{-e}{4\pi\varepsilon_r\varepsilon_0 r}\exp(-r/L_D) \quad (3.1)$$

where L_D is the Debye screening length derived . Use (3.1) to derive the following expression for the differential scattering cross section:

$$\sigma(\theta) = \left[\frac{e^2/(8\pi\varepsilon_r\varepsilon_0 mv^2)}{\sin^2(\theta/2) + \beta^{-2}}\cdot\right]^2 \quad (3.2)$$

where θ is the scattering angle (polar angle of k_f relative to k_i), and $\beta = 2kL_D$

(b) Use (3.2) to derive the energy-dependent relaxation time:

$$\tau(U) = \frac{16\pi \cdot U^{3/2}(2m^*)^{1/2}(\varepsilon_r\varepsilon_0/e^2)^2}{N_I[\ln(1+\beta^2) - \beta^2/(1+\beta^2)]} \quad (3.3)$$

(c) Use (3.3) and the solution to the Boltmann transport (low concentration) to write an expression for <τ> , the ensemble-averaged relaxation time

Problem 4: Electron-acoustic-phonon scattering via the deformation potential
As stated in class, one of the most useful applications of elasticity theory in semiconductor physics (and devices) is the Bardeen-Shockley theorem that relates the change in potential energy for carriers near band-edges to the strain through the relation:

$$\delta U_P = \Xi \, \eta$$

where U_P is the potential energy, η is the strain associated with a long-wavelength acoustical phonon (or lattice wave), and Ξ is the deformation-potential constant - generally a known parameter for most important semiconductors and usually a surprisingly big number, ~ 10 eV.

(a) Use the Bardeen-Shockley theorem to derive the form of the transition Hamiltonian , $|H_{k',k}|$ where k and k' are the electron crystal wave vectors before and after the collision, respectively.

(clue: use plane wave representation for the electronic Bloch waves, and for the phonon).

(b) Use this $|H_{k',k}|$ formula along with the expression derived in class for the momentum relaxation time (via Fermi's Golden rule) to obtain the following expression for electron-phonon scattering (acoustic phonons only) assuming low (non-degenerate) electron concentration (so the Maxwell-Boltzmann distribution is valid):

$$\tau_L(U) = \frac{\hbar \rho v_s^2}{2\pi k_B T g(U) \Xi^2} \tag{4.1}$$

In this expression U is the electron kinetic energy, ρ is the solid density, v_s is the speed of sound, and g is the electronic specific density-of-states in the conduction band where U_C is the conduction band energy (at k = 0) and e is the strain. Assume for this problem that Ξ is independent of direction in the crystal.

(c) Use (4.1) and the solution to the semiclassical Boltzmann transport equation with uniform electric field applied to calculate the electronic mobility at 77 and 300 K assuming m = m* == 0.17 m_0, and Ξ = 10 eV, ρ = 5.3 gm/cm^3 and v_s = 3000 m/s.