ECE215B, Quiz#1 Solutions

1. (Electrostatics Problem):

(1) $E_{in} = E_o - N(P/\varepsilon_0) = E_o - P/\varepsilon_0$ (N = 1 for film or slab). But $P \equiv \chi_e \varepsilon_o E_{in}$ so $E_{in} = E_o - \chi_e E_{in} \Rightarrow E_{in}(1+\chi_e) = E_o$ or $E_{in} = E_o/(1+\chi_e) \equiv E_o/\varepsilon_r$, $\varepsilon_r \equiv 1+\chi_e$

(2) with metal plates, $E_{in} = V_B/d$, so the bias voltage required to get the same E_{in} as in (a) is simply $V_B = E_o \cdot d/\varepsilon_r$

2.(Ferroelectric Problem):

(a) $\mathbf{E_{loc}} = \mathbf{E_{in}} + \mathbf{P_e}/3\varepsilon_0 = \mathbf{E_{in}}(1+\chi_e/3)$

(b) $\mathbf{p} = \alpha\, \mathbf{E_{loc}} = \alpha\, \mathbf{E_{in}}(1+\chi_e/3)$; $\mathbf{P_e} = n\mathbf{p} = n\alpha(\mathbf{E_{in}} + \mathbf{P_e}/3\varepsilon_0)$, so that $\mathbf{P_e} = n\alpha\mathbf{E_{in}}/(1 - n\alpha/3\varepsilon_0)$

(c) The spontaneous polarization occurs when the denominator goes to zero, or $n\alpha/3\varepsilon_0 = 1$. For a body-centered cubic lattice, the volume occupied by each primitive cell is just $a^3/2$, so for a = 5 Ang, $n = 1.6\text{x}10^{28}$ m^{-3}, and $\alpha = 1.66\text{x}10^{-39}$ Cb-m^2/V

3. (Magnetic Problem):

(a) the maximum quantum number is $\gamma = \beta + s = 2 + ½ = 5/2$. The corresponding eigenvalue of J is $\sqrt{\gamma(\gamma+1)} \cdot \hbar = \sqrt{35/4} \cdot \hbar$. There are $2\gamma + 1 = 6$ possible eigenstates of J for this γ, denoted uniquely by their corresponding z components of **J**, m_J = +5/2, +3/2, +1/2, -1/2, -3/2 and -5/2.

(b) For any magnetic dipole $\vec{m} = -g\mu_B \vec{J}/\hbar$, and the corresponding potential energy is $U = -\vec{m}\cdot\vec{B}$. For our given atom, $-\vec{m}\cdot\vec{B} = g\mu_B \vec{J}\cdot\vec{B}/\hbar = g\mu_B m_J \cdot B$ where m_J = +5/2, +3/2, +1/2, -1/2, -3/2, and -5/2. So we have six energy levels $U_1 = (-5/2)g\mu_B B$, $U_2 = (-3/2)g\mu_B B$, $U_3 = (-1/2)g\mu_B B$, $U_4 = (1/2)g\mu_B B$, $U_5 = (3/2)g\mu_B B$, and $U_6 = (5/2)g\mu_B B$. We expect χ_m to be positive → paramagnetic, or perhaps ferromagnetic under special conditions

4.(Kinetic theory)

a) $\Delta z = v_z\, \tau$, where τ is the collision time, so $\kappa = (C'_V)(v_z)^2\cdot\tau$. But if all the carriers have the same magnitude of velocity v_0 and are randomly directed in space, then the statistical average is equivalent to a spatial average, and $\langle\kappa\rangle = (C'_V)\tau \langle(v_z)^2\rangle = (C'_V)\tau\, (1/3)(v_0)^2$

b) From kinetic theory, $\sigma = ne^2\tau/m$, where m is the mass of the electron, or $\tau = m\sigma/ne^2 = 2.8\text{x}10^{-14}$ s = 28 fs. By definition, $T_F \equiv U_F/k_B = 6.4\text{x}10^4$ K. Thus at 300 K, $C_V' = 1.88\text{x}10^4$ J-K^{-1}-m^{-3} , and $\kappa = (C'_V)\tau\, (1/3)(v_0)^2 \approx (C'_V)\tau\, (1/3)(v_F)^2 = 3.4\text{x}10^2$ W/m-K = 3.4 W/cm-K.