1. (Electrostatics Problem):

(1) \[ E_{in} = E_o - N \left( P / \varepsilon_0 \right) = E_o - P / \varepsilon_0 \] (N = 1 for film or slab). But \( P \equiv \chi_e \varepsilon_o E_{in} \) so

\[ E_{in} = E_o - \chi_e E_{in} \Rightarrow E_{in} (1 + \chi_e) = E_o \] or \( E_{in} = E_o / (1 + \chi_e) \equiv E_o / \varepsilon_r \), \( \varepsilon_r \equiv 1 + \chi_e \)

(2) with metal plates, \( E_{in} = \sqrt[3]{V_B/d} \), so the bias voltage required to get the same \( E_{in} \) as in (a) is simply \( V_B = E_o \cdot d / \varepsilon_r \)

2. (Ferroelectric Problem):

(a) \[ E_{loc} = E_{in} + P_e / 3 \varepsilon_0 = E_{in} (1 + \chi_e / 3) \]

(b) \[ p = \alpha E_{loc} = \alpha E_{in} (1 + \chi_e / 3) ; P_e = n \alpha (E_{in} + P_e / 3 \varepsilon_0) \]

(c) The spontaneous polarization occurs when the denominator goes to zero, or \( n \alpha / 3 \varepsilon_0 = 1 \). For a body-centered cubic lattice, the volume occupied by each primitive cell is just \( a^3 / 2 \), so for \( a = 5 \) Ang, \( n = 1.6 \times 10^{28} \) m\(^{-3} \), and \( \alpha = 1.66 \times 10^{-39} \) Cb-m\(^2\)/V.

3. (Magnetic Problem):

(a) the maximum quantum number is \( \gamma = \beta + s = 2 + \frac{1}{2} = 5/2 \). The corresponding eigenvalue of \( J \) is \( \sqrt{\gamma(\gamma + 1)} \cdot \hbar = \sqrt{35 / 4} \cdot \hbar \). There are \( 2 \gamma + 1 = 6 \) possible eigenstates of \( J \) for this \( \gamma \), denoted uniquely by their corresponding \( z \) components of \( J \), \( m_J = +5/2, +3/2, +1/2, -1/2, -3/2, -5/2 \).

(b) For any magnetic dipole \( \vec{m} = - g \mu_B \vec{J} / \hbar \), and the corresponding potential energy is

\[ U = - m \cdot \vec{B} \] For our given atom, \( - m \cdot \vec{B} = g \mu_B \vec{J} \cdot \vec{B} / \hbar = g \mu_B m_J \cdot \vec{B} \) where \( m_J = +5/2, +3/2, +1/2, -1/2, -3/2, -5/2 \). So we have six energy levels \( U_1 = (-5/2)g \mu_B B \), \( U_2 = (-3/2)g \mu_B B \), \( U_3 = (-1/2)g \mu_B B \), \( U_4 = (1/2)g \mu_B B \), \( U_5 = (3/2)g \mu_B B \), and \( U_6 = (5/2)g \mu_B B \). We expect \( \chi_m \) to be positive \( \rightarrow \) paramagnetic, or perhaps ferromagnetic under special conditions

4. (Kinetic theory)

a) \( \Delta z = v_z \tau \), where \( \tau \) is the collision time, so \( \kappa = (C' v) (v_z)^2 \cdot \tau \). But if all the carriers have the same magnitude of velocity \( v_0 \) and are randomly directed in space, then the statistical average is equivalent to a spatial average, and \( \kappa = (C' v) (v_z)^2 \cdot \tau = (C' v) (1/3) (v_0)^2 \)

b) From kinetic theory, \( \sigma = ne^2 / \tau m \), where \( m \) is the mass of the electron, or \( \tau = m \sigma / ne^2 = 2.8 \times 10^{-14} \) s = 28 fs. By definition, \( T_F \equiv v_F / k_B = 6.4 \times 10^4 \) K. Thus at 300 K, \( C' v = 1.88 \times 10^4 \) J-K\(^{-1}\)-m\(^{-3} \), and \( \kappa = (C' v) (1/3) (v_0)^2 \approx (C' v) (1/3) (v_F)^2 = 3.4 \times 10^2 \) W/m-K = 3.4 W/cm-K.