Quiz #2 Solutions, ECE215B, Spring 2008

 # 1: For semiclassical Boltzmann eqn, f is a distribution function of the wave-packet position $\vec{r}$, wave vector $\vec{k}$, and time, t, i.e., $f = f(\vec{r}, \vec{\mathrm{v}}, t)$ so that $\frac{df}{dt} = -\frac{\partial f}{\partial \vec{r}}\frac{d\vec{r}}{dt} - \frac{\partial f}{\partial \vec{k}}\frac{d\vec{k}}{dt}$.(1)

(b) At any point in time, $\int\int f(\vec{r}, \vec{k}, t) d\vec{r} d\vec{k} = N$, the number of particles in the solid. To return to equilibrium, we add a scattering term to (1): $(df/dt)|_{scatt} = -[f(\vec{r}, \vec{k}) - f_0]/\tau(\vec{k})$ where f_0 is the equilibrium distribution function and τ(k) is the relaxation time

c) If $\partial f/\partial \vec{r} = 0$ then in the steady state, we get $0 = -\frac{\partial f}{\partial \vec{k}}\frac{d\vec{k}}{dt} - \frac{f - f_0}{\tau}$ or $f = f_0 - \tau \frac{\partial f}{\partial \vec{k}}\frac{d\vec{k}}{dt}$. For a uniform E field along x axis and carriers in a spherical (constant-energy) valley in k space,

$(\partial f/\partial \vec{k})(\partial \vec{k}/\partial t) = (\partial f/\partial k_x)(\partial k_x/\partial t) = (\partial f/\partial U)(\partial U/\partial k_x)(\partial k_x/\partial t) =$

$(\partial f/\partial U)(\hbar^2 k_x/m^*)(qE_0/\hbar)$ so that $f \approx f_0 - (q\hbar k_x E_0 \tau/m^*)(\partial f/\partial U)$.

#2. (a) For any energy band $\vec{\mathrm{v}}_g = (\hbar)^{-1}\vec{\nabla} U(\vec{k}) \equiv \mathrm{v}_{gx}\hat{x} + \mathrm{v}_{gy}\hat{y} + \mathrm{v}_{gz}\hat{z}$ and $d\vec{r}/dt = \vec{\mathrm{v}}_g$, where $\vec{r}$ defines the location of the wave-packet center. For the specific energy band

$U = (\hbar^2/2)(\alpha k_x^2 + \alpha k_y^2 + \alpha k_z^2 + 2\beta k_x k_y + 2\beta k_x k_z + 2\beta k_y k_z)$, we get

$\vec{\mathrm{v}}_{gx} = \hbar\left(\alpha k_x + \beta(k_y + k_z)\right)$; $\vec{\mathrm{v}}_{gy} = \hbar\left(\alpha k_y + \beta(k_x + k_z)\right)$; $\vec{\mathrm{v}}_{gz} = \hbar\left(\alpha k_z + \beta(k_x + k_y)\right)$, and thus

. $dx/dt = \hbar\left(\alpha k_x + \beta(k_y + k_z)\right)$; $dy/dt = \hbar\left(\alpha k_y + \beta(k_x + k_z)\right)$; $dz/dt = \hbar\left(\alpha k_z + \beta(k_x + k_y)\right)$,

(b) The vector semiclassical equation of motion is $\hbar d\vec{k}/dt = q\left(\vec{E} + \hbar^{-1}\vec{\nabla}_k U \times \vec{B}\right) - \hbar\vec{k}/\tau$. For $\vec{E} = E_0\hat{x}$ and $\vec{B} = B_0\hat{z}$, the component equations become

$$x: \hbar dk_x/dt = q\left[E_0 + B_0[\alpha k_y + \beta(k_x + k_z)]\right] - \hbar k_x/\tau$$

$y: \hbar dk_y/dt = B_0[\alpha k_x + \beta(k_y + k_z)] - \hbar k_y/\tau$ and $z: \hbar dk_z/dt = -\hbar k_z/\tau$.

In the steady state, $dk_x/dt = 0$, so if $B_0 = 0$, the x component has obvious solution $k_x = qE_0\tau/\hbar$

(c) From (a), the motion of the wavepacket along x is $dx/dt = \hbar\left(\alpha k_x + \beta(k_y + k_z)\right)$. Assuming B = 0, the only nonzero ballistic terms from (b) are for k_x (ballistic means collisionless, so τ is set to infinity): $k_x = qE_0 t/\hbar + C$ where C is a constant that must be zero if particle starts at rest (rest means zero velocity and hence $k_x = 0$) . Substitution yields $dx/dt = \hbar\alpha(qE_0 t/\hbar)$ or $x = \hbar\alpha(qE_0 t^2/2\hbar) + D$, where D is a second constant. The distance traveled in real space between t = 0 and t = τ is just Δx = x(t = τ) – x(t = 0) = $\hbar\alpha(qE_0\tau^2/2\hbar)$

#3. (a) for $\tau(U) = \frac{16\pi\sqrt{2m^*}}{CN_I}\left(\frac{\varepsilon_r\varepsilon_0}{q^2}\right)^2 = AU^{-S}$, S = -3/2, $A = \frac{16\pi\sqrt{2m^*}}{CN_I}\left(\frac{\varepsilon_r\varepsilon_0}{q^2}\right)$ and Γ(5/2-S) = Γ(4) = 3!=6. So, $<\tau> = \frac{4\cdot 16\pi\sqrt{2m^*}}{CN_I}\left(\frac{\varepsilon_r\varepsilon_0}{q^2}\right)\frac{6(k_B T)^{3/2}}{3\sqrt{\pi}} = \frac{128\sqrt{2\pi m^*}}{CN_I}\left(\frac{\varepsilon_r\varepsilon_0}{q^2}\right)^2 (k_B T)^{3/2}$

(b) For the given parameters, <τ> = 3.45×10^{-12} and <μ> =e<τ>/m* = 9.05 m^2/V-s @ 77 K.
(c) By inspection, <u> goes as $(m^*)^{-1/2}$, so the reduction in m* will increase the mobility by a factor of 1.5. Physically, this reflects the fact that the decrease in m* increases the thermal velocity and therefore the kinetic energy, making the charged impurity scattering less effective.