1.9 (15 points)
T = 125 ° C
5kT = 173 meV
n = p = 4*10^{18} cm^{-3}

As given in the appendix 2, expression (A2.7), the carrier density for a quantum well material is given by

$$N = \frac{k \cdot T \cdot m^*}{\pi \cdot \hbar^2 \cdot d} \sum_{n_x} \ln\left(1 + e^{\frac{E_F - E_{nx}}{k \cdot T}}\right)$$

Based on the given data, we can solve for

$$\sum_{n_x} \ln\left(1 + e^{\frac{E_F - E_{nx}}{k \cdot T}}\right) = \frac{\pi \cdot \hbar^2 \cdot d}{k \cdot T \cdot m^*} n = \frac{\pi \cdot (6.59 \times 10^{-16} eVs)^2 (80 \times 10^{-8} cm)(4 \times 10^{18} cm^{-3})}{0.034(0.067) \cdot \frac{0.511 \times 10^6 eV}{3 \times 10^{10} cm/s}} = 3.38$$

Now, we need to solve for E_F, and the quantum well energies E_{ni}, such that the left side of the equation is equal to the right side. This will be an iterative process, in which we choose a value for V_0, then find the energies of all possible quantum states, and use the relation between V_0 and E_F, $V_0 = 5kT + E_F$.

Solve for the quantum well energy levels, E_i based on problem 1.3(b). Note that this calculation is independent on the value of V_0, that is, V_0 will only determine the number of energy states in the well. After performing several iterations, we conclude that there will be only two states in the well, and for V_0= 323.6 meV, we get

$$E_1^\infty = 87.68 \text{ meV} \rightarrow n_{max} = 1.92 \rightarrow n_{qw1} = 0.75, \quad n_{qw2} = 1.45$$

E_1 = 49.32 meV
E_2 = 184.47 meV
E_F = 150.6 meV
Plugging in these values, we get

$$\sum_{n_x} \ln\left(1 + e^{\frac{E_F - E_{nx}}{k \cdot T}}\right) = \frac{\pi \cdot \hbar^2 \cdot d}{k \cdot T \cdot m^*} n = 3.38$$

Assume that 2/3 of the bandgap discontinuity occurs in the conduction band,

$$V_0 = \Delta E_C = \frac{2}{3} \Delta E_g \Rightarrow \Delta E_g = 485.4 \text{ meV}$$

We will assume that the bandgaps change by the same amount when heated from 300K to 400K. Then we can use the bandgap values for 300K. Use linear extrapolation between GaAs and $Al_{0.2}Ga_{0.8}As$:

$$\Delta E_g = 0.485 \text{eV} = \frac{x}{0.2}(1.673 \text{ eV} - 1.424 \text{ eV}) \rightarrow x = 0.39 \text{ or } 39\% \text{ Al}$$

1.14:

See Table 1.1 for the indicies of refraction for the materials in the waveguide:

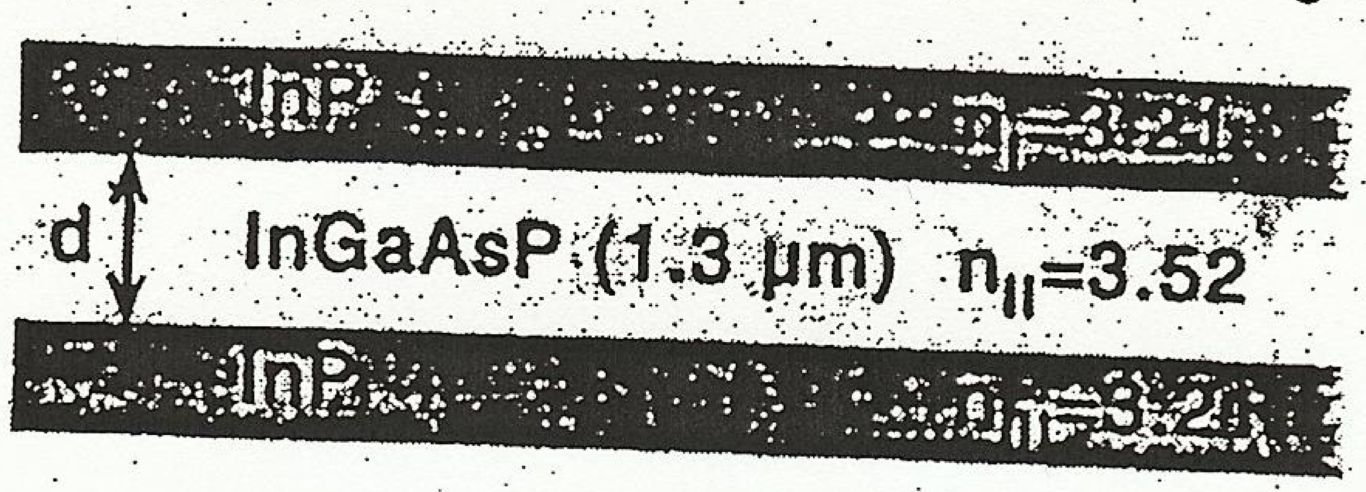

Figure 1.14. InGaAsP waveguide with $d = 0.2\mu m$

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a) Using normalized parameters from Appendix 3, we have

$$\begin{aligned} V &= k_0 d \sqrt{n_{II}^2 - n_{III}^2} \\ &= \frac{2\pi}{1.3\mu m} 0.2\mu m \sqrt{3.52^2 - 3.21^2} \\ &= 1.40 \end{aligned} \qquad (A3.12)$$

Since $V < \pi$, the guide is single-mode (see Figure A3.2).

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b) First, find $\bar{n}$:

Method 1:

Again using normalized parameters, we have for a symmetric guide:

$$a = 0$$

Using Fig. A3.2, we determine

$$b \approx 1 - \frac{\ln(1+V^2/2)}{V^2/2} = 0.303$$

Using b, we can find $\bar{n}$:

$$b = \frac{\bar{n}^2 - n_{III}^2}{n_{II}^2 - n_{III}^2} \qquad (A3.12)$$

$$\bar{n} = \sqrt{0.37(3.52^2 - 3.21^2) + 3.21^2} = 3.307$$

$\beta = \frac{2\pi\bar{n}}{\lambda}$. k_x and γ_x can then be found by Eq. A3.7.

Method 2 (more accurate, but more time consuming):

$\bar{n}$ can also be found by solving the characteristic equation.

$$k_x \tan\left(\frac{k_x d}{2}\right) = \gamma \qquad (A3.9)$$

$$\sqrt{k_0^2 n_{II}^2 - \beta^2} \tan\left(\frac{d}{2}\sqrt{k_0^2 n_{II}^2 - \beta^2}\right) = \sqrt{\beta^2 - k_0 n_{III}^2}$$

$$\sqrt{\left(\frac{2\pi}{1.3\mu m}\right)^2 3.52^2 - \beta^2} \tan\left(\frac{0.2\mu m}{2}\sqrt{\left(\frac{2\pi}{1.3\mu m}\right)^2 3.52^2 - \beta^2}\right) = \sqrt{\beta^2 - \left(\frac{2\pi}{1.3\mu m}\right) 3.21^2}$$

Solve iteratively to find

$$\beta = 15.98\mu\text{m}^{-1} \qquad (\rightarrow \bar{n} = 3.306)$$

$$k_x = 5.84\mu\text{m}^{-1}$$

$$\gamma = 3.83\mu\text{m}^{-1}$$

The electric field of the mode is then given by

$$U_{II} = A\cos(k_x x)$$

$$U_{I,III} = B\ e^{-\gamma|x|}$$

Use boundary conditions to solve for the coefficients A and B. The electric field must be continuous to satisfy Maxwell's equations. Therefore,

$$U_{II}(x = 0.1) = U_{III}(x = 0.1)$$

$$A\cos((5.84)(0.1)) = B\ e^{-(3.83)(0.1)}$$

$$B = 1.22A$$

So

$$U_{II} = A\cos((5.84\mu\text{m}^{-1})x)$$

$$U_{I,III} = 1.22A\ e^{-(3.83\mu\text{m}^{-1})|x|}$$

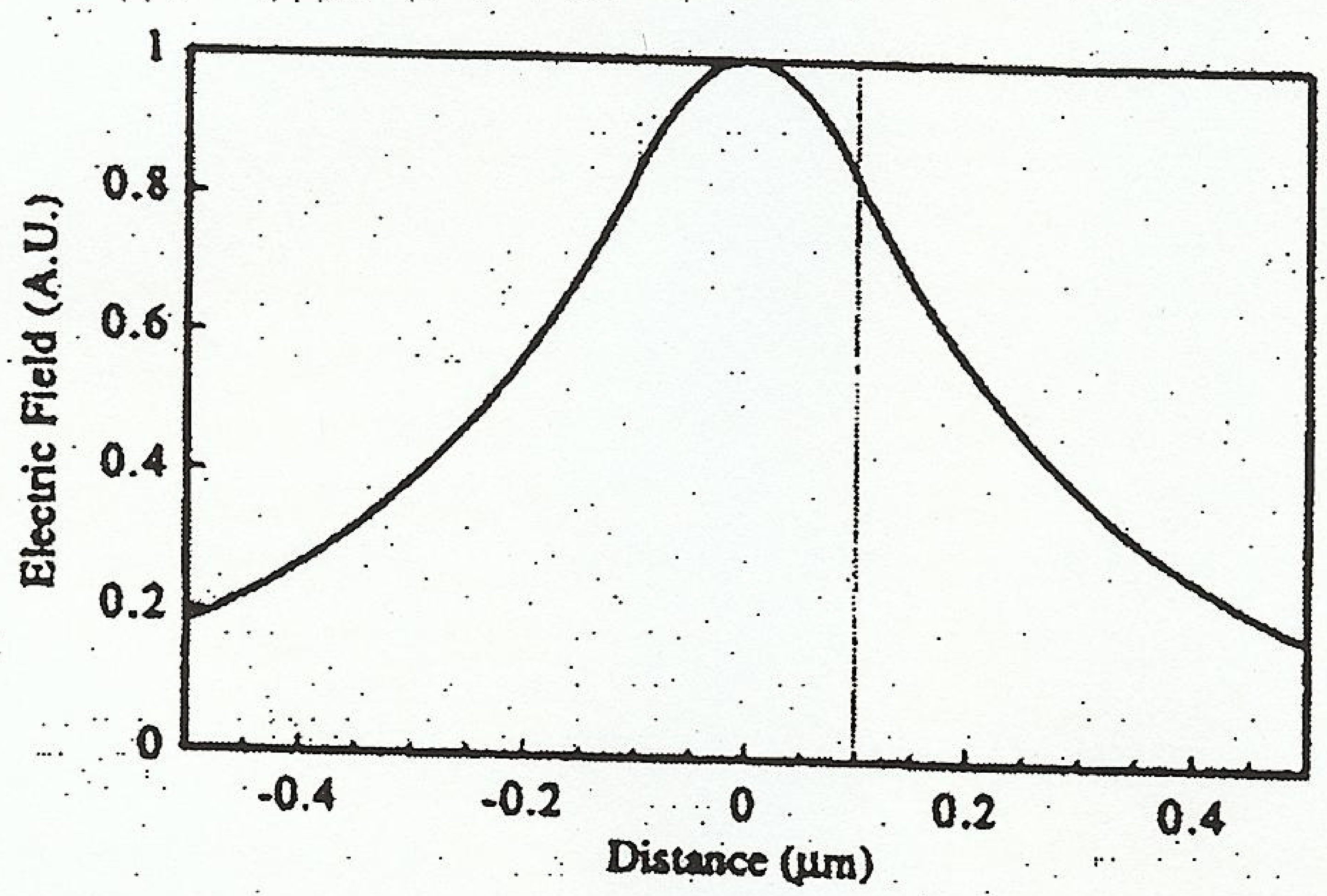

Figure 1.14b. Waveguide electric field profile

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c)

$$E^2(x = 0) = (A\ \ \cos(0))^2 = A^2$$

$$E^2(x = 0.6\mu\text{m}) = (1.22Ae^{-(3.83)(0.6)})^2 = 0.015\,A^2$$

$$\frac{E^2(x = 0.6\mu\text{m})}{E^2(x = 0)} = 1.5\%$$

Note that energy density is really $\frac{\epsilon}{2}E^2$. We ignored the variation in index ($n = \sqrt{\epsilon}$) across the waveguide.

$$\Rightarrow \text{ratio} = 0.015\ \frac{\varepsilon_{core}}{\varepsilon_{cladding}} = 1.8\%$$

d)

$$\beta = \frac{2\pi\bar{n}}{\lambda}$$

$$n_{eff} = \bar{n} = \frac{\lambda\beta}{2\pi} = \frac{(1.3\mu\text{m})(15.98\mu\text{m}^{-1})}{2\pi} = 3.31$$

e) Using equation A3.14 or A3.15, we can solve for the transverse confinement factor:

$$\begin{aligned}\Gamma &= \frac{1+\frac{2\gamma d}{V^2}}{1+\frac{2}{\gamma d}} \\ &= \frac{1+\frac{2(3.83)(0.2)}{1.40^2}}{1+\frac{2}{(3.83)(0.2)}} \\ &= 49.3\%\end{aligned}$$

$$\begin{aligned}\Gamma &\approx \frac{V^2}{2+V^2} \\ &\approx \frac{(1.40)^2}{2+(1.40)^2} \\ &\approx 49.5\%\end{aligned}$$

$$\Gamma = 49.3\%$$

1.16:

$$U(x,y) = U\sin(k_x x)\ \sin(k_y y) \qquad (A3.17)$$

Since the mode is strongly confined, we can approximate the boundary conditions by letting the electric field be zero at the edge of the pillar.

$$U(x,y) = 0 \quad x > 5\mu\text{m or } y > 5\mu\text{m}$$

$k_x = \frac{m\pi}{d}$ where $m = 1, 2, 3, \ldots$

$k_y = \frac{n\pi}{d}$ where $n = 1, 2, 3, \ldots$

$k_z = \beta = constant$

$$k_{mn}^2 = k_x^2 + k_y^2 + \beta^2$$

For the fundamental mode, $m = n = 1$.

$$\lambda_{11} = \frac{2\pi}{k_{11}} = 1.00\mu\text{m}$$

$$\left(\frac{2\pi(3.30)}{1.0\mu\text{m}}\right)^2 = \left(\frac{\pi}{5\mu\text{m}}\right)^2 + \left(\frac{\pi}{5\mu\text{m}}\right)^2 + \beta^2$$

Solve for β^2 to find

$$\beta^2 = 429.13\mu\text{m}^{-2}$$

Now, the mode spectrum can be written analytically as

$$\lambda_{mn} = \frac{2\pi(3.30)}{\sqrt{429.13\mu\text{m}^{-2} + n^2\left(\frac{\pi}{5\mu\text{m}}\right)^2 + m^2\left(\frac{\pi}{5\mu\text{m}}\right)^2}}$$

The first six modes are

$$\lambda_{11} = 1.00\mu\text{m}$$
$$\lambda_{21} = \lambda_{12} = 0.9986\mu\text{m}$$
$$\lambda_{22} = 0.9972\mu\text{m}$$
$$\lambda_{13} = \lambda_{31} = 0.9963\mu\text{m}$$

other modes have shorter wavelengths:

$$\lambda_{23} = \lambda_{32} = 0.9950\mu\text{m}$$
$$\lambda_{33} = 0.9927\mu\text{m}$$

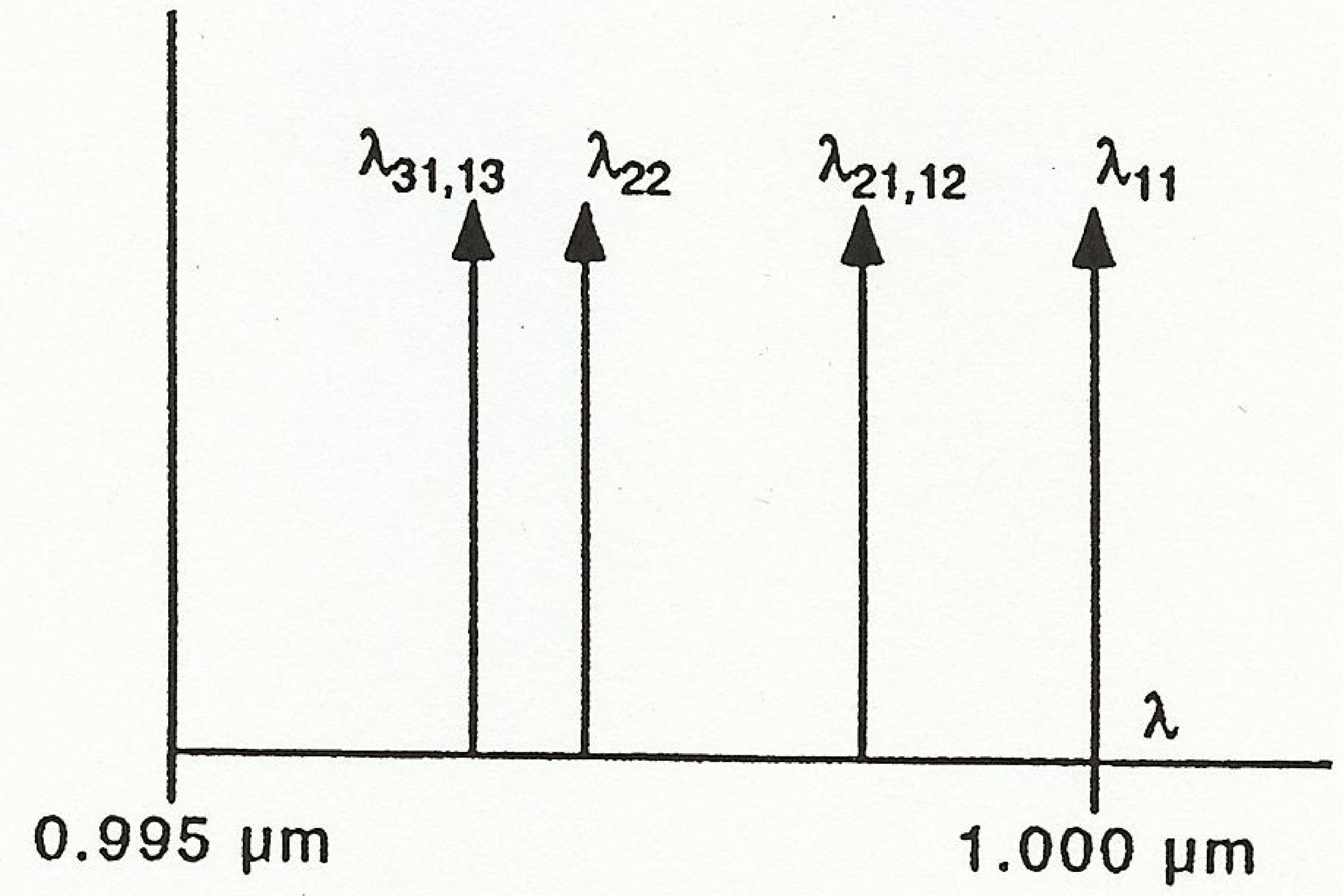

Figure 1.16. Mode spectrum including first six lateral modes.

1.18:

Use the infinite barrier approximation for the lateral direction:

$$E_1^\infty = 3.76 \frac{m_0}{m_e^*} \left(\frac{100\text{Å}}{d} \right)^2 \text{ meV} \tag{A1.14}$$

From Table 1.1 for GaAs, $m_e^* = 0.067 m_0$.

$$\Delta E_1^\infty = 10 \text{ meV} = 3.76 \frac{1}{0.067} \left(\frac{100\text{Å}}{d} \right)^2 \text{ meV}$$

Solve for d:

$$d = \sqrt{\frac{3.76}{(10)(0.067)}} \, (100\text{Å}) = 237\text{Å}$$