2.8:

$$\omega_R = \sqrt{\frac{av_g\Gamma\eta_i(I - I_{th})}{qV}} = \sqrt{\frac{av_g\Gamma\eta_i(J - J_{th})}{qt}} \tag{2.56}$$

From the previous problem:

$$v_g = \frac{c}{\bar{n}_g} = 6.67 \times 10^9 \text{ cm/s}$$

$$\eta_i = 1$$

$$J_{th} = 1 \text{ kA/cm}^2$$

$$N_{th} = 2.5 \times 10^{18} \text{ cm}^{-3}$$

$$t = 0.1 \times 10^{-4} \text{ cm}$$

Assume that the gain can be approximated using the differential gain, a:

$$g \approx a(N - N_{tr}) \tag{2.17}$$

Below threshold:

$$\frac{\eta_i I}{qV} = \frac{\eta_i J}{qt} \approx BN^2$$

$$N_{tr} = \sqrt{\frac{\eta_i J_{tr}}{qtB}} = \sqrt{\frac{(1)\ (0.5 \times 10^3 \text{ A/cm}^2)}{(1.6 \times 10^{-19} \text{ C})\ (0.1 \times 10^{-4} \text{ cm})\ (10^{-10} \text{ cm}^3\text{/s})}} = 1.77 \times 10^{18} \text{ cm}^{-3}$$

$$g_{th} = \frac{<\alpha_i> + \alpha_m}{\Gamma} = \frac{10 \text{ cm}^{-1} + 38 \text{ cm}^{-1}}{0.15} = 320 \text{ cm}^{-1}$$

$$a = \frac{\Delta g}{\Delta N} = \frac{320 \text{ cm}^{-1}}{(2.5 \times 10^{18} \text{ cm}^{-3}) - (1.77 \times 10^{18} \text{ cm}^{-3})} = 4.38 \times 10^{-16} \text{ cm}^2$$

$$\omega_R = \sqrt{\frac{(4.3 \times 10^{-16} \text{ cm}^2)(6.67 \times 10^9 \text{ cm/s})(0.15)(1)(10^3 \text{ A/cm}^2)}{(1.6 \times 10^{-19} \text{ C})(0.1 \times 10^{-4} \text{ cm})}} = 1.56 \times 10^{10}\ rad/s$$

$$f_R = \frac{\omega}{2\pi} = 2.6 \text{ GHz}$$

2.9:

$$R \approx 0.32$$ (p.39)

$L_A = 200\mu\text{m}$ | $L_B = 400\mu\text{m}$

$J_{th_A} = 3\text{ kA/ cm}^2$ | $J_{th_B} = 2\text{ kA/ cm}^2$

$\eta_{dA} = 60\%$ | $\eta_{dB} = 50\%$

$\alpha_{mA} = \frac{1}{L_A}\ln\frac{1}{R} = 57.0\text{ cm}^{-1}$ | $\alpha_{mB} = \frac{1}{L_B}\ln\frac{1}{R} = 28.5\text{ cm}^{-1}$

---

a) Assume that η_i is the same for both lasers.

$$\eta_d = \eta_i \frac{\alpha_m}{<\alpha_i> + \alpha_m}$$

Solve for $<\alpha_i>$:

$$<\alpha_i> = \left(\frac{\eta_i}{\eta_d} - 1\right)\alpha_m$$

Assume that $<\alpha_{iA}> = <\alpha_{iB}>$ and $\eta_{iA} = \eta_{iB}$. Then

$$\left(\frac{\eta_i}{\eta_{dA}} - 1\right)\alpha_{mA} = \left(\frac{\eta_i}{\eta_{dB}} - 1\right)\alpha_{mB}$$

Solve for η_i:

$$\eta_i = \frac{\alpha_{mA} - \alpha_{mB}}{\frac{\alpha_{mA}}{\eta_{dA}} - \frac{\alpha_{mB}}{\eta_{dB}}} = 0.75 = 75\%$$

$$\alpha_i = \left(\frac{\eta_i}{\eta_{dA}} - 1\right)\alpha_{mA}$$

$$\alpha_i = \left(\frac{0.75}{0.6} - 1\right)(62.6\text{ cm}^{-1}) = 14.25\text{ cm}^{-1}$$

---

b) Using the formulae derived in part (a), calculate η_i and $<\alpha_i>$ for each of the four cases that are found by varying η_{dA} and η_{dB} by $\pm 1\%$:

$$\eta_i = \frac{\alpha_{mA} - \alpha_{mB}}{\frac{\alpha_{mA}}{\eta_{dA}} - \frac{\alpha_{mB}}{\eta_{dB}}}$$

$$\alpha_i = \left(\frac{\eta_i}{\eta_{dA}} - 1\right)\alpha_{mA}$$

	case 1	case 2	case 3	case 4
η_{dA}	60.6	60.6	59.4	59.4
η_{dB}	50.5	49.5	50.5	49.5
η_i	0.7575	**0.7812**	0.7211	0.7425
$<\alpha_i>$	14.25	**16.48**	12.19	14.25

$$\Delta\eta_i = \frac{.7812 - .7500}{.7500} = 4.2\%$$

$$\Delta\alpha_i = \frac{16.48 - 14.25}{14.25} = 16\% \quad (= 2.2\text{ cm}^{-1})$$

2.13:

a) Thermal impedance:

$$Z_T = \frac{1}{2\epsilon d} \quad \text{for disk} \tag{2.66}$$

Set the areas of the disk and the square pillars equal to define an equivalent diameter for use in the thermal impedance equation:

$$\pi\left(\frac{d}{2}\right)^2 = s^2$$

$$d = \frac{2}{\sqrt{\pi}}\, s$$

$$Z_T = \frac{\sqrt{\pi}}{4\epsilon s} = \frac{\sqrt{\pi}}{4\,(4.5\times10^{-5}\ \text{W}/\mu\text{m}\ ^\circ\text{C})\ (s)} = \frac{10^4\mu\text{m}}{s}\ \frac{^\circ\text{C}}{\text{W}}$$

Power:

$$P_{in} = I^2R_s + IV_D + IV_s \tag{2.61}$$

$$I = 2I_{th} = 2J_{th}A = 2(1\ \text{kA/cm}^2)\ s^2 = (2\times10^{-5}\ \text{A}/\mu\text{m}^2)\ s^2$$

$R_s = \frac{20\ \text{k}\Omega\mu\text{m}^2}{s^2}$

$V_D = 1.5$ V (typical value)

$V_s \doteq 1$ V

$$P_{in} = (5.8\times10^{-5})\ s^2\ \text{W}$$

$$P_0 = \eta_d\,\frac{h\nu}{q}\,(I - I_{th})$$

For $I = 2\ I_{th}$,

$$P_0 = \eta_d\,\frac{h\nu}{q}\,I_{th} = (0.5)(1.424\ \text{J/C})(10^3\ \text{A/cm}^2)(10^{-8}\ \text{cm}^2/\mu\text{m}^2)\ s^2 = (7.12\times10^{-6}\ \text{W})\left(\frac{s^2}{\mu\text{m}^2}\right)$$

$$P_D = P_{in} - P_0 = 5.1\times10^{-5}\ s^2\ \text{W}$$

$$\Delta T = P_D Z_T = (5.1\times10^{-5}\ s^2\ \text{W})\left(\frac{10^4}{s}\ \frac{^\circ\text{C}}{\text{W}}\right) = 0.51\ s\ ^\circ\text{C}$$

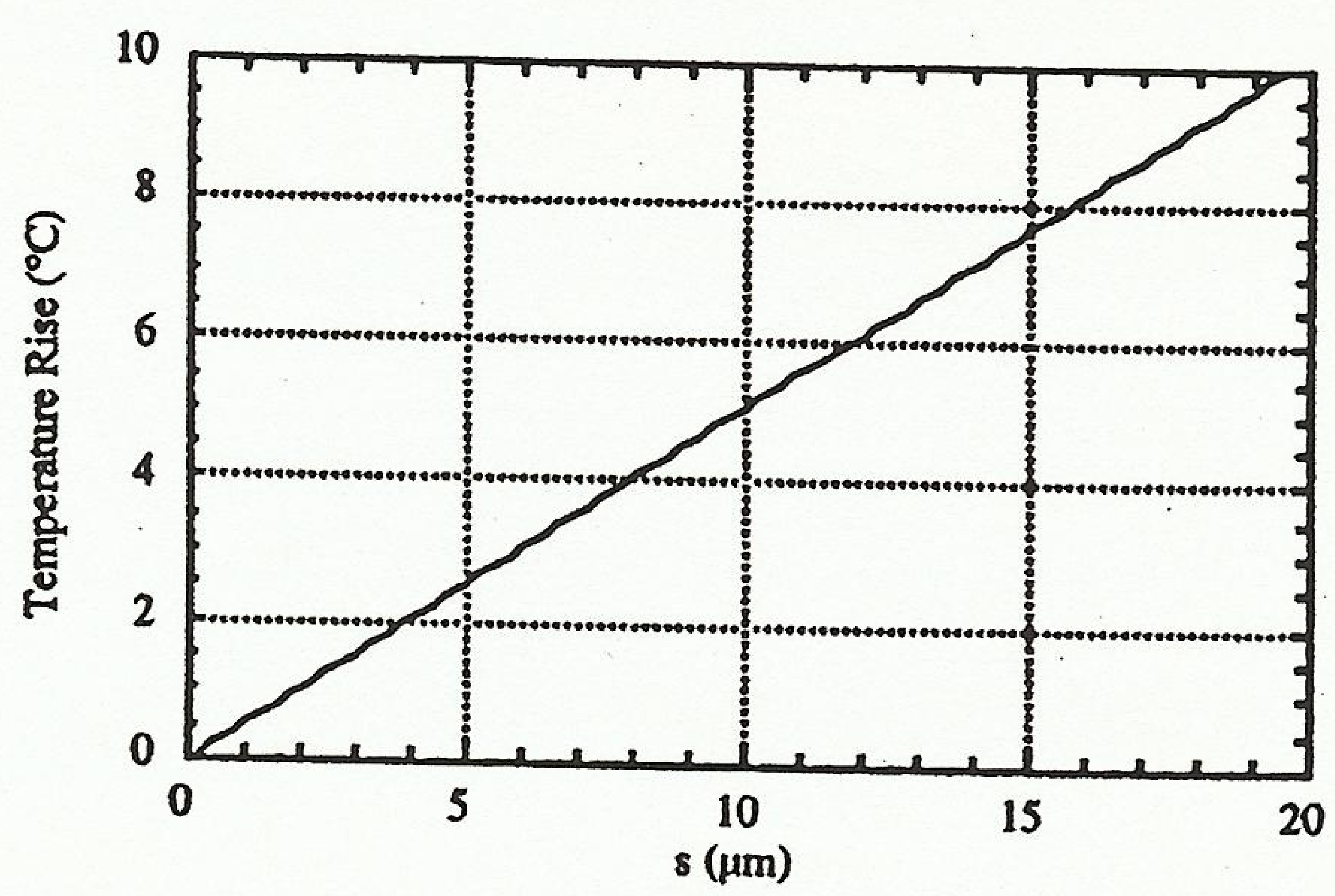

Figure 2.13a. Rise in active region temperature at $2I_{th}$ vs. VCSEL pillar width

b)

$$P_0 = (7.12 \times 10^{-6}\ \text{W}) \left(\frac{s^2}{\mu\text{m}^2}\right)$$

$$I = 2\ I_{th} = (2 \times 10^{-5}\ \text{A}) \left(\frac{s^2}{\mu\text{m}^2}\right)$$

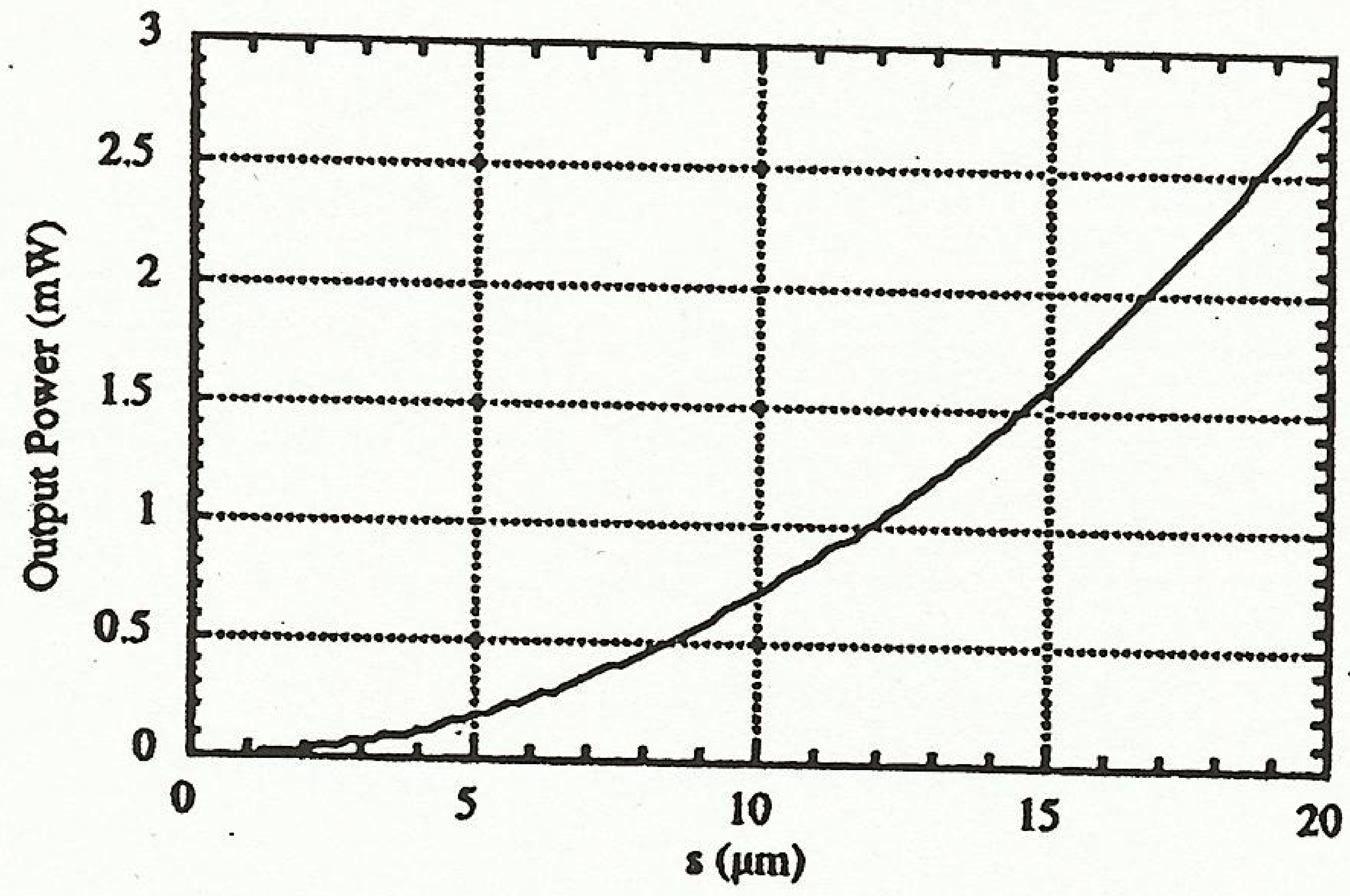

Figure 2.13b. Output power at $2I_{th}$ vs. VCSEL pillar width

2.14:

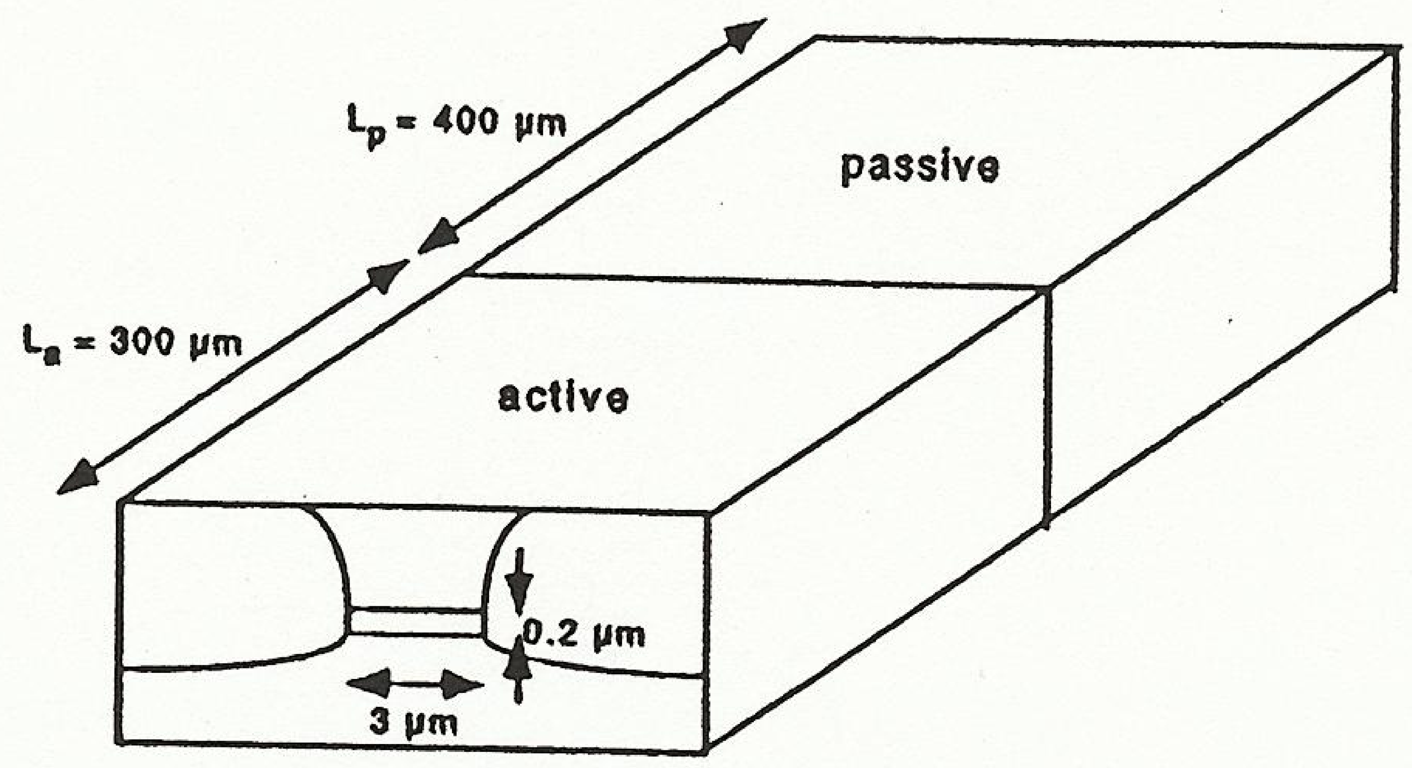

Figure 2.14. Laser Dimensions

$\lambda = 1.55\mu$m for the InGaAsP/InP in-plane laser shown in the figure above.

$\eta_i = 0.70$

$\Gamma_T = 0.20$ and $\Gamma_L = 0.80$

$\alpha_a = 80$ cm^{-1}

$\alpha_p = 20$ cm^{-1}

$\alpha_c = 5$ cm^{-1}

$g = a(N - N_{tr})$, where $a = 5 \times 10^{-16}$ cm^2 and $N_{tr} = 2 \times 10^{18}$ cm^{-3}.

$\Delta\lambda_{sp} = 100$ nm

$$\frac{dN}{dt} = -\frac{N}{\tau} = -BN^2 - C^3 \tag{2.9}$$

Since the recombination rate due to spontaneous emission is equal to the recombination rate due Auger recombination at transparency,

$$BN_{tr}^2 = CN_{tr}^3$$
$$B = CN_{tr}$$

Use a typical value for B as given on p. 31: $B = 10^{-10}$ cm^{-3}/s. Then

$$C = \frac{10^{-10}\text{ cm}^{-3}/\text{s}}{2 \times 10^{18}\text{ cm}^{-3}} = 5 \times 10^{-29}\text{ cm}^6/\text{s}$$

Determine Losses:

$$\alpha_m = \frac{1}{L}\ln\frac{1}{R} = \frac{1}{L_a + L_p}\ln\frac{1}{R} = \frac{1}{700\mu\text{m}}\ln\frac{1}{0.32} = 16.3\text{ cm}^{-1}$$

$$<\alpha_i> = \alpha_{active\ guide} + \alpha_{passive\ guide} + \alpha_{cladding}$$

$$<\alpha_i> = \frac{L_a}{L_a + L_p}(\Gamma_T\Gamma_L\alpha_a) + \frac{L_p}{L_a + L_p}(\Gamma_T\Gamma_L\alpha_p) + (1 - \Gamma_T\Gamma_L)\alpha_c$$

$$<\alpha_i> = \frac{3}{7}(0.16)(80\text{ cm}^{-1}) + \frac{4}{7}(0.16)(20\text{ cm}^{-1}) + (0.84)(5\text{ cm}^{-1}) = 11.5\text{ cm}^{-1}$$

a) P-I Characteristics Below Threshold:

$$P = \eta_r \eta_i \frac{\alpha_m}{<\alpha_i> + \alpha_m} \frac{h\nu}{q} \beta_{sp} I \tag{2.39}$$

$\frac{h\nu}{q} = \frac{1.24\ \text{eV}\mu\text{m}}{1.55\mu\text{m}} = 0.80\frac{\text{J}}{\text{C}}$

$$\begin{aligned} \beta_{sp} &= \frac{\Gamma_z\ \Gamma_{xy}\ \lambda^4}{8\pi\ (d\ w\ L_a)\ n^2\ \bar{n}\ \Delta\lambda_{sp}} \\ &= \frac{(3/7)(0.2)(0.8)(1.55\mu\text{m})^4}{8\pi(0.2\mu\text{m})(3\mu\text{m})(300\mu\text{m})(3.55)^2\ (4)(100 \times 10^{-3})} \\ &= 1.73 \times 10^{-5} \end{aligned} \tag{A4.10}$$

$$P_0 = \eta_r\ (0.70) \left(\frac{16.3}{16.3 + 11.5}\right)\ (0.8\ \text{J/ C})\ (1.73 \times 10^{-5})\ I$$

See note at the end of this problem about Γ_c vs. Γ.

Thus, the power out of both facets below threshold is

$$P = (5.70 \times 10^{-6}\ \text{J/C})\ \eta_r I$$

Find Threshold Current:

$$I_{th} = \frac{qV}{\eta_i} \frac{N_{th}}{\tau} \tag{2.30}$$

The condition for threshold is

$$\Gamma g_{th} = <\alpha_i> + \alpha_m \tag{2.24}$$

$\Gamma = \Gamma_z \Gamma_x \Gamma_y = (3/7)(0.8)(0.2) = 0.069$

$$g_{th} = a(N_{th} - N_{tr}) \tag{2.17}$$

Combine these two equations and solve for N_{th}:

$$N_{th} = N_{tr} + \frac{<\alpha_i> + \alpha_m}{\Gamma\ a} = 2 \times 10^{18}\ \text{cm}^{-3} + \frac{11.5\ \text{cm}^{-1} + 16.3\ \text{cm}^{-1}}{(0.069)(5 \times 10^{-16}\ \text{cm}^2)} = 2.8 \times 10^{18}\ \text{cm}^{-3}$$

$\frac{N_{th}}{\tau} = BN_{th}^2 + CN_{th}^3 = (10^{-10})(2.8 \times 10^{18})^2 + (5 \times 10^{-29})(2.8 \times 10^{18})^3 = 1.9 \times 10^{27}\ \text{cm}^{-3}\ \text{s}^{-1}$

$V = (0.2 \times 10^{-4}\ \text{cm})(3 \times 10^{-4}\ \text{cm})(300 \times 10^{-4}\ \text{cm}) = 1.8 \times 10^{-10}\ \text{cm}^{-3}$

$$I_{th} = \frac{q\ V}{\eta_i} \frac{N_{th}}{\tau} = \frac{(1.6 \times 10^{-19}\ \text{C})(1.8 \times 10^{-10}\ \text{cm}^{-3})}{(0.7)}\ 1.9 \times 10^{27}\ \text{cm}^{-3}\ \text{s}^{-1}$$

$$I_{th} = 79\ \text{mA}$$

Find Output Power at Threshold: From the previous two sections, we know the output power at threshold if we determine the radiative efficiency, η_r.

$$\eta_r = \frac{R_{sp}}{R_{sp} + R_{nr}} \tag{2.6}$$

$R_{sp} = BN_{th}^2 = 10^{-10}\ \text{cm}^{-3}\ \text{s}^{-1}\ (2.8 \times 10^{18}\ \text{cm}^{-3})^2 = 7.84 \times 10^{26}\ \text{cm}^{-3}\ \text{s}^{-1}$

Since Auger recombination is much larger than non-radiative recombination at transparency, it will be much larger at threshold as well. (Note: This is a good approximation for long wavelength lasers where the Auger coefficient is large due to the small bandgap.)

$R_{nr} = R_{Auger} = CN_{th}^3 = BN_{tr}^{-1}N_{th}^3 = 1.10 \times 10^{27}\ \text{cm}^{-3}\ \text{s}^{-1}$

$$\eta_r(I = I_{th}) = 0.42$$

Now solve for power at threshold:

$$P_o(I = I_{th}) = (5.70 \times 10^{-6}\ \text{J/C})\ (0.42)\ (78\ \text{mA}) = 0.19\ \mu\text{W}$$

<u>Find Output Power Above Threshold:</u> Above threshold,

$$P = P_o(I = I_{th}) + \eta_d \frac{h\nu}{q} (I - I_{th}) \qquad (2.36)$$

$$\eta_d = \frac{\eta_i \alpha_m}{<\alpha_i> + \alpha_m} = \frac{(0.7)(16.3\ \text{cm}^{-1})}{(11.5\ \text{cm}^{-1}) + (16.3\ \text{cm}^{-1})} = 0.41 \qquad (2.35)$$

$$P = 0.19\ \mu\text{W} + (0.41)(0.80\ \text{J/C})(I - 79\ \text{mA}) = 0.19\ \mu\text{W} + (0.326\ \text{J/C})(I - 79\ \text{mA})$$

Note that this is the total power emitted by the laser (i.e. from both facets).

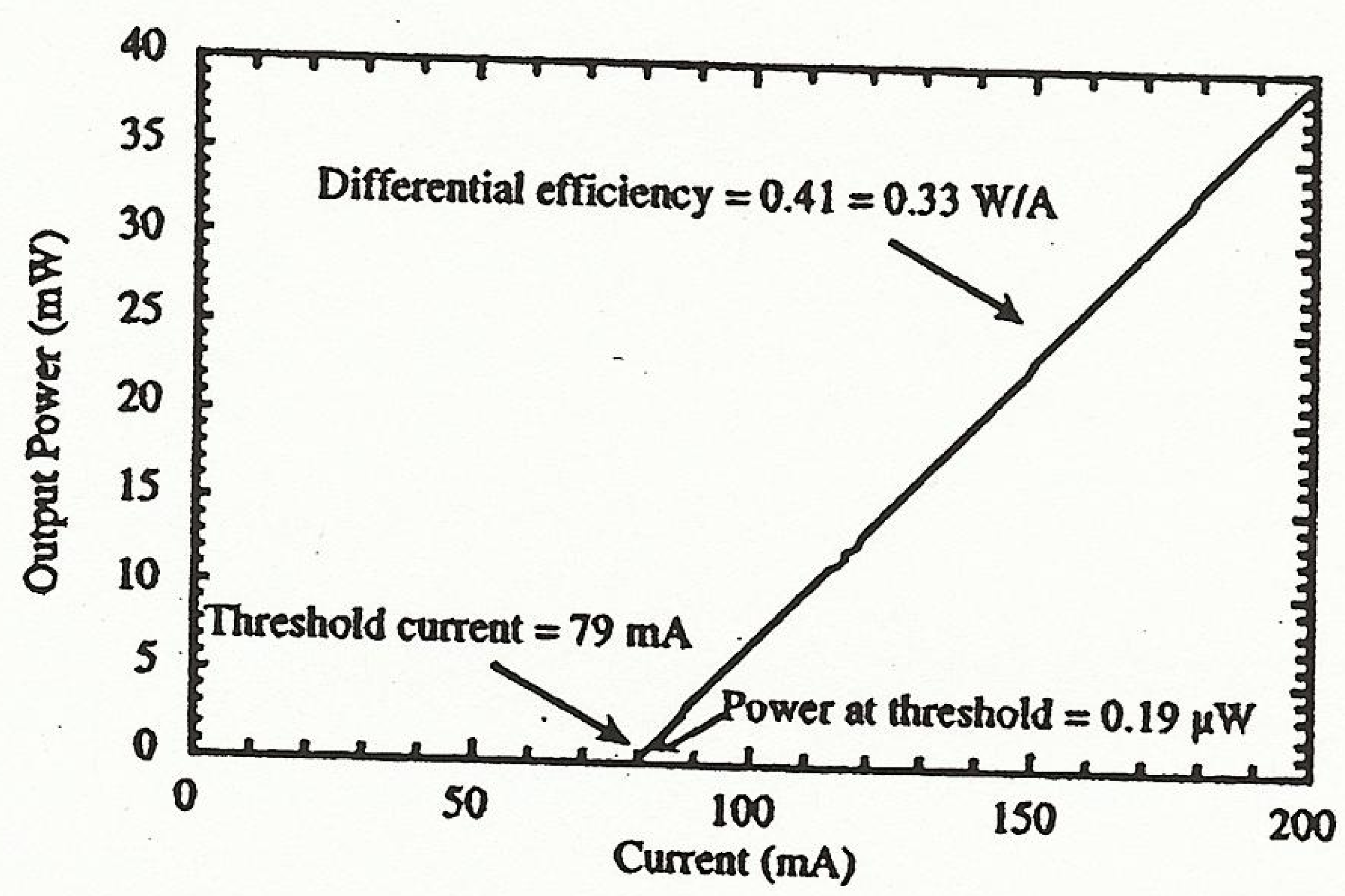

Figure 2.14a. Light vs. Current characteristics (L-I Curve)

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b)

$$\frac{P_{ac}(\omega)}{I(\omega)} = \frac{\eta_i\ \alpha_m\ a\ v_g^2\ N_{p0}\ (h\nu/q)}{\frac{a\ v_g\ N_{p0}}{\tau_p} + j\omega\left(\frac{1}{\tau} + a\ v_g\ N_{p0}\right) - \omega^2} \qquad (2.53)$$

$\overline{n}_g = 4$ from p. 40 for InGaAsP/InP DH structures

$v_g = \frac{c}{\overline{n}_g} = \frac{3\times10^{10}\ \text{cm/s}}{4} = 7.5 \times 10^9\ \text{cm/s}$

$N_{p0} = \frac{\eta_i(I - I_{th})}{q\ g_{th}\ v_g\ d\ w\ L_a} = \frac{(0.7)(80\times10^{-3}A)}{(1.6\times10^{-19}C)(405\ \text{cm}^{-1})(7.5\times10^9\ \text{cm/s})(0.2)(3)(300)(10^{-12}\ \text{cm}^3)} = 6.40 \times 10^{14}\ \text{cm}^{-3}$

(2.32)

$\tau_p = \frac{1}{v_g(<\alpha_i> + \alpha_m)} = 4.80 \times 10^{-12}\ \text{s}$ (2.24)

$C = \frac{B}{N_{tr}} = 5 \times 10^{-29}$ cm^6/s

$\tau = \frac{1}{BN_{th} + CN_{th}^2} = \frac{1}{(10^{-10}\ \text{cm}^3/\text{s})(2.81\times10^{18}\ \text{cm}^{-3}) + (5\times10^{-29}\ \text{cm}^6/\text{s})(2.81\times10^{18}\ \text{cm}^{-3})^2} = 1.47 \times 10^{-9}$ s

$$\frac{P_{ac}(\omega)}{I(\omega)} = \frac{1.64 \times 10^{20}\ \text{W/(C s)}}{5.0 \times 10^{20}\ \text{s}^{-2} - \omega^2 + j\omega\ (3.08 \times 10^9\ \text{s}^{-1})}$$

$$\left|\frac{P_{ac}(\omega)}{I(\omega)}\right|^2 = \frac{2.69 \times 10^{40}}{2.5 \times 10^{41} - 1.0 \times 10^{21}\omega^2 + \omega^4}$$

$$\left|\frac{P_{ac}(0)}{I(0)}\right|^2 = 0.108$$

Let I be constant (i.e. $I(\omega) = I(0)$).

$$\left|\frac{P_{ac}(\omega)}{P_{ac}(0)}\right| = \sqrt{\frac{2.5 \times 10^{41}}{2.5 \times 10^{41} - 1.0 \times 10^{21}\omega^2 + \omega^4}}$$

Plot response in dB:

$$\left|\frac{P_{ac}(\omega)}{P_{ac}(0)}\right|_{dB} = 20\ log_{10}\left(\left|\frac{P_{ac}(\omega)}{P_{ac}(0)}\right|\right)$$

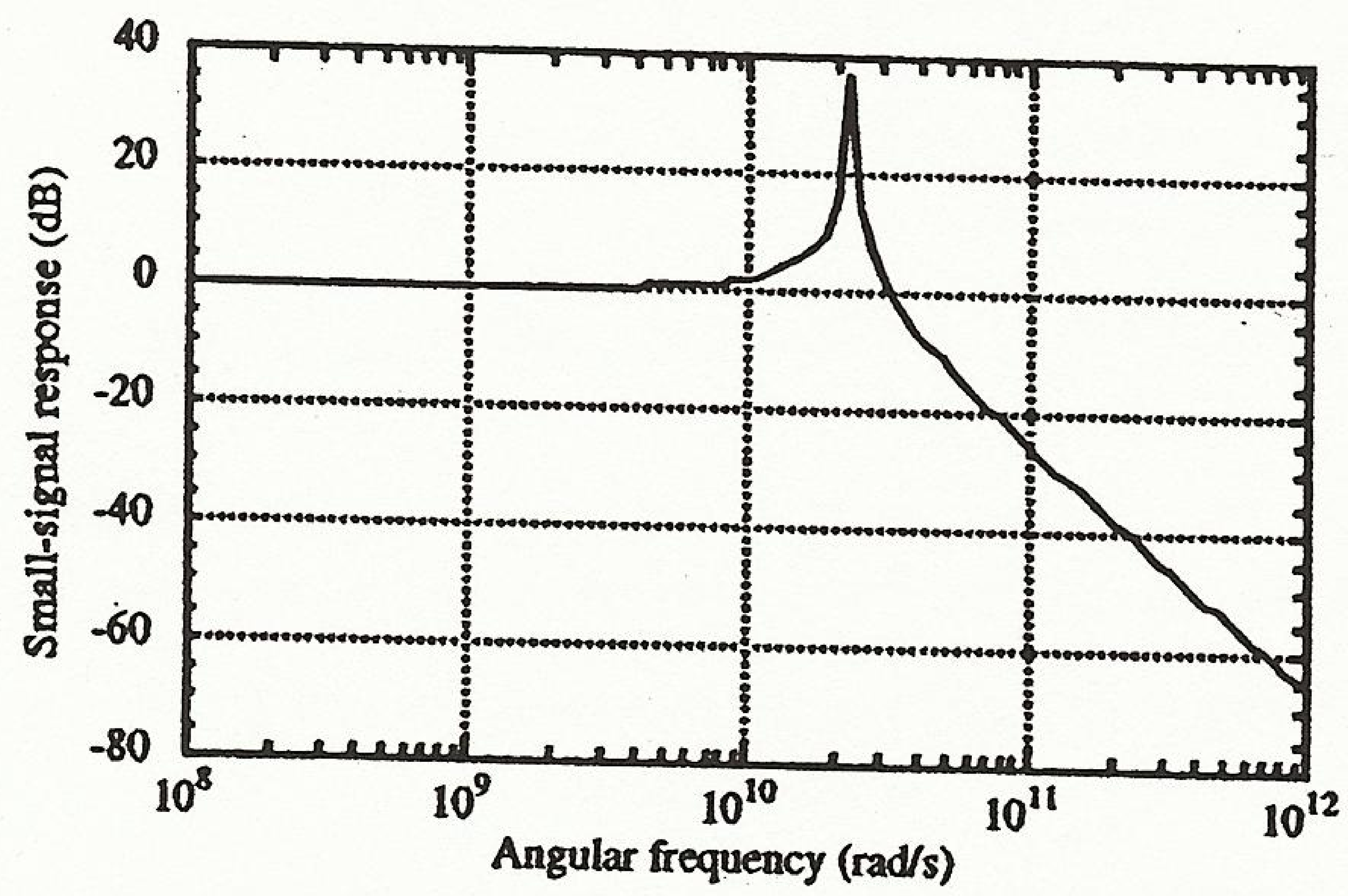

Figure 2.14b. Small-signal frequency response of laser at $I = 2I_{th}$

Note from part (a)

In the calculation of β_{sp}, we assumed that $\Gamma_c \approx \Gamma$, where Γ is the confinement factor for the fundamental mode and Γ_c is the cavity confinement factor of Appendix 4. This a reasonable approximation because only those modes that are guided by the epitaxial layer structure (i.e. have approximately the same modal size as the fundamental mode) will have any significant overlap with the active region. Modes that do not have such an overlap will not receive any of the spontaneous emission output and therefore should not be included in the calculation of the spontaneous emission factor, β_{sp}.