3.1:

a)

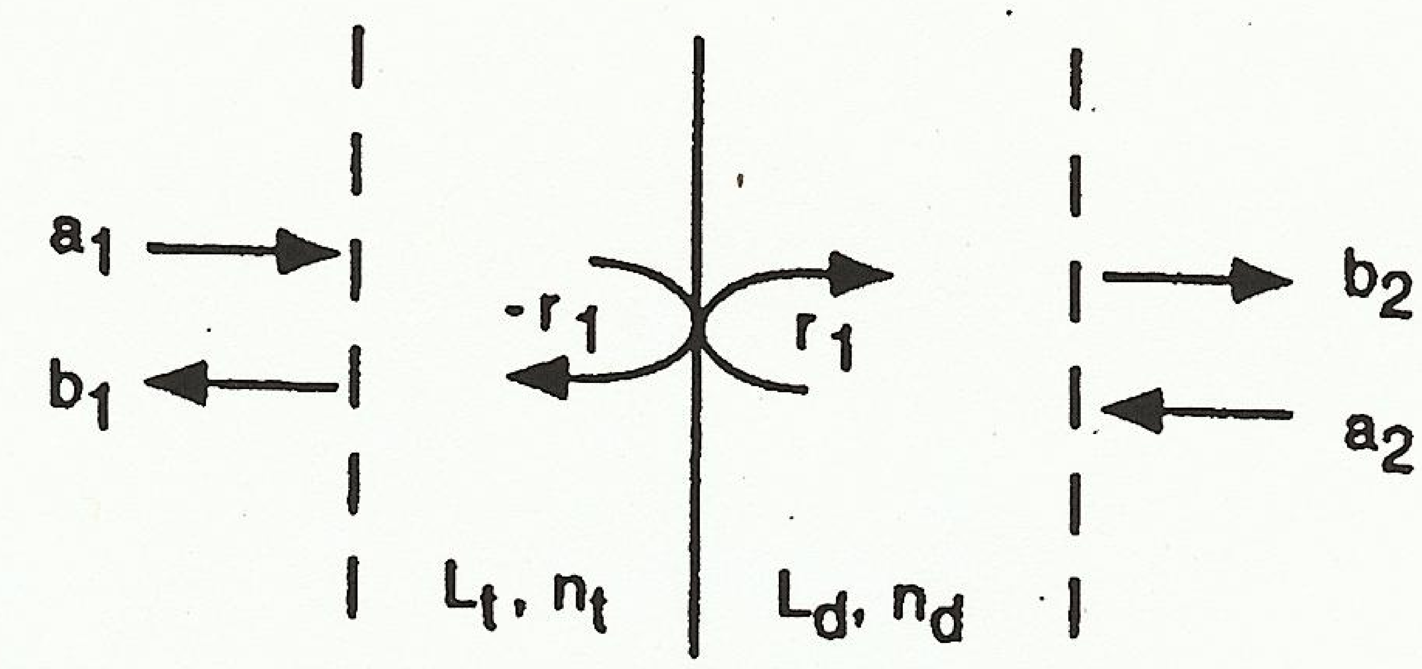

Figure 3.1a. Definition of variables

<u>S-matrix</u>

$$S_{11} = \frac{b_1}{a_1}\bigg|_{a_2=0} = -r_1\, e^{-2j\beta_t L_t}$$

$$S_{12} = \frac{b_1}{a_2}\bigg|_{a_1=0} = t\, e^{-j(\beta_t L_t + \beta_d L_d)}$$

$$S_{21} = \frac{b_2}{a_1}\bigg|_{a_2=0} = t\, e^{-j(\beta_t L_t + \beta_d L_d)}$$

$$S_{22} = \frac{b_2}{a_2}\bigg|_{a_1=0} = r_1\, e^{-2j\beta_d L_d}$$

<u>T-matrix</u>

$$T_1 = \begin{bmatrix} e^{j\beta_t L_t} & 0 \\ 0 & e^{-j\beta_t L_t} \end{bmatrix}$$

$$T_2 = \frac{1}{t}\begin{bmatrix} 1 & -r_1 \\ -r_1 & 1 \end{bmatrix}$$

$$T_3 = \begin{bmatrix} e^{j\beta_d L_d} & 0 \\ 0 & e^{-j\beta_d L_d} \end{bmatrix}$$

$$\begin{aligned} T &= T_1 T_2 T_3 \\ &= \begin{bmatrix} e^{j\beta_t L_t} & 0 \\ 0 & e^{-j\beta_t L_t} \end{bmatrix} \frac{1}{t}\begin{bmatrix} 1 & -r_1 \\ -r_1 & 1 \end{bmatrix} \begin{bmatrix} e^{j\beta_d L_d} & 0 \\ 0 & e^{-j\beta_d L_d} \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{t}\, e^{j(\beta_t L_t + \beta_d L_d)} & \frac{-r_1}{t}\, e^{j(\beta_t L_t - \beta_d L_d)} \\ \frac{-r_1}{t}\, e^{-j(\beta_t L_t - \beta_d L_d)} & \frac{1}{t}\, e^{-j(\beta_t L_t + \beta_d L_d)} \end{bmatrix} \end{aligned}$$

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b) Find and plot S_{11} and S_{21}:

$$S_{11} = -r_1\, e^{-2j\beta_t L_t}$$

$$|S_{11}| = r_1 = \frac{n_t - n_d}{n_t + n_d} = \frac{1 - 3.5}{1 + 3.5} = -0.\text{[redacted]}556$$

$$\text{angle of } S_{11} = \pi - 2\beta_t L_t = \pi - 2\left(\frac{2\pi}{\lambda}\right)(1)(10\mu\text{m}) = \pi - \frac{125.7\mu\text{m}}{\lambda}$$

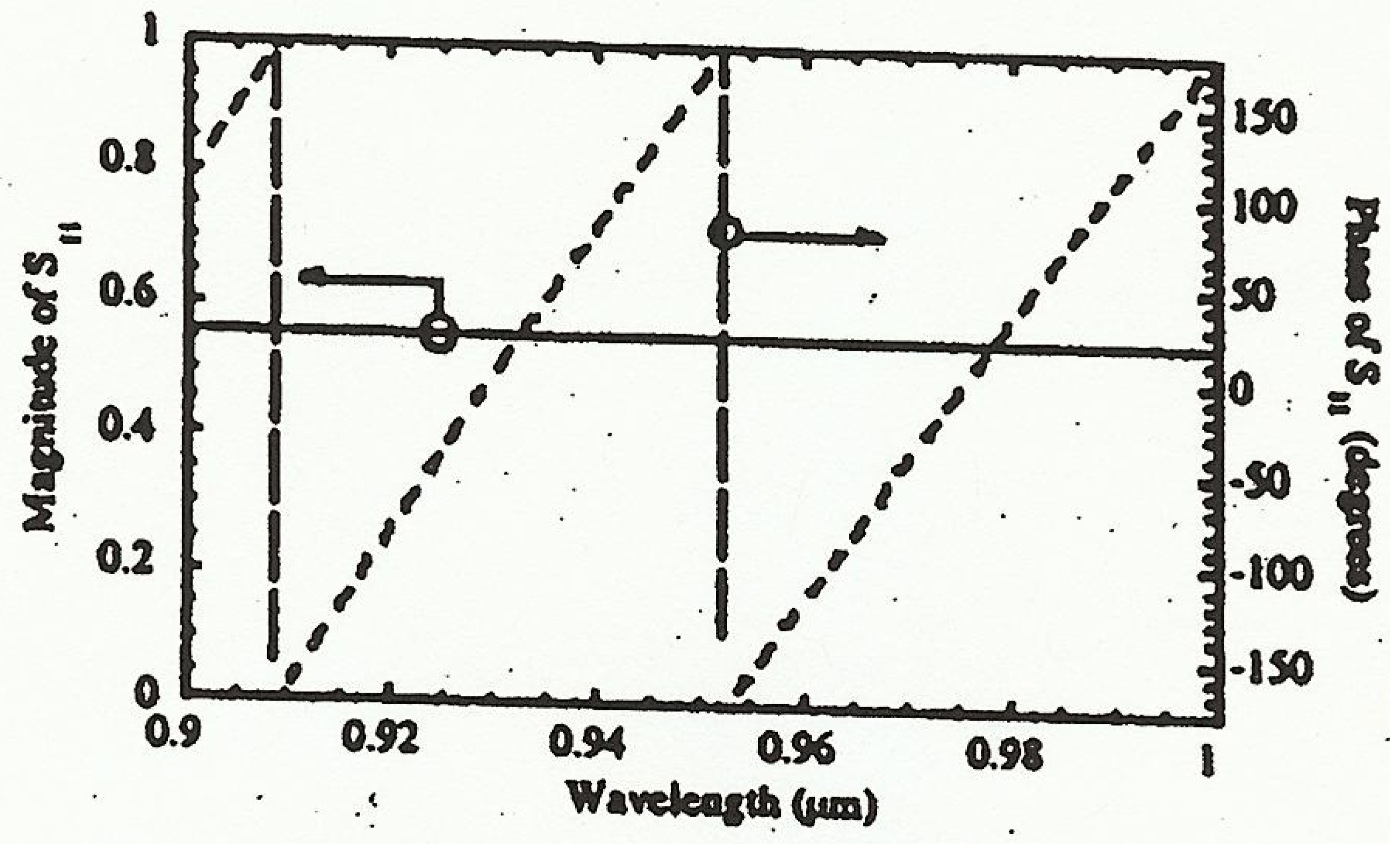

Figure 3.1b. Phase and magnitude of S_{11}

$$S_{21} = t\ e^{-j(\beta_t L_t + \beta_d L_d}$$

$$|S_{21}| = t = \sqrt{1 - r_1^2} = 0.831$$

$$\text{angle of } S_{11} = -(\beta_t L_t + \beta_d L_d) = -\frac{2\pi}{\lambda}[(1)(10\mu\text{m}) + (3.5)(5\mu\text{m})] = -\frac{173.8\mu\text{m}}{\lambda}$$

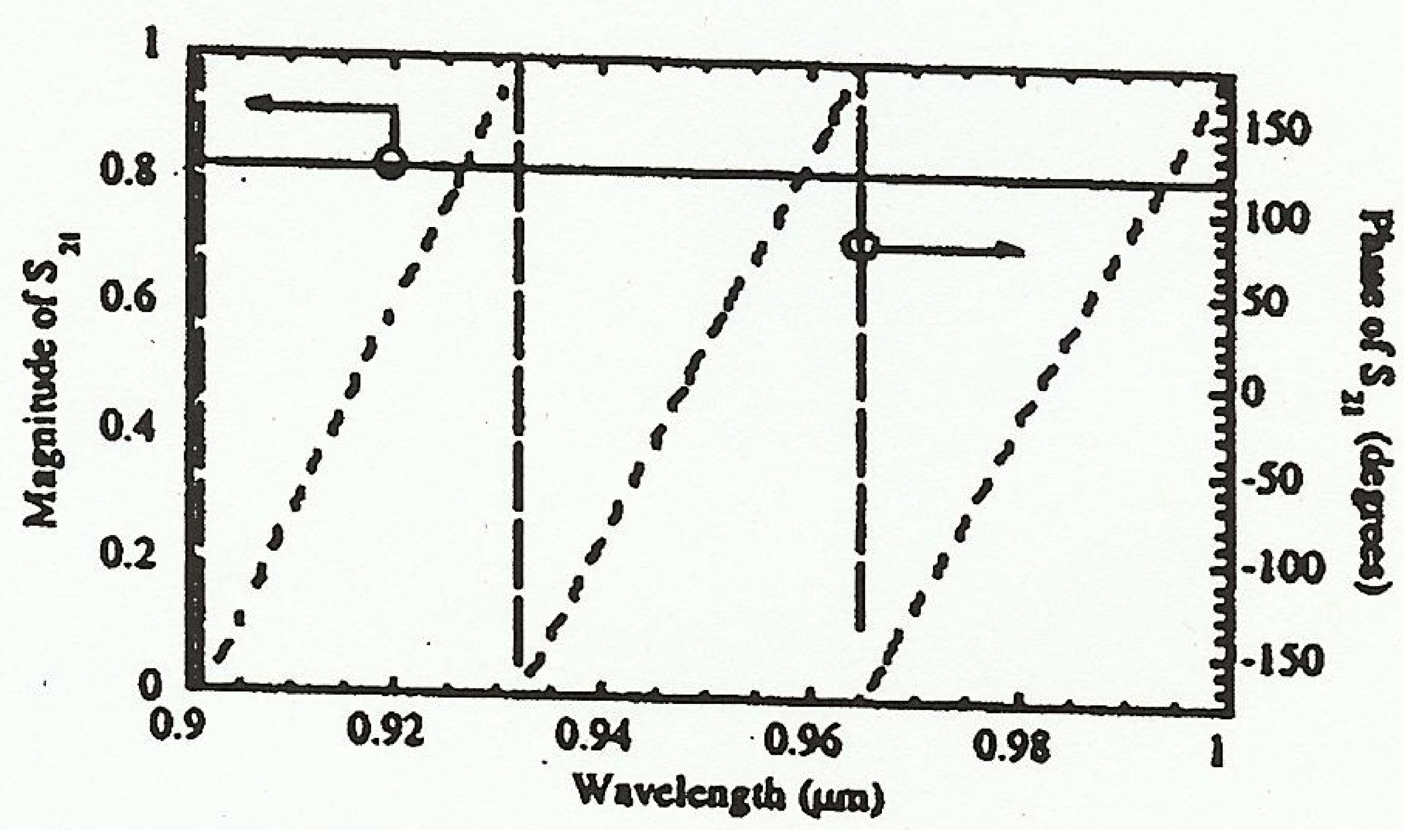

Figure 3.1b. Phase and magnitude of S_{21}