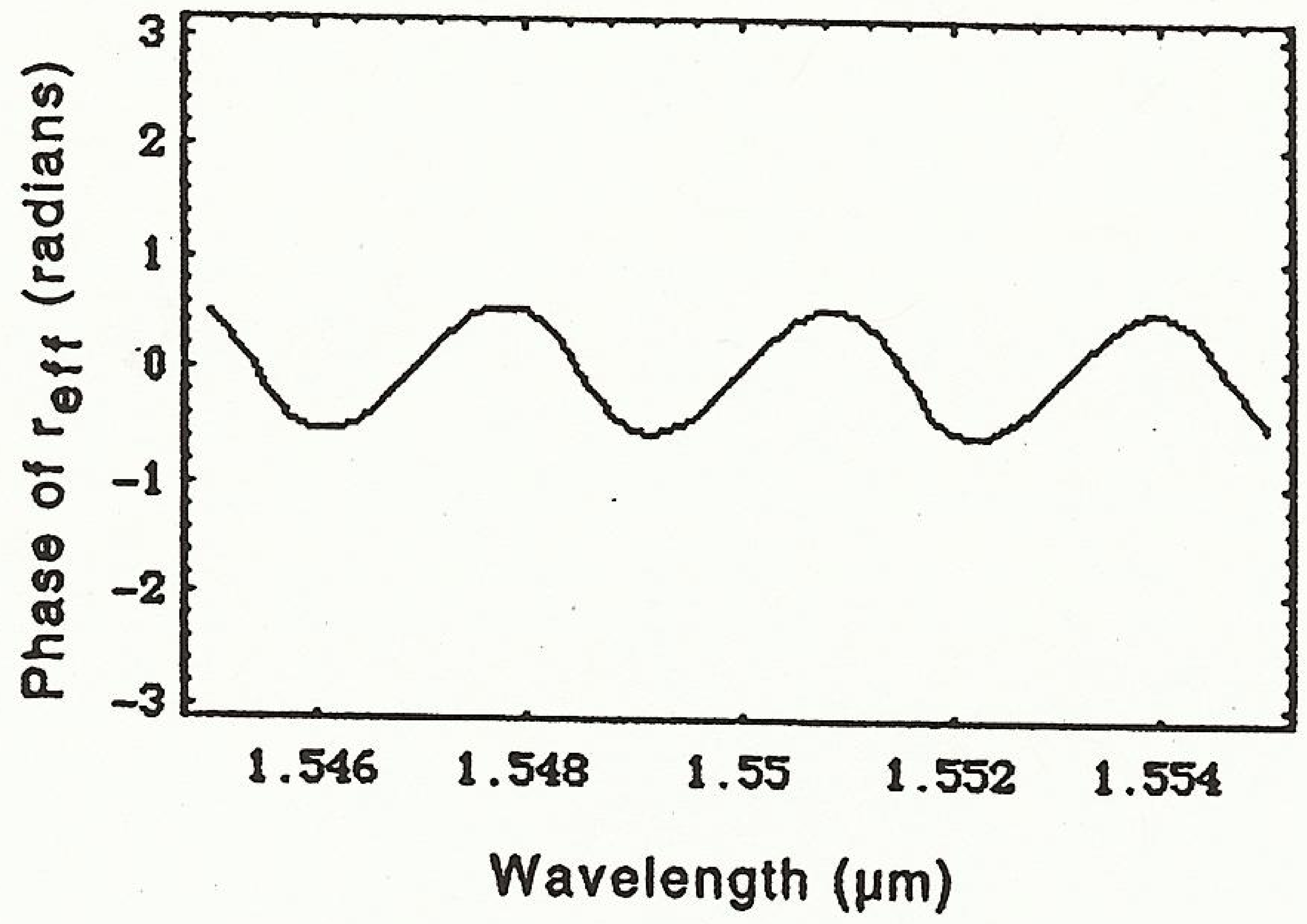

Figure 3.7c. Phase of r_{eff}

3.8:

First, verify the equalities on the left side of Eqs. 3.43:

As described on p. 87, the values of the transmission elements for each period of the grating are obtained by multiplying transmission matrices for the two reflections and two transmission segments of each grating period.

$$T = (T_1 T_2)(T_3 T_4)$$

From Table 3.3

$$T_1 T_2 = \frac{1}{t_{21}} \begin{bmatrix} e^{j\tilde{\beta}_1 L_1} & r_{21}\, e^{-j\tilde{\beta}_1 L_1} \\ r_{21}\, e^{j\tilde{\beta}_1 L_1} & e^{-j\tilde{\beta}_1 L_1} \end{bmatrix}$$

$$T_3 T_4 = \frac{1}{t_{12}} \begin{bmatrix} e^{j\tilde{\beta}_2 L_2} & r_{12}\, e^{-j\tilde{\beta}_2 L_2} \\ r_{12}\, e^{j\tilde{\beta}_2 L_2} & e^{-j\tilde{\beta}_2 L_2} \end{bmatrix}$$

Define variables:

$t = t_{12} = t_{21}$

$r = -r_{12} = r_{21}$

$\phi_1 = \tilde{\beta}_1 L_1$

$\phi_2 = \tilde{\beta}_2 L_2$

$\phi_\pm = \phi_1 \pm \phi_2$

Substituting these values in the matrices above, and multiplying the resulting matrices together to form T gives us the equations on the left side of Eq. 3.43.

Second, verify the right side of Eqs. 3.43:

For lossless gratings,

$$\phi_+ = \pi + \delta\Lambda$$
$$\phi_- = 0 \qquad (3.46)$$

At the Bragg Frequency,

$$\delta = 0 \quad \rightarrow \quad \phi_+ = \pi$$

Using these values for $\phi_\pm$ in the left side of Eqs. 3.43 produces the right side:

$$T_{11} = \frac{1}{t^2}\left[e^{j\pi} - r^2\, e^0 \right] = -\frac{1+r^2}{t^2}$$

$$T_{21} = \frac{r}{t^2}\left[e^{j\pi} - e^0 \right] = -\frac{2r}{t^2}$$

$$T_{12} = \frac{r}{t^2}\left[e^{-j\pi} - e^0 \right] = -\frac{2r}{t^2}$$

$$T_{22} = \frac{1}{t^2}\left[e^{-j\pi} - r^2\, e^0 \right] = -\frac{1+r^2}{t^2}$$

3.10:

a)

$$r_{eff} = S_{11} + \frac{S_{21}S_{12}r_3\ e^{-2j\beta L}}{1 - S_{22}r_3\ e^{-2j\beta L}} \tag{3.40}$$

$r_3 = 0.95$

$L = 0$

$$r_{eff} = S_{11} + \frac{S_{21}S_{12}r_3}{1 - S_{22}r_3}$$

$S_{11} = r_g = \tanh(2mr) = \tanh(0.6) = 0.537$

$|S_{21}| = \sqrt{1 - |S_{11}|^2} = 0.8436$ (for lossless case)

$e^{j\beta L} = -1 \rightarrow S_{21} = -0.8436$

$S_{12} = S_{21}$

$S_{22} = -S_{11} = -0.537$

$$r_{eff} = 0.537 + \frac{(0.8436)^2(0.95)}{1-(-0.537)(0.95)} = 0.9847$$

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b)

$$L_{eff} = -\frac{1}{2}\frac{\delta\phi}{\delta\beta}\bigg|_{\lambda_B}$$

So, by definition, L_{eff} depends only on $\frac{\delta\phi}{\delta\beta}$.

If there is only a grating (i.e. no metal coating), then $L_{eff} = L_g/2$. If there is metal only, then L_{eff} is located at the surface of the metal. So, for a grating coated with metal, L_{eff} should satisfy $L_g/2 < L_{eff} < L_g$. Since the metal is the dominant reflector in this case, L_{eff} will be closer to the metal surface. Therefore, a reasonable approximation is

$$L_{eff} \approx L_m$$

where $x = L_m$ is the location of the mirror.