3.11:

a) A VCSEL with InGaAs quantum wells is likely to have a nominal lasing wavelength of $\lambda = 980$ nm. From Table 1.1, the indices of refraction for the VCSEL materials are $n_{GaAs} = 3.52$ and $n_{AlAs} = 2.95$.

$$r_g = \frac{1-(n_1/n_2)^{2m}}{1+(n_1/n_2)^{2m}} \tag{3.54}$$

m = 18 = number of mirror periods
n_1 = 2.95 = lower index (AlAs)
n_2 = 3.52 = higher index (GaAs)

$$r_g = \frac{1-(2.95/3.52)^{2\times18}}{1+(2.95/3.52)^{2\times18}} = 0.99654$$

This mirror is then metalized to give a reflectivity of 0.999

---

b)

$$L_{eff} = \frac{1}{2\kappa}\tanh(\kappa L_g) = \frac{1}{2\kappa}\tanh(2mr)$$

m = 18

$$r = \frac{3.52-2.95}{3.52+2.95} = 0.0881$$

$$L_g = 18\left(\frac{\lambda}{4n_1}+\frac{\lambda}{4n_2}\right) = 2.748 \text{ µm}$$

$$r_g \approx \tanh(2mr) = \tanh(\kappa L_g) \tag{3.53}$$

Using r_g from part (a), we find $\kappa = 2mr/L_g = 1.154$ µm^{-1}

$$L_{eff} = \frac{1}{2(1.154\mu m^{-1})}(0.99654) = 0.4317 \text{ µm}$$

Assume that L_{eff} is approximately the same when a mirror is added. (Since very little power reaches the metal mirror, this is a reasonable approximation)

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c)

$$\Gamma g_{th} = \langle\alpha_i\rangle + \alpha_m = \langle\alpha_i\rangle + \frac{1}{L_{DBR}}\ln\left(\frac{1}{|r_g||r_{gm}|}\right)$$

Where r_g is the grating reflectivity without the metal coating (light emitting side) and r_{gm} is the grating reflectivity with the metal coating. The cavity length (L_a+L_p) is one wavelength long, so

$$\begin{aligned} L_{DBR} &= L_a + L_p + 2L_{eff} \\ &= \lambda/n_2 + 2L_{eff} \\ &= 0.98\mu m/3.52 + 2(0.4317\mu m) \\ &= 1.142\mu m \end{aligned}$$

$$\Gamma g_{th} = 20cm^{-1} + \frac{1}{1.142 \times 10^{-4} cm} \ln\left(\frac{1}{|0.999||0.99654|}\right) = 59.11cm^{-1}$$

---

d) The light leaving the VCSEL is attenuated along the entire grating length. So, additional loss must be taken into account, which is done through the use of the F parameter.

$$\eta_{d1} = \eta_i \frac{\alpha_m}{\langle \alpha_i \rangle + \alpha_m} F_1$$

$$F_1 = \frac{t_1^2}{\left(1 - r_1^2\right) + \frac{r_1}{r_2}\left(1 - r_2^2\right)} = \frac{\left(1 - |r_1|^2\right) e^{-\langle \alpha_i \rangle L_{eff}}}{\left(1 - r_1^2\right) + \frac{r_1}{r_2}\left(1 - r_2^2\right)}$$

$$F_1 = \frac{\left(1 - (0.99654)^2\right) e^{-20cm^{-1}\left(0.4317 \times 10^{-4} cm\right)}}{\left(1 - (0.99654)^2\right) + \frac{0.99654}{0.999}\left(1 - (0.999)^2\right)} = 0.775$$

$\eta_i = 1.0$

$$\eta_{d1} = (0.775)(1) \frac{39.11cm^{-1}}{39.11cm^{-1} + 20cm^{-1}} = 0.513 = 51.3\%$$

3.13:

** Correction of stated problem: $\partial \bar{n} / \partial N = 10^{-21} cm^{-3}$

a) There are two effects that must be considered:
   shift of the axial mode wavelength
   shift of the DBR peak reflectivity wavelength

First, consider the shift of the axial mode wavelength due to current in the grating only

$$\frac{\Delta\lambda_m}{\lambda_m} = \frac{\Delta\bar{n}_g}{\bar{n}} \frac{L_{eff}}{L_{total}}$$

Where $\Delta\bar{n}_g$ in this case is the shift in effective index of the grating, not group index.

$$L_{eff} = \frac{1}{2\kappa}\tanh(\kappa L_g)$$

$$= \frac{1}{2(100cm^{-1})}\tanh(100cm^{-1} \cdot 200\times10^{-4} cm)$$

$$= 48.2\mu m$$

$$\Delta\bar{n}_g = \frac{\partial\bar{n}}{\partial N}\frac{\eta_i \tau}{qV} I_g \quad (3.66)$$

$$= (-10^{-21} cm^{-3}) \frac{(0.7)(3\times10^{-9}s)}{(1.6\times10^{-19}C)(2\times10^{-2}cm)(3\times10^{-4}cm)(0.2\times10^{-4}cm)} I_g$$

$$= -0.1092\frac{I_g}{A} = -1.092\times10^{-4}\frac{I_g}{mA}$$

The mode shift due to the current to the DBR mirror is then given by

$$\frac{\Delta\lambda_m}{\lambda_m} = \frac{\Delta\bar{n}_g}{\bar{n}} \frac{L_{eff}}{L_{total}} \quad (3.65)$$

$$= \frac{(-1.092\times10^{-4} mA^{-1}) I_g (48.2\mu m)}{(3.4)(448.2\mu m)}$$

$$= -3.45\times10^{-6}\frac{I_g}{mA}$$

$$\Delta\lambda_m = (1.57\mu m)(-3.45\times10^{-6} mA^{-1}) I_g = 0.0542 \text{ Å} \frac{I_g}{mA}$$

Second, calculate the shift of the Bragg mirror's central wavelength:

$$L_1 = \frac{\lambda_{B0}}{4n_1}$$

$$L_2 = \frac{\lambda_{B0}}{4n_2}$$

$$\Lambda = \frac{\lambda_{B0}}{2} = n_1 L_1 + n_2 L_2$$

$$\rightarrow \lambda_{B0} = 2\bar{n}\Lambda$$

$$\lambda_B = 2(\bar{n} + \Delta\bar{n})\Lambda = \left(\frac{\bar{n} + \Delta\bar{n}}{\bar{n}}\right)\lambda_{B0}$$

$$\Delta\lambda_{B0} = \frac{\Delta\bar{n}}{\bar{n}}\lambda_{B0} = \frac{\left(-1.092\times10^{-3}\,\frac{I_g}{mA}\right)}{3.4}(1.57\,\mu m) = -0.504\,Å\,\frac{I_g}{mA}$$

Now that we know how the mode shifts, we must find out where the mode hops occur. The mode spacing is

$$\delta\lambda = \frac{\lambda^2}{2\bar{n}_g L_{total}} = \frac{(1.57\,\mu m)^2}{2(4)(448\,\mu m)} = 6.9\,Å \qquad (2.27)$$

where $\bar{n}_g$ is the effective group index in this case

Assume that the gain spectrum is flat. This is a reasonable approximation since we are only interested in a very narrow region and the mirror reflectivity varies much more rapidly than does the gain spectrum.

As current I_g is injected into the DBR, the modes shift to shorter wavelengths, following the shift of the DBR mirror peak reflectivity. Whichever mode is the closest to the minimum in α_m will be the lasing mode. The Bragg wavelength shifts about 10 times faster than the modes themselves. For this reason, there are discontinuities in the dominant mode wavelength vs. current profile as shown below.

The difference in DBR current between each axial mode jump depends on the difference in the rate of change of the modes and the rate of change of the Bragg frequency of the grating:

$$\Delta I_g = \frac{\delta\lambda}{\Delta\lambda_m - \Delta\lambda_{B0}} = \frac{6.9\,Å}{\left(-0.0542\,Å/mA\right)-\left(-0.504\,Å/mA\right)} = 15.3\,mA$$

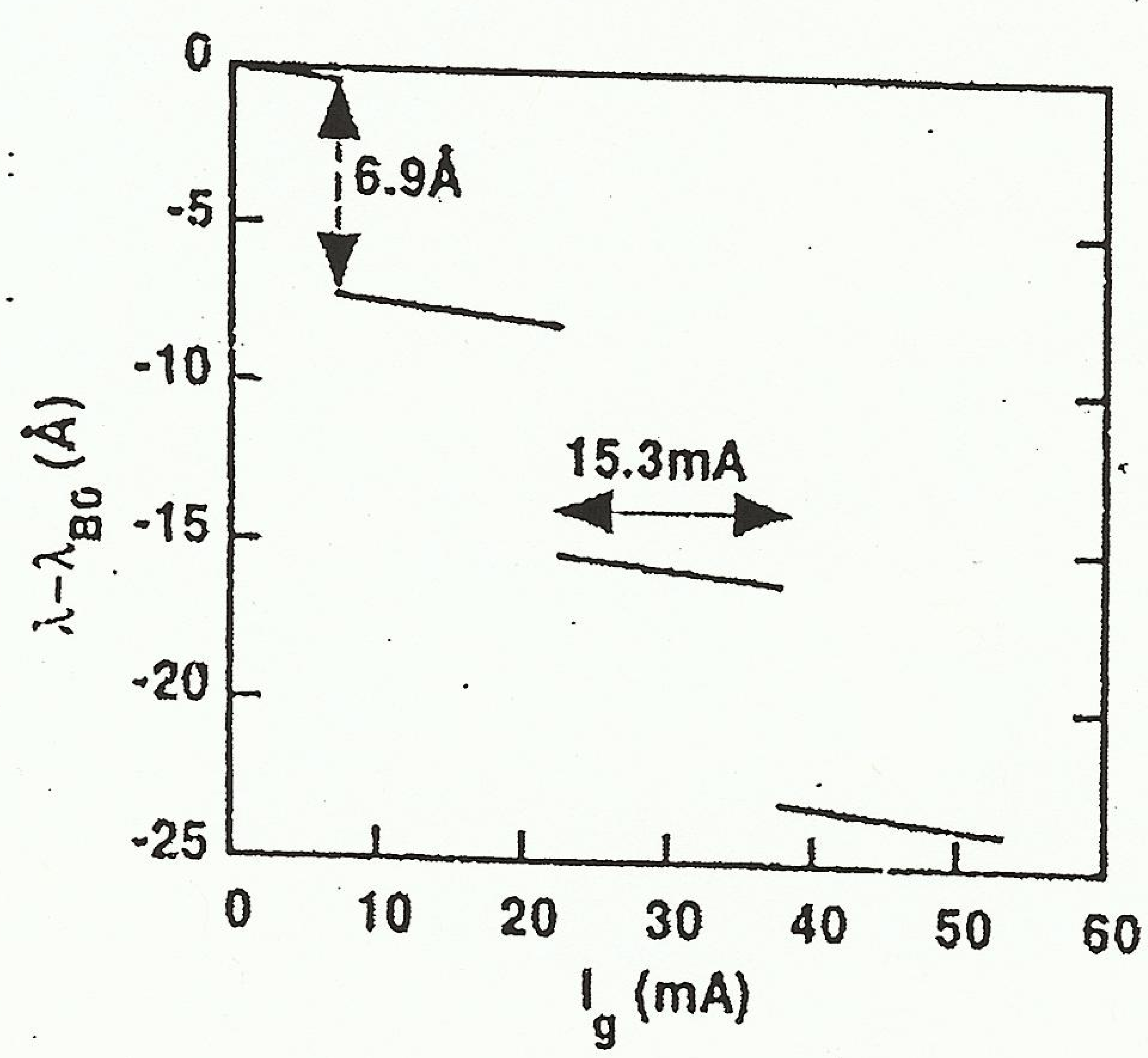

Figure 3.13a. Dominant laser mode wavelength vs. DBR current

b) In order to maintain the lasing wavelength at the peak of the Bragg mirror's reflectivity, we need the modes to move at the same rate as the central Bragg frequency of the grating.

$$\Delta\lambda_m = \Delta\lambda_B$$

To move the modes at the proper rate, current is injected into the passive (phase) waveguide section. Using Eq. 3.65 as above.

$$\Delta\lambda_m = \lambda_m \frac{\Delta\bar{n}_p}{\bar{n}}\frac{L_p}{L_{total}}$$

$$= (1.57\mu m)\frac{\left(-1.092\times10^{-4}\frac{I_g}{mA}\right)(200\mu m)}{(3.4)(448\mu m)} = -0.23\text{Å}\ \frac{I_p}{mA}$$

Including the shift from the passive and grating sections.

$$\Delta\lambda_{mtotal} = \left(-0.0542\,\mathring{A}\right)\frac{I_g}{mA} + \left(-0.23\,\mathring{A}\right)\frac{I_p}{mA}$$

This must be equal to the shift of the central frequency of the Bragg mirror:

$$\Delta\lambda_{mtotal} = \Delta\lambda_B$$

$$\Delta\lambda_{mtotal} = \left(-0.0542\,\mathring{A}\right)\frac{I_g}{mA} + \left(-0.23\,\mathring{A}\right)\frac{I_p}{mA} = \left(-0.504\,\mathring{A}\right)\frac{I_g}{mA}$$

$$I_p = 1.96 I_g \approx 2.0 I_g$$

Now the laser can be tuned continuously,

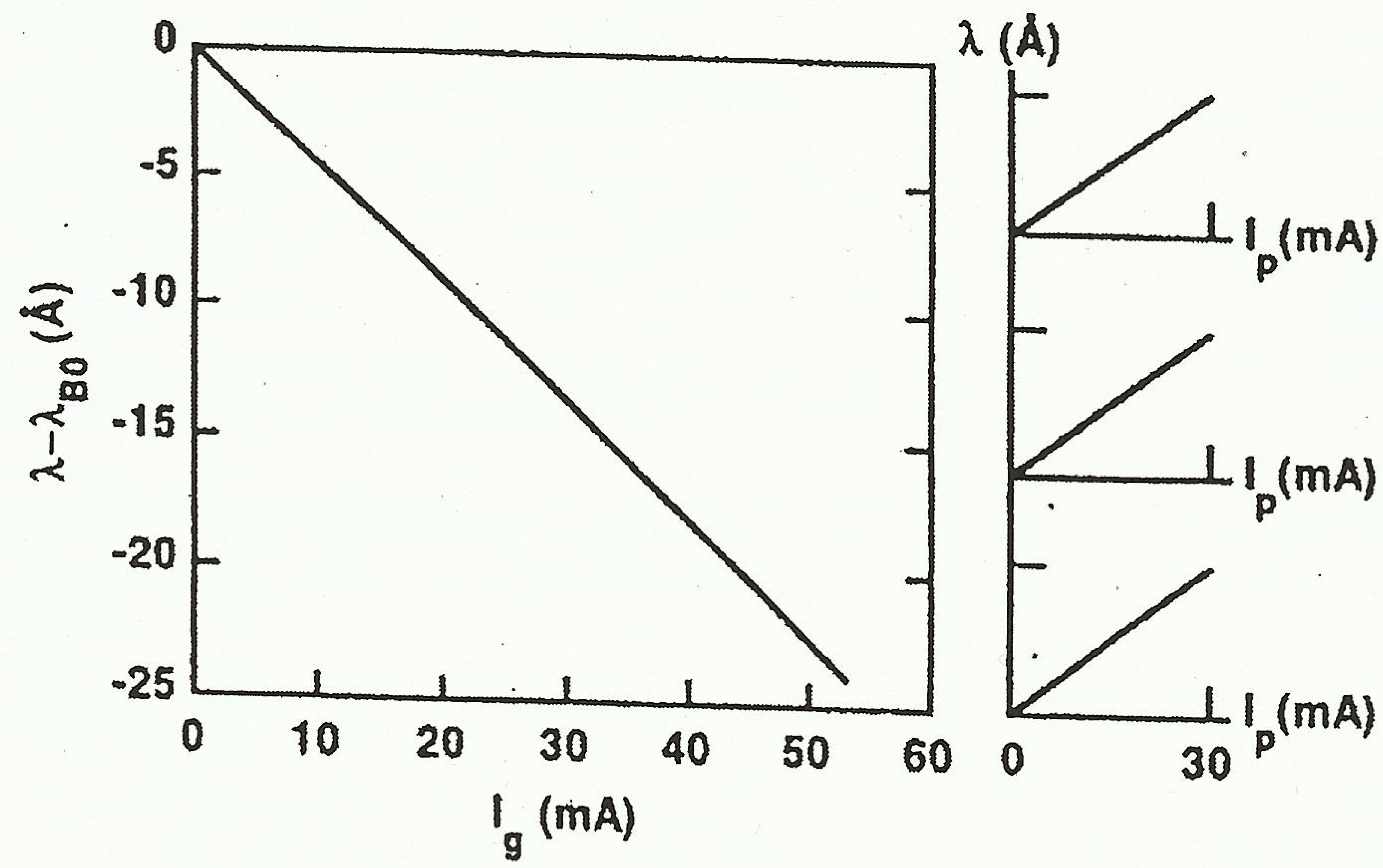

Figure 3.13b. Dominant laser mode wavelength vs. DBR current and phase current

The phase current is applied cyclically. Current is injected until the wavelength is equal to the wavelength where a mode hop would occur without phase tuning. The current in the phase section is not simply increased monotonically because heating would eventually counteract the change in index due to current injection.

3.14:

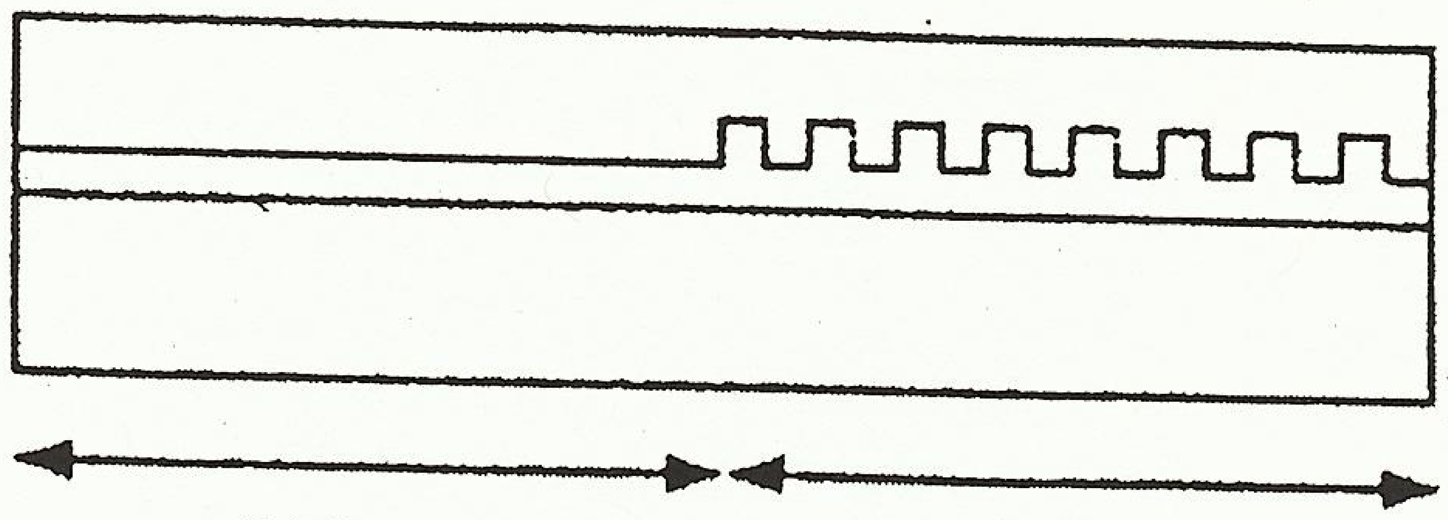

Figure 3.14. Laser structure

$\kappa L_g = 1$

$\eta_i = 0.7$

$<\alpha_i> = 20\ \text{cm}^{-1}$

Use the effective mirror model to replace the grating reflector with a hard mirror at a distance of L_{eff}.

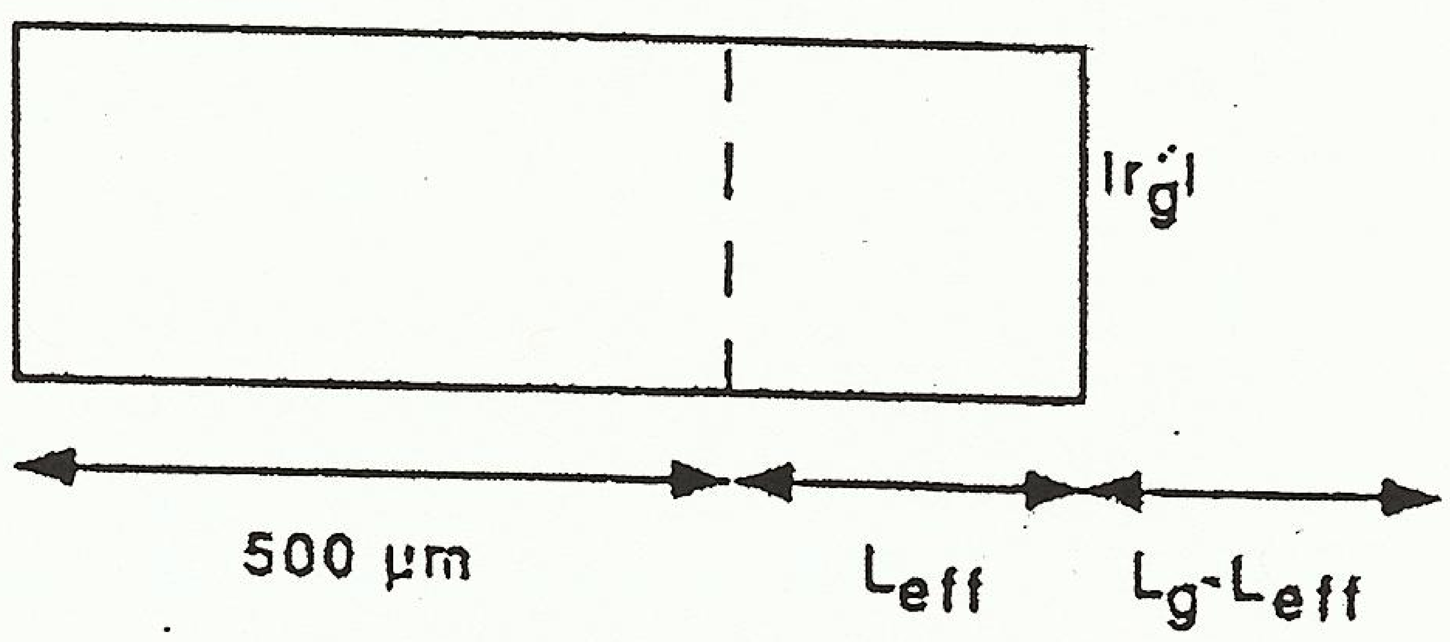

Figure 3.14. Laser with effective mirror model

Using Eq. 3.30, we can write

$$F_1 = \frac{(1-r_1^2)}{(1-r_1^2)+\frac{r_1}{|r_g'|}(1-|r_g'|^2)}$$

$$F_2 = \frac{(1-|r_g'|^2)\ e^{-<\alpha_i>(\ L_{eff}\ )}}{(1-|r_g'|^2)+\frac{|r_g'|}{r_1}(1-r_1^2)}$$

$L_{eff} = \frac{1}{2\kappa}tanh(\kappa L_g) = \frac{500um}{2}tanh(1) = 190\mu m$

$|r_g'| = tanh(2mr) = tanh(\kappa L_g) = 0.76$

$$F_2 = \frac{(1-0.76^2)\ e^{-20(\ 0.190\ )\times 10^{-4}}}{(1-0.76^2)+\frac{0.76}{0.56}(1-0.56^2)} = 0.2133$$

$$F_1 = \frac{(1-0.56^2)}{(1-0.56^2)+\frac{0.56}{0.76}(1-0.76^2)} = 0.6880$$

$$\alpha_m = \frac{1}{L_a+L_{eff}}\ln\left(\frac{1}{r_1|r_g'|}\right) = \frac{1}{690um}\ln\left(\frac{1}{(0.56)(0.76)}\right) = 12.4\ \text{cm}^{-1}$$

$$\eta_{d1} = \eta_i\frac{\alpha_m}{\alpha_m+<\alpha_i>}F_1$$
$$= 0.7\frac{12.4}{20+12.4}0.688 = 18.4\%$$

$$\eta_{d2} = \eta_i\frac{\alpha_m}{\alpha_m+<\alpha_i>}F_2$$
$$= 0.7\frac{12.4}{20+12.4}0.2133 = 5.71\%$$

3.16:

To solve this problem, we have used a number of clarifications or assumptions:

1) $L_g = 500\mu$m
2) $\eta_r = 1$
3) change in gain with wavelength is negligible in comparison to change in mirror loss

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a) From Fig. 3.18b, we find that for $\kappa L_g = 1$, we have $(\Gamma g_{th} - <\alpha_i>)L_g = 3.1$. If we assume that $L_g = 500\mu$m, then

$$\Gamma g_{th} = \frac{3.1}{0.05\text{ cm}} + 10\text{ cm}^{-1} = 62\text{ cm}^{-1} + 10\text{ cm}^{-1} = 72\text{ cm}^{-1}$$

---

b)

$$\begin{aligned} MSR &= \frac{<\alpha_i> + \alpha_m(\lambda_1) - \Gamma g(\lambda_1)}{<\alpha_i> + \alpha_m(\lambda_0) - \Gamma g(\lambda_0)} \frac{F_1(\lambda_0)\alpha_m(\lambda_0)}{F_1(\lambda_1)\alpha_m(\lambda_1)} \\ &= \left(\frac{\Delta\alpha + \Delta g}{\delta_g} + 1\right) \frac{F_1(\lambda_0)\alpha_m(\lambda_0)}{F_1(\lambda_1)\alpha_m(\lambda_1)} \end{aligned} \qquad (3.73, 3.74)$$

$\Delta\alpha + \Delta g = (\alpha(\lambda_1) - (\Gamma g(\lambda_0)) - (\alpha(\lambda_0) - \Gamma g(\lambda_1)) = (\Delta(\Gamma g_{th})$

For the modes on either side of the peak mode (i.e. modes ± 1), we have (see part (a) for method):

$$(\Gamma g_{th}(\lambda_1) - <\alpha_i>))L_g = 4.3$$
$$\rightarrow \Gamma g_{th} = 96\text{ cm}^{-1}$$

Then $\Delta\alpha + \Delta g = \Delta(\Gamma g_{th}) = 96\text{ cm}^{-1} - 72\text{ cm}^{-1} = 24\text{ cm}^{-1}$

$$\delta_g = \eta_r \beta_{sp}(<\alpha_i> + \alpha_m)\frac{I_{th}}{(I - I_{th})} = (1)(10^{-4})(72\text{ cm}^{-1})(1) = 7.2 \times 10^{-3}\text{ cm}^{-1} \qquad (3.77)$$

Assume that the gain spectrum is approximately constant over the range of interest while the mirror loss varies with wavelength. So we can write $\alpha_m(\lambda) = \Gamma g_{th} - <\alpha_i>$.

$\alpha_m(\lambda_0) = 62\text{ cm}^{-1}$

$\alpha_m(\lambda_1) = 86\text{ cm}^{-1}$

The DFB laser has distributed gain and loss. These have both been taken into account by the calculations above. So, $F_1(\lambda_0) = F_1(\lambda_1) = 1/2$.

$$MSR = \left(\frac{24\text{ cm}^{-1}}{7.2 \times 10^{-3}\text{ cm}^{-1}}\right)(1)\frac{62\text{ cm}^{-1}}{86\text{ cm}^{-1}} = 2404 = 34\text{ dB}$$

---

c)

$$\eta_{d1} = \eta_{d2} = F\eta_i \frac{\alpha_m}{<\alpha_i> + \alpha_m} = \frac{1}{2}(0.6)\frac{62\text{ cm}^{-1}}{72\text{ cm}^{-1}} = 0.26$$