Final

ECE 227B

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Name SOLUTIONS

(30 pts)

1. A cleaved-facet, 400 μm long, edge-emitting GaAs multiple-quantum-well (MQW) laser has an output wavelength of 860 nm, a threshold current of 7 mA, and mode and active region volumes of 200 and 5 μm^3, respectively. The effective spontaneous emission bandwidth is 110 nm. We assume internal and radiative efficiencies of unity and a threshold inversion factor, n_{sp}, equal to 1.3.

(a) What is the optical mode density?
(b) What is the spontaneous emission rate per unit energy per unit active volume in $(\text{s-eV-cm}^3)^{-1}$?
(c) What are the threshold modal and material gains for the TE mode?
(d) What is the spontaneous emission factor, β_{sp}?
(e) What is the internal modal loss?

For a current of 14 mA the laser linewidth is measured to be 3 MHz with no feedback:
(f) What is the linewidth enhancement factor?
(g) What linewidth would be expected at a bias of 21 mA?
(h) If the worst possible level of feedback is now applied, what would be the maximum linewidth the laser might have?

1a. $$\rho_0(\nu) = \frac{8\pi}{c^{\cancel{3}}} n^2 n_g \frac{\cancel{\nu^2}}{\lambda^2} = \frac{8\pi}{3\times10^{10}\,cm/s} \frac{(3.5)^2(4)}{(0.86\mu m \times 10^{-4}\,cm/\mu m)^2}$$

$$\boxed{\rho_0(\nu) = 5.55\ s/cm^3}$$

1b. $$R_{sp} = R_{sp}^{21}(\nu_0)\,\Delta\nu_{sp} = R_{sp}^{21}(E)\,\Delta E_{sp}$$

$$R_{sp}^{21}(E) = \frac{R_{sp}}{\Delta E_{sp}} = \frac{\frac{\eta_i \eta_r I}{qV}}{\Delta E_{sp}} = \frac{\frac{(1)\ 7\times10^{-3}\,C/s}{1.6\times10^{-19}\,C \cdot 5\times10^{-12}\,cm^3}}{1.24\,eV\mu m\,\frac{0.11\mu m}{(0.86\mu m)^2}}$$

$$\Delta E_{sp} = 1.24\,\frac{\Delta\lambda_{sp}}{\lambda^2}$$

$$\boxed{R_{sp}^{21}(E) = 4.74\times10^{28}\ (s\text{-}eV\text{-}cm^3)^{-1}}$$

1c. $$\bar{g}_{21} = \frac{h R_{sp}^{21}(E)}{\rho_0(\nu)\, v_g\, n_{sp}} = \frac{6.626\times10^{-34}\,J\cdot s\,(4.74\times10^{28}/s\cdot eV\cdot cm^3)}{(5.55\,s/cm^3)(\tfrac{3}{4}\times10^{10}\,cm/s)(1.3)(1.6\times10^{-19}\,J/eV)}$$

$$\bar{g}_{21} = 3{,}631.\ cm^{-1}$$

$$\bar{g}_{21} = \tfrac{1}{3}\left(2g_{21}^{TE} + g_{21}^{TM}\right)$$

@ $k_t \sim 0$ only CB-HH,

$g_{21}^{TM} = 0$

$\therefore\ \bar{g}_{21} = \tfrac{2}{3}\, g_{21}^{TE}$

$$\boxed{g_{21}^{TE} = \tfrac{3}{2}(3631\,cm^{-1}) = 5{,}446.\ cm^{-1} = g_{th}}$$

$\Gamma = \frac{5}{200}$; $\therefore\ \boxed{\Gamma g_{th}^{TE} = 136.2\ cm^{-1}}$

1d. $\frac{R_{sp}'}{R_{sp}} = \beta_{sp} = \frac{\Gamma g\, v_g\, n_{sp}}{\eta_i \eta_r I/q} = \frac{(136.2\,cm^{-1})\,3/4\times10^{10}\,cm/s\,(1.3)\,(1.6\times10^{-19}C)}{(1)\;7\times10^{-3}\,C/s}$

$$\boxed{\beta_{sp} = 3.03\times10^{-5}}$$

1e. $\langle\alpha_i\rangle = \Gamma g_{th} - \alpha_m = 136.2\,cm^{-1} - \underbrace{\frac{1}{0.04\,cm}\ln\frac{1}{0.32}}^{28.5}$

$$\boxed{\langle\alpha_i\rangle = 107.7\,cm^{-1}}$$

Now: $I = 14\,mA$; $\Delta\nu = 3\,MHz$; $f_R = 3\,GHz$

1f.

$\Delta\nu = \frac{\Gamma R_{sp}'}{4\pi N_p}(1+\alpha^2)$; $R_{sp}' = \frac{\Gamma g\, v_g\, n_{sp}}{V}$

$\alpha^2 = \frac{4\pi N_p \Delta\nu}{\Gamma \underbrace{R_{sp}'}_{\frac{\Gamma g v_g n_{sp}}{V}}} - 1 = \frac{4\pi \overbrace{\cancel{P_o}}^{(\frac{h\nu}{q})\cancel{\eta}(\Delta I)} (\Delta\nu)}{(\Gamma v_g g_{th})^2 \cancel{h\nu}\, n_{sp} \cancel{\eta_d}} - 1$

$P_o = \frac{h\nu}{q}\eta_d(I - I_{th}) = \frac{1.24\,eV\cdot\mu m}{0.86\,\mu m}\left(\frac{28.5}{136.2}\right)(7\times10^{-3}\,A) = \underline{2.11\,mW}$

$\alpha^2 = -1 + \underbrace{\frac{4\pi(7\times10^{-3}\,C/s)(3\times10^6/s)}{(136.2\,cm^{-1}\times 3/4\times10^{10}\,cm/s)^2\;1.3\;(1.6\times10^{-19}C)}}_{1.216} = \underline{0.216}$

$$\boxed{\alpha = 0.465}$$

1g. $\Delta\nu \propto \frac{1}{P_0}$; $\boxed{\Delta\nu\big|_{21mA} = 1.5\,MHz}$

1h. $\Delta\nu\big|_{max} = f_R\sqrt{1+\alpha^2}\sqrt{\ln 4} = 3GHz\sqrt{1.216}\,\underbrace{\sqrt{\ln 4}}_{1.177}$

$$\boxed{\Delta\nu\big|_{max} = 3.9\,GHz}$$

(50pts)

2. A 500 μm long, cleaved-facet, edge-emitting MQW InGaAsP/InP laser chip is obtained. It is known that the gain characteristics of the quantum-wells are as given by Fig. 4.25, and four QWs are included in this device. Most of the threshold current is assumed to be due to Auger recombination, internal losses are known to be 10 cm^{-1}, and the confinement factor is 5%. Direct current modulation measurements over a range of frequencies indicate a K-factor and damping factor offset of 0.3 ns and 0.4/ns, respectively. These measurements also indicate an FM/IM modulation index ratio, M/m, that initially decreases linearly and then saturates at a value of 3 as the modulation frequency is increased.

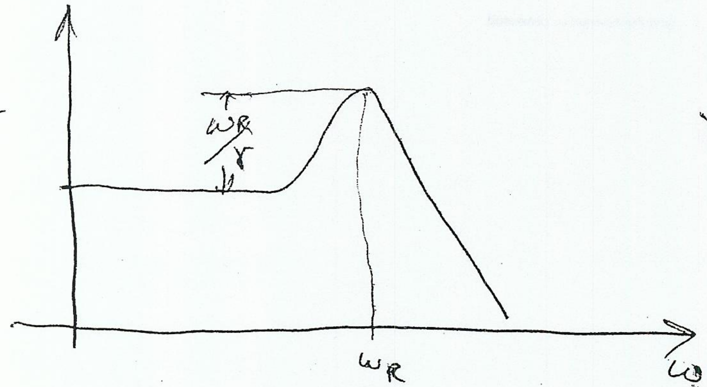
The laser is now biased such that the output power (both facets) is 20 mW and the relaxation resonance is at 8 GHz.

(a) In the small signal frequency response, P_1/I_1, what is the magnitude of the resonance peak relative to the low frequency value?
(b) What is the gain compression factor, ε?
(c) What is the linewidth enhancement factor?
(d) What is M/m at 300 MHz?
(e) What is the laser linewidth?
(f) What is the laser RIN/Δf at 8GHz?
(g) What is the maximum possible 3 dB modulation bandwidth, if it is biased higher and no parasitics are included?

If a small step in current is suddenly added on top of the original bias to increase the steady-state output to 25mW:

(h) By how much would the output power overshoot,
(i) What time would it take to settle down to within 0.5 mW of its steady-state value?
(j) Make a plot of the output power vs. time for this situation—i.e., stepping from 20mW to 25mW
(k) Make a plot of the chirp vs. time for the output shown in (j). Label the maximum chirp excursion value.

7a



$$\gamma = K f_R^2 + \gamma_0$$

$$\omega_R = 2\pi \cdot 8\,\text{GHz}$$

~~2a~~

$$\gamma = (0.3\times10^{-9}\,s)(8\times10^{9}/s)^2 + 0.4\times10^{9}/s = \underline{1.96\times10^{10}/s}$$

$$\boxed{\frac{\omega_R}{\gamma} = \frac{2\pi\cdot 8\times10^{9}/s}{1.96\times10^{10}/s} = 2.56} = \frac{P_1/I_1(\omega_R)}{P_1/I_1(0)}$$

2b. $\langle\alpha_i\rangle = 10\,cm^{-1}$; $\Gamma = 0.05$; $\Gamma g_{th} = 10\,cm^{-1} + \frac{1}{0.05\,cm}\ln\frac{1}{0.32} = \underline{32.8\,cm^{-1}}$

$$g_{th} = 32.8/0.05 = \underline{656\,cm^{-1}}$$

$$J_{th}/\text{well} = 81\,A/cm^2\, e^{656/503} = \underline{249.5\,A/cm^2} \Rightarrow 998\,A/cm^2 \text{ for 4 wells}$$

$$K = 4\pi^2\tau_p\left[1 + \frac{\Gamma a_p}{a}\right] \;;\; \frac{\Gamma a_p}{a} = \frac{\frac{\Gamma\epsilon g}{\cancel{1+\epsilon N_p}}}{\frac{g}{\cancel{1+\epsilon N_p}}} = \underline{\frac{\Gamma g_m}{a}\epsilon}$$

$$\tau_p = \frac{1}{\Gamma g_m v_g} = \frac{1}{3/4\times10^{10}\,cm/s\,(32.8\,cm^{-1})}$$

$$\underline{\tau_p = 4.07\,ps}$$

$$\frac{\Gamma a_p}{a} = \frac{\Gamma g_{th}}{a}\epsilon = \frac{K}{4\pi^2\tau_p} - 1 = \frac{\overbrace{0.3\times10^{-9}\,s}^{1.867}}{4\pi^2\,4.07\times10^{-12}\,s} - 1 = \underline{0.867}$$

NEED $a = g_{0N}/N$

⇒ CRUDE ESTIMATE — ASSUME ALL AUGER ; $J_A = q\,d\,C N^3$

$$N_{th} = \left(\frac{J_A}{q\,d\,C}\right)^{1/3} = \left(\frac{249.5\,A/cm^2 \text{ per well}}{(1.6\times10^{-19})(\underset{\uparrow\ 1\text{ well}}{7\times10^{-7}\,cm})(6\times10^{-29}\,cm^6/s)}\right)^{1/3} = \underline{3.34\times10^{18}\,cm^{-3}}$$

2b cont'd Crude estimate of a cont'd : $J \propto N^3$

$g_{0n} = 3 g_{0J} \ln \frac{J_{th}}{J_{tr}}\left(\frac{N_{t}}{N_{tr}}\right)^3 \quad g_{0N} \ln \frac{N_{th}}{N_{tr}} \; ; \; g_{0N} \approx 3 g_{0J} = 3.583\ cm^{-1}$

$g_{0N} \approx 1749\ cm^{-1}$

$\therefore \underline{a} \approx \frac{1749\ cm^{-1}}{3.34 \times 10^{18}\ cm^{-3}} = \underline{\underline{524 \times 10^{-18}\ cm^2}}$

→ Estimate #2 — use chart for $d = 60Å$ & multiply g_{0N} by $\frac{6}{7}$

Table 4.4 g_{0N} for unstrained 60Å InGaAs $= 1800\ cm^{-1}$

$\therefore g_{0N}'(70Å\ well) = 1800 \, 6/7 = \underline{\underline{1543\ cm^{-1}}}$

use N_{th} for only Auger portion — from Fig 4.25 @ $g = 656\ cm^{-1}$

$J_{sp} = 40\ A/cm^2 \; ; \; \therefore J_A = 210\ A/cm^2 = q d C N_{th}^3$

Auger only

$\therefore N_{tn}' = \left(\frac{210}{250}\right)^{1/3} 3.34 \times 10^{18}\ cm^{-3} = \underline{\underline{3.15 \times 10^{18}\ cm^{-3}}}$ correct value

$\therefore a' = \frac{1543\ cm^{-1}}{3.15 \times 10^{18}\ cm^{-3}} = 490. \times 10^{-18}\ cm^2$

→ Estimate #3

Use correct value of N_{th} & solve $J_T = J_{rad} + J_A = J_{tr_r} e^{g/g_{0r}} + J_{tr_A} e^{g/g_{0A}}$

for several values of gain — $g = 0$, g_{th}, others from F. 4.25.

Then set $g_{0N}'' = 3 \cdot g_{0J}''$ $= g_{0J}''$

I will use Estimate #2 as the best value → correct N_{th}
→ calc. g_{0N} for similar well corrected for d.

$\epsilon = \frac{a}{\Gamma g_{th}}(0.867) = \frac{490 \times 10^{-18}\ cm^2}{32.8\ cm^{-1}}(0.867)$

$\boxed{\epsilon = 1.29 \times 10^{-17}\ cm^3} \Rightarrow 1.38$ for estimate #1

2c.

$$\frac{M}{m} = \frac{\alpha}{2}\sqrt{\left(\frac{\gamma_{pp}}{\omega}\right)^2 + 1} \implies 3 \text{ for } \omega \text{ large}$$

$$\therefore \boxed{\alpha = 6}$$

2d.

$$\gamma_{pp} = \Gamma v_g a_p N_p = \frac{\Gamma a_p}{a}\frac{(2\pi)^2 f_R^2 \cancel{\tau_p}}{\cancel{v_g(\alpha_i + \alpha_m)}}$$

$$\gamma_{pp} = (0.867)\,\frac{(2\pi)^2 (8\times10^9/s)^2\ 4.07\times10^{-12}\,s}{1} = 8.92\times10^9/s$$

@300MHz

$$\left(\frac{\gamma_p}{\omega}\right)^2 = \left(\frac{\gamma_p\ 8.92\times10^9/s}{2\pi\cdot 300\times10^6/s}\right)^2 = (4.73)^2 = 22.37$$

$$\boxed{\left.\frac{M}{m}\right|_{300\,MHz} = 3\sqrt{23.37} = 14.5}$$

2e.

$$\Delta\nu = \frac{\Gamma R_{sp}'}{4\pi N_p}(1+\alpha^2) = \frac{(\Gamma v_g g_{th})^2\, \eta_0}{4\pi P_0}\, n_{sp}\, h\nu\,(1+\alpha^2)$$

assume 1.5 from Fig 4.11

$$\Delta\nu = \frac{\left[32.8\,cm^{-1}\left(\tfrac{3}{4}\times10^{10}\,cm/s\right)\right]^2_{/s}\left(\frac{22.8}{32.8}\right)1.5\,(0.8\,eV)\,1.6\times10^{-19}\,J/eV\,(37)}{4\pi\,(20\times10^{-3}\,J/s)}$$

$$\boxed{\Delta\nu = 1.19\text{ MHz}}$$

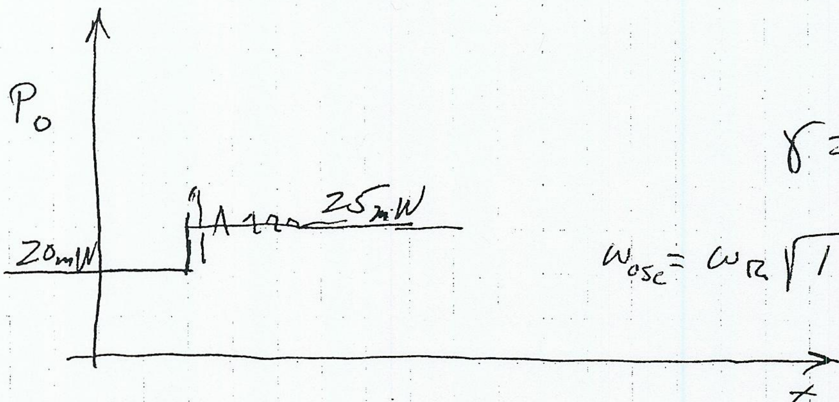
2f.

$$\left.\frac{RIN}{\Delta f}\right|_{f_R} = \frac{16\pi(\Delta\nu)_{ST}}{\gamma^2} = \frac{16\pi\,(1.19\times10^6/s)\,\frac{1}{37}}{(1.96\times10^{10}/s)^2}$$

$$\boxed{\left.\frac{RIN}{\Delta f}\right|_{f_R} = 4.21\times10^{-15}/Hz \quad \text{or} \quad -143.8\,dB/Hz}$$

2g. $\boxed{f_{3dB}\big|_{max} = \sqrt{2}\,\frac{2\pi}{\underset{0.3\times10^{-9}s}{K}} = \underline{\underline{29.6\,GHz}}}$

2h.



$\gamma = 1.96\times10^{10}/s$ @ 8GHz

$$\omega_{osc} = \omega_R\sqrt{1-\left(\frac{\gamma}{2\omega_R}\right)^2}$$

$$\frac{\Delta P(t)}{\cancel{dN_p(t)}} = \frac{\Delta P(\infty)}{\cancel{dN_p(\infty)}}\left[1 - e^{-\gamma t/2}\cos\omega_{osc}t - \underbrace{\frac{\gamma}{2\omega_{osc}}e^{-\gamma t/2}\sin\omega_{osc}t}_{\text{correction for high damping}}\right]$$

$$\Delta N(t) = \Delta N_0 e^{-\gamma t/2}\sin\omega_{osc} + \underbrace{\frac{\cancel{\gamma_{NP}}}{\cancel{\gamma_{PN}}}\left(\frac{q_p}{a}\right)\Delta N_p(t)}_{\text{correction}}$$

could use $\Delta\nu(t) = \frac{\alpha}{4\pi}\Gamma v_g a\Delta N(t)$ (5.75)

or better??

$\Delta\nu(t) = \frac{\alpha}{4\pi}\frac{1}{P_0(t)}\frac{dP_0(t)}{dt}$ (5.97)

→ use γ @ 25mW, not 20 because after first rise we are at this photon density, etc & have this ω_{osc} & γ.
($f_R^2 \propto P_0$)

$$\therefore\ \gamma(25) = K\cdot(8GHz)^2\frac{25}{20} + \gamma_0 = (0.3\times10^{-9}s)(8\times10^9/s)^2\frac{25}{20} + 0.4\times10^9/s$$

$$\gamma(25) = \underline{2.44\times10^{10}/s}$$

$$\omega_{osc}(25) = 2\cdot\pi\,8\times10^9\sqrt{\frac{25}{20}}\sqrt{1-\left(\frac{2.44\times10^{10}/s}{2\cdot2\cdot\pi\cdot8\times10^9\cdot\sqrt{25/20}}\right)^2} = \underline{\underline{5.49\times10^{10}/s}}$$

2k:
$$\Delta\nu(t) = \frac{\alpha}{4\pi}\frac{1}{P_0(t)}\frac{dP_0(t)}{dt}$$

$$P_0(t) = 20\,mW + 5\,mW\left[1 - e^{-\frac{\gamma}{2}t}\cos\omega_{osc}t - \cancel{small}\right]$$

$$\frac{dP_0(t)}{dt} = +5\,mW\left(\frac{\gamma}{2}\right)e^{-\gamma/2\,t}\cos\omega_{osc}t - 5\,mW\,e^{-\gamma/2\,t}(-\omega_{osc})\sin\omega_{osc}t$$

$$= 5\,mW\,e^{-\gamma/2\,t}\left[\gamma/2\cos\omega_{osc}t + \omega_{osc}\sin\omega_{osc}t\right]$$

$$\Delta\nu(t) = \frac{\alpha}{4\pi}\,\frac{5\,mW\,e^{-\gamma/2\,t}\left[\gamma/2\cos\omega_{osc}t + \omega_{osc}\sin\omega_{osc}t\right]}{20\,mW + 5\,mW\left[1 - e^{-\gamma/2\,t}\cos\omega_{osc}t\right]}$$

t (ps)	$\Delta\nu(t)$	$\omega_{osc}t$	$e^{-\gamma/2\,t}$
0	$1.46\times10^9/s$	0	1
28.6	$3.9\times10^9/s$	$\pi/2$	~.75
57.2	$-5.3\times10^9/s$	π	.5
85.8	$-1.8\times10^9/s$	$3\pi/2$	.35
114.4	$2.6\times10^8/s$	2π	.22
143		2.5π	.17

$\alpha = 6$
$\gamma = 2.44\times10^{10}/s$
$\omega_{osc} = 5.49\times10^{10}/s$
) at 25 mW

$t=0$
$$\Delta\nu(0) = \frac{6}{4\pi}\,\frac{5\,mW(1)\left[1.22\times10^{10}/s(+1) + 0\right]}{\cancel{20 + 5[(1 - 1(1))]\,mW}\ \cancel{30\,mW}\ 20}$$

$$= \cancel{1.9.7\times10^8/s} \times \frac{30}{20} = \underline{1.455\times10^9/s}$$

2h cal

$t = 28.6$

$$\delta\nu(\pi/2) = \frac{6}{4\pi} \frac{5mW(.75)\left[\cancel{1.22\times10^{10}/s(0)} + 5.49\times10^{10}/s(1)\right]}{\cancel{20+5\left[1-.75(0)\right]}}$$

$25\,mW$

$= 3.9\times10^{9}/s$

$$\Delta\nu(\pi) = \frac{6}{4\pi} \frac{5mW(.5)\left[1.22\times10^{10}/s(-1) + 0\right]}{20mW + 5m\left[1-(.5)(-1)\right]}$$

1.5

$= -5.3\times10^{8}/s$ 27.5 $-5.30\times10^{8}/s$

$$\delta\nu(3\pi/2) = \frac{6}{4\pi} \frac{5mW(.35)\left[0 + 5.49\times10^{10}/s(-1)\right]}{\cancel{20}+5\left[1-.35(0)\right]}$$

$25\,mW$

$\Delta\delta(3\pi/2) = -1.83\times10^{9}/s$

$$\Delta\nu(2\pi) = \frac{6}{4\pi} \frac{5mW(.22)\left[1.22\times10^{10}/s(+1) + 0\right]}{20 + 5\left[1-(.22)(+1)\right]}$$

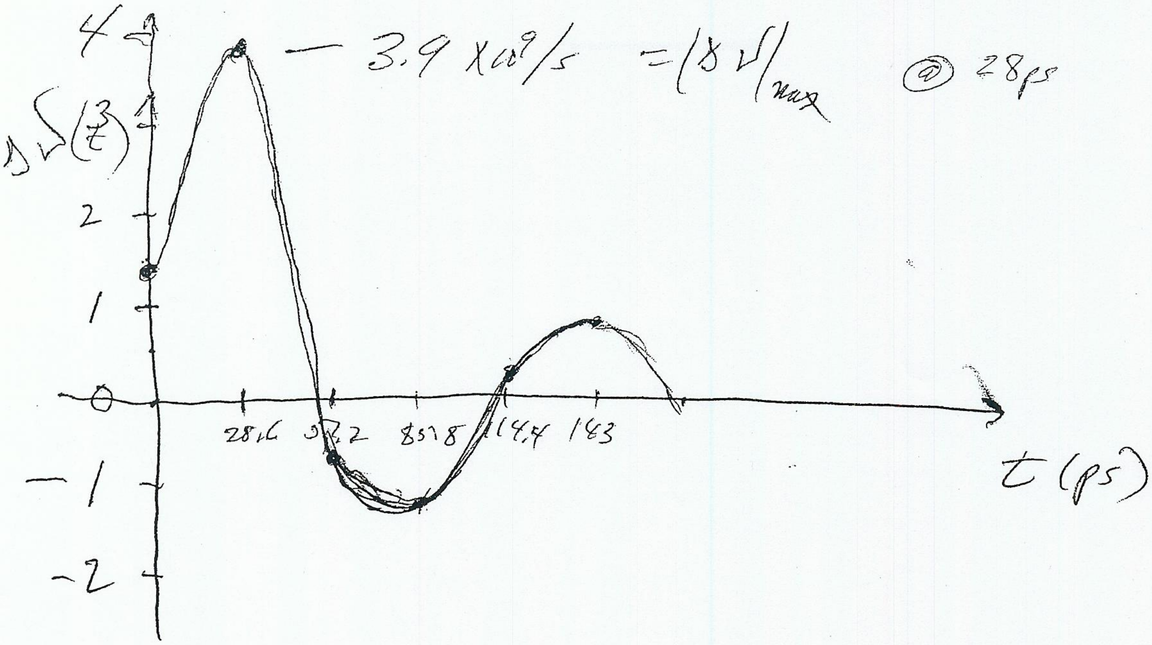
23.9

$= 2.63\times10^{8}/s$

$$\delta\nu(2.5\pi) = \frac{6}{4\pi} \frac{5mW(.17)\left[\cancel{1.22\times10^{10}(0)} + 5.49\times10^{10}/s(1)\right]}{\cancel{20+5\left[1-.17(0)\right]}}$$

25

$\Delta\nu = 8.9\times10^{8}/s$



→ probably easier to use approx $\Delta N(t)$

— should be roughly in quadrature

(20pts)

3. Another wafer of the same material as in problem #2 is now grown, but a large undoped SCH region is included, and this acts as a second carrier reservoir. The time constant for the transfer of carriers from the SCH to the MQW active is found to be 80 ps, while the loss of carriers from the active back to the SCH has a time constant of 240 ps. The modulation properties of these lasers are not as good as the previous batch, although the injection efficiency is almost the same. If biased at the same value as for 20 mW output in Prob. #2,

(a) What is the relaxation resonance frequency in this case?
(b) What is the new K-factor and maximum possible 3 dB modulation bandwidth?
(c) By how much would P_1/I_1 vs. frequency be reduced at 1 GHz relative to Prob. #2?
(d) Same question as (c) for the relaxation resonance frequency?
(e) What is the RIN/Δf at the relaxation resonance frequency?

3ai: $\omega_{tR}^2 = \omega_R^2/\chi$

$\tau_s = 80ps$; $\tau_e = 240ps$

$\chi \approx 1 + \tau_s/\tau_e = 1 + \frac{80}{240} = 1.333$

$f_{tR}^2 = \frac{(8\,GHz)^2}{1.333} = 48.0$

$$\boxed{f_{tR} = 6.93\,GHz}$$

3b:

0.867 × 1.333

$$\boxed{K_t = 4\pi^2\tau_p\left[1 + \frac{a_p}{a/\chi}\right] = \cancel{K}\left(\frac{\overbrace{1+.867(1.333)}^{1.1546}}{1.867}\right) = \underline{0.346ns}}$$

0.3ns

$$\boxed{f_{3dB}\Big|_{max} = \sqrt{2}\,\frac{2\pi}{0.346ns} = 2.56\times10^{10}/s = \underline{25.6\,GHz}}$$

3c: @ 1GHz add the low freq cutt-off

$$\left|\frac{P_1}{I_1}\right|_t = \left|\frac{1}{1 + j\omega\tau_s}\right|\left|\frac{P_1}{I_1}\right|$$

$2\pi 10^9/s$ 80ps

$$\frac{\left|\frac{P_1}{I_1}\right|_t}{\left|\frac{P_1}{I_1}\right|} = \frac{1}{\sqrt{1+\omega^2\tau_s^2}} = \frac{1}{\underbrace{\sqrt{1+(2\pi 10^9 \cdot 80\times10^{-12})^2}}_{1.119}}$$

= 89.3%

3(d): ⓐ f_{R_t} peak reduced by $\left(\frac{\omega_{Rt}}{\gamma_t}\right) / \left(\frac{\omega_R}{\gamma}\right) \times \frac{1}{\sqrt{1+\omega_{R_t}^2\tau_S^2}}$

$$\gamma_t = \frac{\gamma\,(1.1546)}{1.333} = 1.96\times10^{10}\,\frac{1.1546}{1.333} = \underline{1.697\times10^{10}/s}$$

$$\frac{\left(\frac{\omega_{Rt}}{\gamma_t}\right)}{\frac{\omega_R}{\gamma}\sqrt{1+\omega_{R_t}^2\tau_S^2}} = \frac{\overset{2.56}{\left(\frac{2\pi\cdot 6.93\,GHz}{1.697\times10^{10}}\right)}}{\cancel{2.56}\,\underbrace{\sqrt{1+(2\pi\cdot 6.93\times10^9\cdot 80\times10^{-12}s)^2}}_{3.62}}$$

$$=27.5\%$$

3(e):

$$\frac{RIN}{\Delta f} = \frac{16\pi\,(\Delta\nu)_{ST}}{\gamma_t^2} = 4.21\times10^{-15}/Hz\left(\frac{1.333}{1.1546}\right)^2$$

$$= \underline{5.61\times10^{-15}/Hz} \Rightarrow -142.5\,dB/Hz$$