Final

ECE 227B

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Name Solutions

(35 pts)

1. A 1300 nm wavelength, cleaved facet, edge-emitting, strained-layer quantum-well laser, which has a threshold current of 10 mA, has active and modal volumes of 20 and 400 μm^3, respectively. The injection efficiency is 0.7; the internal modal loss is 20 cm^{-1}; the radiative efficiency is 0.5; the fully inverted gain (or unpumped electronic absorption) coefficient is 7000 cm^{-1}; and electron and hole effective masses are 0.06 m_0 and 0.20 m_0, respectively. The effective spontaneous emission bandwidth [integral of SE spectrum/$R_{sp}^{21}(\lambda_0)$] is measured to be 200 nm.

(a) What is the total spontaneous emission rate per unit active volume above threshold?

(b) What is the spontaneous emission rate per unit energy (in eV) per unit active volume above threshold near the lasing mode?

~~(c) What is the spontaneous emission rate into the lasing mode?~~

(d) What are the threshold modal and material gains?

(e) What are the values of the individual Fermi functions, f_1 & f_2, and the population inversion factor, n_{sp}?

(f) What is the laser length and differential efficiency?

(g) What is the spontaneous emission factor, β_{sp}?

① (a) $\boxed{R_{sp}} = \dfrac{\eta_i \eta_r I}{qV} = \dfrac{0.7\,(0.5)(10\times10^{-3}\,A)}{1.6\times10^{-19}\,C\,(20\times10^{-12}\,cm^3)} = \underline{\underline{1.09\times10^{27}/s\cdot cm^3}}$

$R_{sp}^{21}(\lambda_0) = \dfrac{R_{sp}}{200\,nm} = \dfrac{1.09\times10^{27}/s\cdot cm^3}{200\,nm} = 5.47\times10^{24}/s\cdot cm^3\cdot nm$

(b) $\boxed{R_{sp}^{21}(E) = \dfrac{R_{sp}}{\Delta E_{sp}} = \dfrac{1.09\times10^{27}/s\cdot cm^3}{1.24\,eV\cdot\mu m\left(\dfrac{0.2\,\mu m}{(1.3\,\mu m)^2}\right)} = 7.45\times10^{27}/s\cdot cm^3\cdot eV}$

$\rho_r(\nu) = \dfrac{8\pi n^2 n_g}{c\lambda^2} = \dfrac{8\pi}{3\times10^{10}\,cm/s}\,\dfrac{(3.4)^2(4)}{(1.3\times10^{-4}\,cm)^2} = \underline{2.29\,s/cm^3}$

(4.56) $\bar{g}_{21} = \dfrac{h\,R_{sp}^{21}(E)}{\rho(\nu)\,v_g\,n_{sp}}$; but need to guess @ n_{sp} & check later for consistency. Let $n_{sp} = 1.3$

$$\bar{g}_{21} = \frac{(6.62\times10^{-34}\,J\cdot s)\,(7.45\times10^{27}/s\cdot cm^3\cdot eV)}{(2.29\,s/cm^3)\left(\tfrac{3}{4}\times10^{10}\,cm/s\right)(1.3)\,1.6\times10^{-19}\,J/eV}$$

$\underline{\bar{g}_{21} = 1379\ cm^{-1}}$; now $g_{21}^{TM} = 0$

$\bar{g}_{21} = \tfrac{1}{3}\left(2g_{21}^{TE} + g_{21}^{TM}\right) = \tfrac{2}{3}g_{21}^{TE}$

$g_{21}^{TE} = \tfrac{3}{2}\bar{g}_{21} = \tfrac{3}{2}(1379\,cm^{-1}) = \underline{2069\,cm^{-1}} = g_{th}^{TE}$

$\Gamma = 1/20$; $\therefore\ \Gamma g_{th} = 103\ cm^{-1}$

TO be corrected w/ new n_{sp}

$$(f_2 - f_1) = \frac{g_{th}}{g_{max}} = \frac{2069}{7000} = 0.2956$$

$$f_1(E_{Fn_1}) = (1 - f_2(E_G))^{1/D} \quad ; \quad D = \frac{0.2}{0.06} = 3.33$$

$$f_2 = f_1 + 0.2956$$

$$f_1 = (0.7044 - f_1)^{0.30}$$

(e) Iterate $\Rightarrow \boxed{f_1 = 0.56 \;;\; f_2 = 0.86}$

$$n_{sp} = \frac{f_2(1-f_1)}{f_2 - f_1} = \frac{\overset{0.3784}{\cancel{0.86(1-0.56)}}}{0.86 - 0.56} = 1.26$$

close enough to 1.3 as used, but can correct to 1st order

(d) g_{th} should be a little larger, say $\frac{2069}{1.26}\, 1.3 = 2135\ cm^{-1}$

corrected

$$\Gamma g_{th} = 107\ cm^{-1} = \alpha_i + \alpha_m = 20 + \alpha_m$$

$$\alpha_m = 87\ cm^{-1} = \frac{1}{L} \ln \frac{1}{R}$$

(f) $L = \frac{1}{87\,cm^{-1}} \ln \frac{1}{0.32} = 131\ \mu m$

$$\eta_d = \eta_i \frac{\alpha_m}{\alpha_i + \alpha_m} = 0.7\ \frac{87}{107} = 0.569$$

$$\beta_{sp} = \frac{\Gamma v_g\, g\, n_{sp}}{\eta_i \eta_r I/q} = \frac{R_{sp}'}{R_{sp}}$$

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$$R_{sp}' = \frac{\Gamma_g v_g n_{sp}}{V} = \frac{(107\,cm^{-1})(3/4 \times 10^{+10}\,cm/s)(1.26)}{20 \times 10^{-12}\,cm^3}$$

(c) $$\boxed{R_{sp}' = 5.06 \times 10^{22} /s\cdot cm^3}$$

(g) $$\beta_{sp} = \frac{5.06 \times 10^{22}}{1.09 \times 10^{27}} = \boxed{4.64 \times 10^{-5}}$$

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(45 pts)

2. Etched mesa GaAs-based VCSELs with active regions consisting of 3-8nm thick $In_{0.2}Ga_{0.8}As$ strained-layer quantum-wells placed at an E^2 standing wave maximum are formed. For a deeply-etched 3 μm diameter mesa, the threshold current is 300 μA, due in part to surface recombination. Using the effective mirror model, the lossless mean mirror power reflectivity is 0.994, the one pass modal power loss is 0.5%, and the effective cavity length is 1.3 μm. The enhancement in the confinement factor is assumed to be 1.8, and the injection efficiency is 0.9.

Small-signal rf modulation with bias currents of 1.0, 2.0 and 4.0 mA above threshold show relaxation resonance frequencies of 8, 11.3 and 16 GHz, and damping factors of 2.1, 4.1 and 8.1 x 10^{10}/s at the three respective biases. Measurements of the FM/IM ratio, M/m, further show a $1/f_m$ dependence at low frequencies and a saturation to a common value of 2.5 at high frequencies. At 50 MHz, M/m = 80, 160, and 320 at the three respectively biases. Neglect heating.

From these measurements:

(a) What is the differential gain, a?

(b) What is the differential gain compression, a_p?

(c) What is the linewidth enhancement factor?

(d) What is the K-factor and damping factor offset, γ_0?

(e) Compute the cavity lifetime from the K-factor and compare to that obtained from the cavity losses?

(f) What is the linewidth at the three biases?

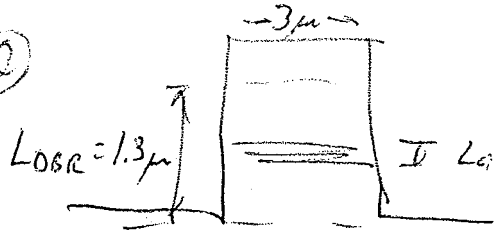
(g) At what critical level of external feedback might we expect coherence collapse at the three biases, assuming all transmission is at one output mirror?

(h) What maximum linewidths would one expect under coherence collapse?

(i) What are the $RIN/\Delta f$ values at the resonance peak for all three biases?

(j) What are the $RIN/\Delta f$ values if we can assume bias conditions to give the standard quantum limit at these biases?

(2)



$I_{th} = 0.3\ \text{mA}$

$\sqrt{R_1 R_2} = R = 0.994$

$$\alpha_i L_{DBR} = 0.005$$

$$\alpha_i = \frac{0.005}{1.3\times10^{-4}\,\text{cm}} = 38.5\ \text{cm}^{-1}$$

$$\alpha_m = \frac{1}{1.3\times10^{-4}} \ln\frac{1}{0.994} = 46.3\ \text{cm}^{-1}$$

$$\Gamma g_{th} = 38.5\ \text{cm}^{-1} + 46.3\ \text{cm}^{-1} = \underline{84.8\ \text{cm}^{-1}}$$

$$\Gamma = 1.8\left(\frac{24\ \text{nm}}{1300\ \text{nm}}\right) = \underline{0.0332}$$

$$\therefore\ g_{th} = 2554.\ \text{cm}^{-1}$$

$I - I_{th}$	f_R	γ
1.0 mA	8 GHz	$2.1\times10^{+10}$ /s
2.0	11.3	4.1
4.0	16	8.1

$$a = \frac{(2\pi f_R)^2\, q V}{\Gamma v_g \eta_i (I - I_{th})}$$

Use $I - I_{th} = 1\,mA$

$$V = 1.697 \times 10^{-13}\,cm^3$$

$$a = \frac{(2\pi 8 \times 10^{9}/s)^2 (1.6 \times 10^{-19}\,C) \overbrace{(3 \times 8 \times 10^{-3}\,\mu m \times \pi \times 1.5^2\,\mu m)}^{V = 1.697\times10^{-13}\,cm^3} \times 10^{-12}\,cm^2/\mu m^2}{(0.0332)(3/4 \times 10^{10}\,cm/s)(0.9)(1 \times 10^{-3}\,A)}$$

$$\boxed{a = 3.06 \times 10^{-16}\,cm^2}$$

(c) $\boxed{\alpha = 5}$, from $\frac{M}{m} \to \alpha/2 = 2.5$ @ high f

(b) $\frac{M}{m} = \frac{\alpha}{2}\sqrt{\left(\frac{\omega_{R}}{\omega}\right)^2 + 1}$ (5.78)

@ 50 MHz can neglect 1 if heating is also neglected

$$\frac{M}{m} = \frac{\alpha}{2}\frac{\omega_R}{\omega} = \frac{\alpha}{2}\frac{\Gamma v_g a_p N_p}{\omega}$$

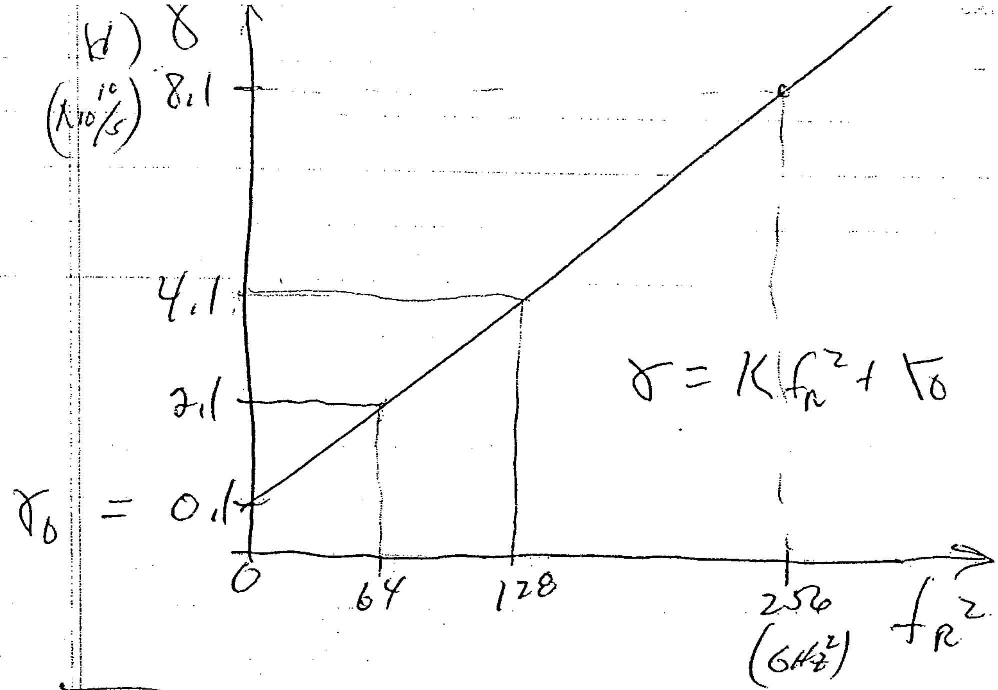
$$N_p = \frac{\eta_i (I - I_{th})}{q v_g g_{th} V} \qquad (2.32)$$

$$\therefore a_p = \frac{M}{m}\frac{2}{\alpha}\frac{q\,\omega\,g_{th}\,V}{\eta_i (I - I_{th})\,\Gamma}$$

$$a_p = 80\,\frac{2}{5}\,\frac{(1.6 \times 10^{-19}\,C)(2\pi \cdot 50 \times 10^{6}/s)(2.554\,cm^{-1})(1.697 \times 10^{-13}\,cm^3)}{0.9\,(1 \times 10^{-3}\,A)\,(0.0332)}$$

$$\boxed{a_p = 2.33 \times 10^{-14}\,cm^2}$$

(d)



$$\boxed{K = \frac{\Delta\gamma}{\Delta(f_R^2)} = \frac{8\times10^{10}/s}{256\times10^{18}/s^2} = 3.125\times10^{-10}\,s = 0.313\,ns}$$

$$\boxed{\gamma_0 = 1\times10^{9}/s}$$

FROM LOSS

(e) $$\boxed{\tau_p = \frac{1}{v_g(\cancel{\alpha_i+\alpha_m})} = 1.57\times10^{-12}\,s}$$

$(3/4\times10^{10}\,cm/s)\ 84.8\,cm^{-1}$

FROM K-factor

$$\tau_p = \frac{K}{4\pi^2\left[1+\frac{\Gamma a_p}{a}\right]} = \frac{3.125\times10^{-10}\,s}{4\pi^2\left[1+\frac{0.0332\cdot 2.33\times10^{-16}\,cm^2}{3.06\times10^{-16}\,cm^2}\right]}$$

for K

$$\tau_p' = 2.24\times10^{-12}\,s$$

$$\Delta\nu = \frac{\overbrace{\Gamma R_{sp}'}^{\omega r_{ST}}}{4\pi N_p}(1+\alpha^2)$$

$$\Delta\nu = \frac{\Gamma\left(\frac{\Gamma v_g g\, n_{sp}}{V}\right)(1+\alpha^2)}{4\pi\left(\frac{\eta_i (I-I_{th})}{q\, v_g\, g_{th} V}\right)} = \frac{\Gamma^2 v_g^2 g_{th}^2 n_{sp} q (1+\alpha^2)}{4\pi\, \eta_i (I-I_{th})}$$

$$\Delta\nu = \frac{(0.0332)^2(.75\times10^{10}\,\mathrm{cm/s})^2(2554\,\mathrm{cm}^{-1})^2(1.1)(1.6\times10^{-19}\mathrm{C})(1+25)}{4\pi\ (0.9)\ 10^{-3}A}$$

(f) $$\boxed{\begin{aligned} \Delta\nu\Big|_{\Delta I = 1mA} &= 164\ \mathrm{MHz} \\ \Delta\nu\Big|_{\Delta I = 2} &= 81.8\ \mathrm{MHz} \\ \Delta\nu\Big|_{\Delta I = 4} &= 40.9\ \mathrm{MHz} \end{aligned}}$$

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(g) $$f_{ext}\Big|_{crit} = \frac{\left(\frac{\beta Y}{v_g}\cancel{t_1}\right)^2 (2r_2)^2}{16\, t_2^4}\left(\cancel{K f_r^2 + \gamma_0}\right)^2\left(\frac{1+\alpha^2}{\alpha^4}\right)$$

γ^2

$r_1 r_2 = 0.994$; $r_1 = 1$, $r_2 = 0.994$

$t_2^2 = 0.01196$

1mA $$f_{ext}\Big|_{crit} = \frac{\left[2(1.3\times10^{-4}\,cm)\right]^2 \left(2(0.994)\right)^2}{\left[3/4\times10^{10}\,cm/s\right]^2 16\,(0.01196)^2}\left(2.1\times10^{10}/s\right)^2\left(\frac{26}{25^2}\right)$$

$f_{ext}\Big|_{crit}(1mA) = \underline{3.81\times10^{-5}} \Rightarrow \underline{-44.2\,dB}$

2mA $f_{ext}\Big|_{crit}(2mA) = 3.81\times10^{-5}\times\left(\frac{4.1}{2.1}\right)^2 = \underline{1.45\times10^{-4}}$
$\Rightarrow \underline{-38.4\,dB}$

4mA $f_{ext}\Big|_{crit}(4mA) = 3.81\times10^{-5}\times\left(\frac{8.1}{2.1}\right)^2 = \underline{5.67\times10^{-4}}$
$\Rightarrow \underline{-32.5\,dB}$

(h) $\Delta\nu\Big|_{max} = f_R\sqrt{1+\alpha^2}\sqrt{\ln 4}$

$$\boxed{\begin{aligned} \Delta\nu\Big|_{max} &= 8\,GHz\,\overbrace{\sqrt{26}\sqrt{\ln 4}}^{6.00} = 48.0\,GHz \ @\ 1mA \\ &\ 11.3\ (6) = 67.8\,GHz\ @\ 2mA \\ &\ 16\ (6) = 96\,GHz\ @\ 4mA \end{aligned}}$$

(i) $\left.\frac{RIN}{\Delta f}\right|_{\omega=\omega_e} = \frac{16\pi(\Delta\nu)_{ST}}{\gamma^2}$

(i) $I - I_{th} = 1\,mA$:

$(\Delta\nu)_{ST} = 164\,MHz/26 = \underline{6.3\,MHz}$

@ 2mA: $(\Delta\nu)_{ST} = 81.8\,MHz/26 = \underline{3.15\,MHz}$

@ 4mA: $(\Delta\nu)_{ST} = \underline{1.57\,MHz}$

$\left.\frac{RIN}{\Delta f}\right|_{\Delta I = 1mA} = \frac{16\pi\,(6.3\times10^6/s)}{(2.7\times10^{10}/s)^2} = 7.18\times10^{-13}/Hz \Rightarrow \underline{-121.4\,dB/Hz}$

$\left.\frac{RIN}{\Delta f}\right|_{\Delta I = 2mA} = 9.39\times10^{-14}/Hz \Rightarrow \underline{-130.2\,dB/Hz}$

$\left.\frac{RIN}{\Delta f}\right|_{\Delta I = 4} = 1.20\times10^{-14}/Hz \Rightarrow \underline{-139.2\,dB/Hz}$

(j) $\underline{SQL}$: $\frac{RIN}{\Delta f} = \frac{2h\nu}{P_0}$

$P_0(\Delta I = 1mA) = \frac{h\nu}{q}\eta_i\left(\frac{\alpha_m}{\alpha_i + \alpha_m}\right)(I - I_{th}) = \frac{1.24}{0.98}V(0.9)\frac{46.3}{84.5}(1mA)$

$= \underline{0.622\,mW}$

$P_0(2mA) = \underline{1.24\,mW}$

$P_0(4mA) = \underline{2.49\,mW}$

SQL (j) cal

$$\left.\frac{RIN}{\Delta f}\right|_{1\,mA} = \frac{2\left(\frac{1.24}{0.98}\right)eV\ 1.6\times10^{-19}\,J/eV}{0.622\times10^{-3}\,J/s} \cong 6.51\times10^{-16}/Hz$$

$\Rightarrow -151.9\,dB/Hz$

$$\left.\frac{RIN}{\Delta f}\right|_{2\,mA} = 3.27\times10^{-16}/Hz \Rightarrow -154.9\,dB/Hz$$

$$\left.\frac{RIN}{\Delta f}\right|_{4\,mA} = 1.63\times10^{-16}/Hz \Rightarrow -157.9\,dB/Hz$$

(20 pts)

3. The turn-on characteristics of the VCSEL of Prob. #2 are now explored. Assume only radiative recombination. The laser is initially biased with a current of 200 μA (100 μA below threshold). At time $t = 0$, the bias current is suddenly increased to 2 mA

(a) What is the turn-on delay, t_d, for the carrier density to first reach its steady-state value?

For delays larger than the turn-on delay of part (a), the output power will overshoot and oscillate back to some steady state value.

(b) Make a sketch of the output power vs. time, assuming all output is from one mirror?

(c) What is the oscillation period for $t > t_d$?

~~(d) How long will it take this oscillation to damp to 1/e of it's initial magnitude?~~

(e) Approximately how much peak overshoot in output power will be observed?

(3) for all radiative

$$t_d = \tau_f \left[\tanh^{-1}\sqrt{\frac{I_{th}}{I_f}} - \tanh^{-1}\sqrt{\frac{I_i}{I_f}} \right]$$

$I_f = 2\,mA$
$I_{th} = 0.3\,mA$
$I_i = 0.2\,mA$

$$\tau_f = \sqrt{\frac{qV}{\eta_i I_f B}}$$

Let $B = 0.8 \times 10^{-10}\ cm^3/s$

$$\tau_f = \sqrt{\frac{1.6 \times 10^{-19} C \cdot (1.697 \times 10^{-13}\ cm^3)}{(0.9)(2 \times 10^{-3} A)(0.8 \times 10^{-10}\ cm^3/s)}}$$

$$\tau_f = 4.34 \times 10^{-10}\ s$$

useful trig identity

$$\boxed{\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x}}$$

$$\tanh^{-1}\sqrt{\frac{\cancel{I_{th}}\ 0.3}{\cancel{I_f}\ 2}} = \tanh^{-1}(0.387) = \frac{1}{2}\ln\left(\frac{1.387}{0.6127}\right)$$

$$= 0.4085$$

$$\tanh^{-1}\sqrt{\frac{0.2}{2}} = \tanh^{-1}(0.316) = \frac{1}{2}\ln\left(\frac{1.316}{0.684}\right)$$

$$= 0.3274$$

$$\boxed{t_d = 0.434\,ns \underbrace{\left[0.4085 - 0.3274\right]}_{0.0811} = 35.2\,ps}$$

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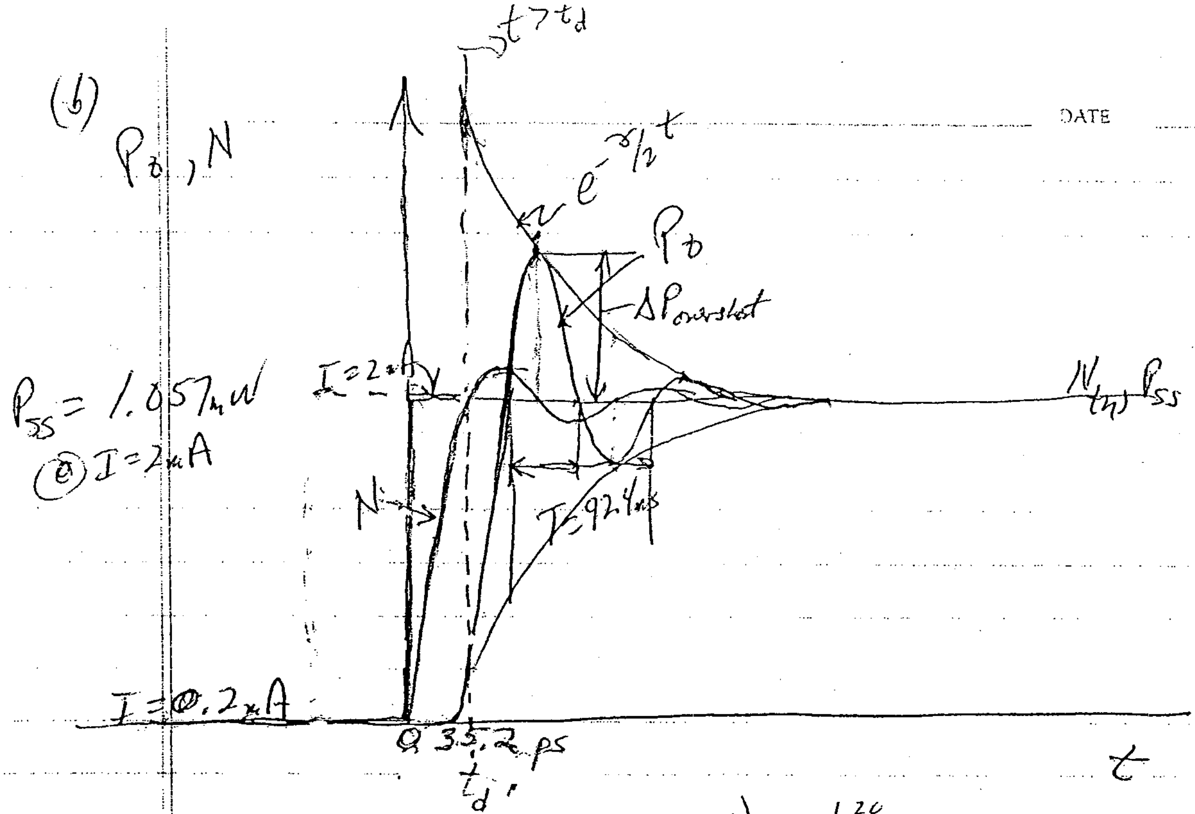
(b)



$$P_o(2mA) = \frac{h\nu}{q}\eta_i \frac{\alpha_m}{\alpha_i+\alpha_m}(I - I_{th}) = 0.9\left(\frac{1.24}{1.3}\right)\frac{46.3}{84.8}(2-0.3)mA = \underline{1.057\,mW}$$

$$\omega_{osc} = \omega_R\sqrt{1-\left(\frac{\gamma}{2\omega_R}\right)^2} = \omega_R\sqrt{1-0.08337}$$

$\omega_R = 11.3\times 2\pi \times 10^9/s$, $\gamma = 4.1\times10^{10}/s$, $\sqrt{1-0.08337} = 0.957$

$$f_{osc} = 0.957(11.3\,GHz) = 1.0819\times10^{10}/s$$

(c) osc. period $T = \frac{1}{f_{osc}} = \underline{9.24\times10^{-11}\,s} = \underline{92.4\,ps}$

(d) Damping rate $\equiv e^{-\frac{\gamma t}{2}}$; $\frac{\gamma}{2}t = 1$ @ $\frac{1}{e}$

$$\boxed{t_{decay} = \frac{2}{\gamma} = \frac{2}{4.1\times10^{10}/s} = \underline{4.88\times10^{-11}\,s} = 48.8\,ps}$$

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