4.1:

a) 5

From Table 1.1:

$$E_g(GaAs) = 1.424 \text{ eV}$$
$$E_g(AlGaAs) = 1.673 \text{ eV}$$

$$m_C^* = 0.067m_0$$
$$m_{HH}^* = 0.38m_0$$
$$m_{LH}^* = 0.09m_0$$

$$m_{rHH}^* = \frac{m_C^* m_{HH}^*}{m_C^* + m_{HH}^*} = 0.057m_0$$
$$m_{rLH}^* = \frac{m_C^* m_{LH}^*}{m_C^* + m_{LH}^*} = 0.038m_0 \quad (4.27)$$

Define an equivalent effective mass that can be used in the reduced density of states equation: $m_{rEQ}^* = \left(m_{rHH}^{*\,3/2} + m_{rLH}^{*\,3/2}\right)^{2/3} = 0.076\ m_0$

From Table 4.3, the reduced density of states is

$$\rho_r(E_{21}) = \frac{1}{2\pi^2}\left(\frac{2m_{rHH}^*}{\hbar^2}\right)^{3/2}(E_{21}-E_g)^{1/2} + \frac{1}{2\pi^2}\left(\frac{2m_{rLH}^*}{\hbar^2}\right)^{3/2}(E_{21}-E_g)^{1/2}$$
$$= \frac{1}{2\pi^2}\left(\frac{2m_{rEQ}^*}{\hbar^2}\right)^{3/2}(E_{21}-E_g)^{1/2}$$
$$= \frac{1}{2\pi^2}\left(\frac{2(0.076)(9.11\times10^{-31}\text{ kg})}{(1.054\times10^{-34}\text{ J s})^2}\right)^{3/2}(E_{21}-E_g)^{1/2}$$
$$= 2.25\times10^{54}\,(E_{21}-E_g)^{1/2}\ (\text{m}^{-3}\ \text{J}^{-1}) \qquad E_{21} \text{ and } E_g \text{ in J}$$
$$= 1.44\times10^{20}\,(E_{21}-E_g)^{1/2}\ (\text{cm}^{-3}\ \text{eV}^{-1}) \qquad E_{21} \text{ and } E_g \text{ in eV}$$

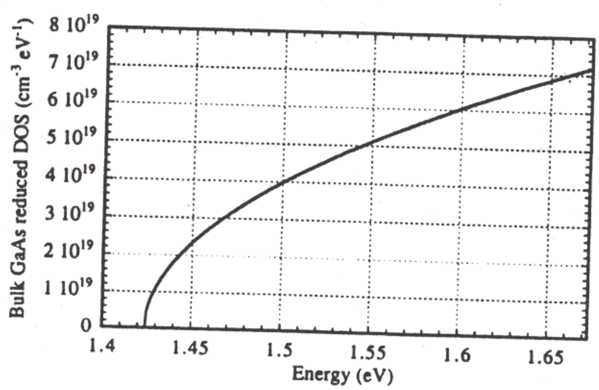

Figure 4.1a. Bulk GaAs reduced density of states

5 b) The primary differences between the quantum well and the bulk cases is that the energy levels are quantized, which leads to a step like behavior in the density of states function. First calculate the energy levels in the conduction and valence bands:

Conduction band :

$$E_1 = \frac{\pi^2\hbar^2}{2mL^2} = 3.76 \left(\frac{m_0}{m^*}\right)\left(\frac{100Å}{L}\right)^2 \text{ meV}$$
$$= \frac{3.76}{0.067} \text{ meV}$$
$$= 56.12 \text{ meV}$$

$$E_2 = 4E_1 = 224.5 \text{ meV}$$

Heavy hole valence band :

$$E_1 = \frac{3.76}{0.38} \text{ meV}$$
$$= 9.89 \text{ meV}$$

$$E_2 = 4E_1 = 39.57 \text{ meV}$$

Light hole valence band :

$$E_1 = \frac{3.76}{0.09} \text{ meV}$$
$$= 41.78 \text{ meV}$$

$$E_2 = 4E_1 = 167.1 \text{ meV}$$

$\Delta E_g = (1.673 - 1.424)$ eV $= 249$ meV

$E_{C2} - E_{HH2} \geq E_g + \Delta E_g$. So the C2–HH2 and C2–LH2 transitions do not need to be considered. By the selection rules, C2–LH1, C2-HH1, C1-LH2, etc. transitions are not valid transitions. Therefore, only consider the C1–HH1 and C1–LH1 transitions.

C1–HH1:

$E_g = (1.424 + 0.056 + 0.010)$ eV $= 1.490$ eV

From Table 4.3, the reduced density of states is

$$\rho_r(E_{21}) = \frac{m^*_{rHH}}{\pi\hbar^2 d}$$
$$= \frac{(0.057)(9.11\times 10_{-31}\text{ kg})}{\pi(1.054\times 10^{-34}\ J\,s)^2\ (10^{-8}\text{ m})}$$
$$= 1.50\times 10^{44}\text{ m}^{-3}\text{ J}^{-1}$$
$$= 2.40\times 10^{19}\text{ cm}^{-3}\text{ eV}^{-1}$$

C1–LH1:

$E_g = (1.424 + 0.056 + 0.042)$ eV $= 1.522$ eV

$$\rho_r(E_{21}) = \frac{m^*_{rLH}}{\pi\hbar^2 d}$$
$$= \frac{(0.038)(9.11\times 10_{-31}\text{ kg})}{\pi(1.054\times 10^{-34}\ J\,s)^2\ (10^{-8}\text{ m})}$$
$$= 9.98\times 10^{43}\text{ m}^{-3}\ J^{-1}$$
$$= 1.60\times 10^{19}\text{ cm}^{-3}\ eV^{-1}$$

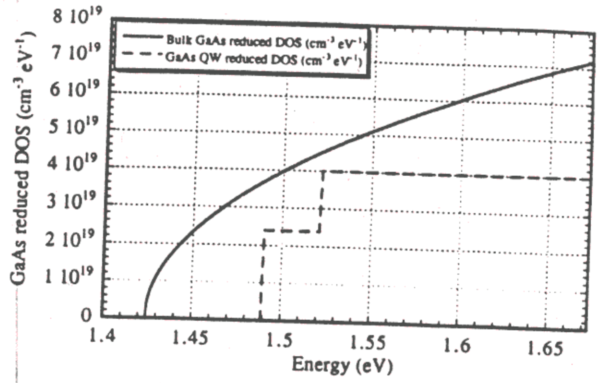

Figure 4.1b. GaAs (infinite barrier) QW reduced density of states

The bulk case density of states is greater than the quantum well density of states. As we will see in part (c), the bulk case is the upper limit for the infinite barrier quantum well case.

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5 c) Repeat parts (a) and (b), but using only a single hole mass at a time:

C1 – HH1 :

bulk :

$$\rho_r(E_{21}) = \frac{1}{2\pi^2}\left(\frac{2m^*_{rHH}}{\hbar^2}\right)^{3/2}(E_{21}-E_g)^{1/2}$$

$$= 9.36\times 10^{19}\,(E_{21}-E_g)^{1/2}\ \mathrm{cm^{-3}}\,eV^{-1} \qquad E_{21},\ E_g \text{ in eV}$$

QW :

$$\rho_r(E_{21}) = 2.40\times 10^{19}\ \mathrm{cm^{-3}}\,eV^{-1} \qquad (E_g = 1.490 \text{ eV})$$

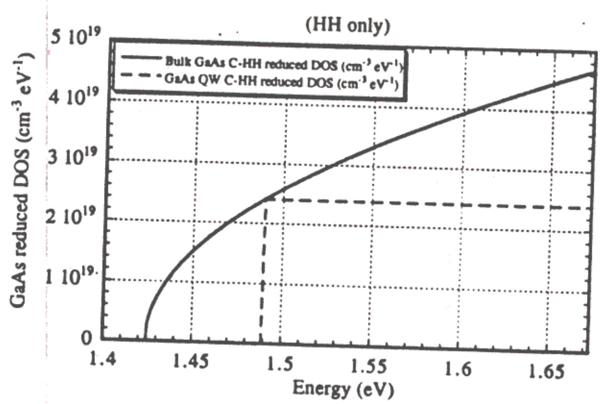

Figure 4.1c. GaAs (infinite barrier) QW reduced density of states (C–HH only)

C1 – LH1 :

bulk :

$$\rho_r(E_{21}) = \frac{1}{2\pi^2}\left(\frac{2m^*_{rLH}}{\hbar^2}\right)^{3/2}(E_{21}-E_g)^{1/2}$$

$$= 5.09\times 10^{19}\,(E_{21}-E_g)^{1/2}\ \mathrm{cm^{-3}}\,eV^{-1} \qquad E_{21},\ E_g \text{ in eV}$$

QW :

$$\rho_r(E_{21}) = 1.60\times 10^{19}\ \mathrm{cm^{-3}}\,eV^{-1} \qquad (E_g = 1.522 \text{ eV})$$

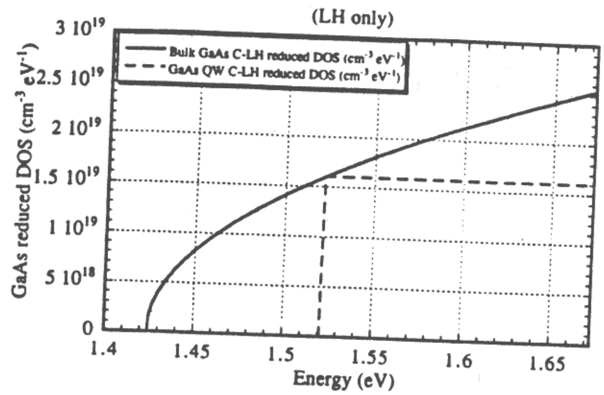

Figure 4.1c. GaAs (infinite barrier) QW reduced density of states (C–LH only)

As mentioned in part (b), the bulk density of states is the limiting case of the infinite barrier quantum well density of states. This is especially clear when looking at the contributions from only one of the hole bands as in this part of the problem. In this type of comparison, the density of states for the bulk and quantum well cases are equal at the step edges of the quantum well.

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5 d)

In the bulk limit, electrons can move in all three dimensions, whereas in the quantum well limit, they are confined to moving in the plane of the well. As a result, the smooth density of states function in bulk material is gradually transformed into a "step-like" density of states function in the quantum well. Each step can be associated with the subband of a quantized energy level, E_n, as described in Table 4.3. For the conduction band - heavy hole transition, we have, for each quantum well energy level,

$$\rho_{QW} = \frac{m^*_{rHH}}{\pi\hbar^2 d} \qquad \text{(Table 4.3)}$$

The effective mass and well width determine the height of each step. At any given energy above the band edge, the total quantum well density of states function is the step height times the number of contributing energy levels. Thus, for $E_n \leq E < E_{n+1}$:

$$\rho_{QW,total} = \frac{m^*_{rHH}}{\pi\hbar^2 d} n$$

where n is defined by

$$E_n = \frac{\hbar^2}{2m^*}\left(\frac{\pi}{d}\right)^2 n^2$$

Letting the well width go to infinity reduces the step height to zero, but also moves the energy levels infinitely close together. Writing n in terms of E_n and noting that E is approximately equal to one of the energy levels, E_n, we get the bulk density of states function:

$$\rho_{QW,total}\big|_{d\to\infty} \quad \Rightarrow \quad \rho_{bulk} = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}\sqrt{E}$$

5 e) Repeat part (b) with new values of quantum well levels that are calculated for finite barrier heights. Then compare with the results from part (a).

Conduction band :

$$m_C^* = 0.067m_0$$
$$E_1^\infty = 56.12 \text{ meV}$$
$$\Rightarrow n_{max} = 1.63 \Rightarrow 2 \text{ bound states}$$

From Fig. A1.14,

$$n_{QW1} = 0.72 \Rightarrow E_{C1} = 29 \text{ meV}$$
$$n_{QW2} = 1.38 \Rightarrow E_{C2} = 107 \text{ meV}$$

Heavy hole valence band :

$$m_{HH}^* = 0.38m_0$$
$$E_1^\infty = 9.89 \text{ meV}$$
$$\Rightarrow n_{max} = 3.17 \Rightarrow 4 \text{ bound states}$$

From Fig. A1.14,

$$n_{QW1} = 0.83 \Rightarrow E_{HH1} = 6.8 \text{ meV}$$
$$n_{QW2} = 1.65 \Rightarrow E_{HH2} = 27 \text{ meV}$$
$$n_{QW3} = 2.43 \Rightarrow E_{HH3} = 58 \text{ meV}$$
$$n_{QW4} = 3.12 \Rightarrow E_{HH4} = 96 \text{ meV}$$

Light hole valence band :

$$m_{LH}^* = 0.09m_0$$
$$E_1^\infty = 41.78 \text{ meV}$$
$$\Rightarrow n_{max} = 1.54 \Rightarrow 2 \text{ bound states}$$

From Fig. A1.14,

$$n_{QW1} = 0.70 \Rightarrow E_{LH1} = 20.5 \text{ meV}$$
$$n_{QW2} = 1.38 \Rightarrow E_{LH2} = 80 \text{ meV}$$

Allowed transitions:

C1–HH1 (1.460 eV) C2–HH2 (1.558 eV)

C1–LH1 (1.474 eV) C2–LH2 (1.610 eV)

$\rho_r(E_{21})$ step for C–HH:

$$\frac{m_{rHH}^*}{\pi\hbar^2 d} = 2.40 \times 10^{19} \text{ cm}^{-3} \text{ eV}^{-1}$$

$\rho_r(E_{21})$ step for C–LH:

$$\frac{m_{rLH}^*}{\pi\hbar^2 d} = 1.60 \times 10^{19} \text{ cm}^{-3} \text{ eV}^{-1}$$

$E_{21} \geq 1.673$ eV for bulk AlGaAs (x=0.2)

$$\rho = \frac{1}{2\pi^2}\left(\frac{2m'_{rEQ}}{\hbar^2}\right)(E_{21} - E'_g)^{1/2}$$

$$E'_g = 1.673 \text{ eV}$$

$$m'_{rHH} = \frac{(0.084)(0.39)}{(0.084+0.39)}\, m_0 = 0.069\, m_0$$

$$m'_{rLH} = \frac{(0.084)(0.1)}{(0.084+0.1)}\, m_0 = 0.046\, m_0$$

$$m'_{rEQ} = \left(0.069^{3/2} + 0.046^{3/2}\right)^{2/3} m_0 = 0.092\, m_0$$

$\Rightarrow \rho_r = 1.92 \times 10^{20}\,(E_{21} - E_g)^{1/2}$ $(cm^{-3}\,eV^{-1})$ E_{21} and E_g in eV

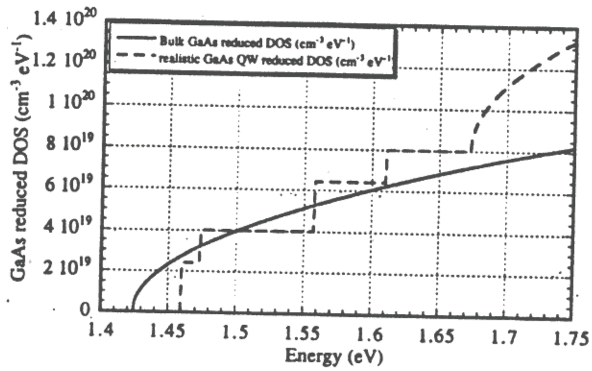

Figure 4.1e. QW with finite barrier and bulk reduced density of states

Overall, the total quantum well and bulk density of states curves are comparable in magnitude, which supports a "conservation of states" viewpoint.

4.4: 10

symmetry $\Rightarrow f_1 = (1 - f_2)$

For a modal gain of $\frac{g_{max}}{2} = 200\ \text{cm}^{-1}$, we have

$$g_{21} = g_{max}(f_2 - f_1) = g_{max}\left(\frac{1}{e^{\frac{E_{C1}-E_{FC}}{kT}}+1} - \frac{1}{e^{\frac{E_{HH1}-E_{FV}}{kT}}+1}\right) = g_{max}\frac{1}{2} = 200\ \text{cm}^{-1}$$

Since the conduction and valence bands are completely symmetric, $E_{C1} - E_{FC} = -(E_{HH1} - E_{FV})$. Plugging this into the equation for g_{21}, we find

$$E_{FC} - E_{C1} = 28.56\ \text{meV}$$

A key point to recognize is that for $E_{21} = E_{C2} - E_{HH2}$, gain from the C1–HH1 and C2–HH2 transitions are equal for symmetric bands. Therefore, in order for the gain at $E_{21} = E_{C1} - E_{HH1}$ to be greater than the gain at $E_{21} = E_{C2} - E_{HH2}$, the gain from the C1–HH1 transition must drop by 50% from $E_{21} = E_{C1} - E_{HH1}$ to $E_{21} = E_{C2} - E_{HH2}$.

$$g_{21}(C1 - HH1) = g_{max}\left(\frac{1}{e^{\frac{E_{C2}-E_{FC}}{kT}}+1} - \frac{1}{e^{\frac{E_{HH2}-E_{FV}}{kT}}+1}\right) = g_{max}\frac{1}{4} = 100\ \text{cm}^{-1}$$

Since $E_{FC} - E_{C2} = -(E_{HH2} - E_{FV})$, we find that $E_{C2} - E_{FC} = 13.28$ meV. Therefore,

$$E_{C2} - E_{C1} = (28.56 - 13.28)\ \text{meV} = 15.28\ \text{meV}.$$

This is the minimum subband spacing that we can use.