ECE 227A - Diode Lasers
Course Notes

Larry A. Coldren, Scott W. Corzine, Milan L. Mashanovitch

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Example 1.1

An InP/InGaAsP double heterostructure laser cross section consists of a 320 nm wide InGaAsP separate confinement heterostructure waveguide region with the bandgap corresponding to 1.3 μm (1.3Q), clad by InP on both sides

Problem: 1) Determine the effective index of the fundamental transverse mode of this waveguide. 2) Determine the rate of decay of the normalized electric field U.

Solution : To solve this problem, we utilize the tools from the Appendix 3. Since the optical waveguide structure is symmetric, we can utilize the expression (A3.14) to solve for the effective index. Then, we can compute the wave vector component along x, k_x, and the decay constant γ using equation (A3.7). From the problem statement, the refractive index of the InGaAsP region can be found in the Table **??**, $n_{\text{II}} = 3.4$. For the cladding, the refractive index value at 1.55 μm is $n_{\text{I}} = n_{\text{III}} = 3.17$. From equation (A3.12), the normalized frequency, V is given by

$$V = k_0 d \left(n_{\text{II}}^2 - n_{\text{III}}^2\right)^{\frac{1}{2}} = \frac{2\pi}{1.55\mu\text{m}} 0.32\mu\text{m} \left(3.4^2 - 3.17^2\right)^{\frac{1}{2}} = 1.594 \tag{1}$$

Using the equation (A3.14), we can compute the value of the normalized propagation parameter b,

$$b = 1 - \frac{ln\left(1 + \frac{V^2}{2}\right)}{\frac{V^2}{2}} = 1 - \frac{ln\left(1 + \frac{1.594^2}{2}\right)}{\frac{1.594^2}{2}} = 0.354 \tag{2}$$

and the effective index value,

$$\bar{n} = \left(n_{\text{II}}^2 b + n_{\text{I}}^2 (1-b)\right)^{\frac{1}{2}} = \left(3.4^2 \cdot 0.354 + 3.17^2 \cdot (1 - 0.354)\right)^{\frac{1}{2}} = 3.253 \tag{3}$$

In order to determine the minimum thickness of the top p doped cladding, we can compute the decay constant γ using equation (A3.7), remembering that the propagation constant $\beta = k_0 \bar{n}$,

$$\gamma = k_0 \left(\bar{n}^2 - n_{\text{I}}^2\right)^{\frac{1}{2}} = \frac{2\pi}{1.55\mu\text{m}} \left(3.253^2 - 3.17^2\right)^{\frac{1}{2}} = 2.9591\mu\text{m}^{-1} \tag{4}$$

Thus, for a 1 μm thick cladding, the optical energy decays to $\exp(-2.9591) = 0.0027$ at the top surface. We can observe that the rate of decay is strongly dependent on the refractive index difference between the waveguide and the cladding - for a larger difference, the field intensity outside the waveguide region decays faster. In a real laser, the active region would probably be defined by a set of quantum wells in the center of the InGaAsP double heterostructure region. This would complicate solving for the effective index – and this case will be treated in Chapter 6, when we talk about the perturbation theory.

End Example

Example 1.2

InP-based 1550nm vertical cavity surface emitting lasers have been made with AlAsSb/AlGaAsSb multi-layer mirrors.

Problem: Calculate the fraction of As in the AlAsSb mirror layers for lattice matching to InP.

Solution :

To solve this problem, we will use the Vegard's law. The composition of any AlAsSb alloy can be specified by value x, where x is the percentage of As in the alloy $\text{AlSb}_{1-x}\text{As}_x$. From Table **??**, the lattices constants of InP, AlAs and AlSb are $a_{\text{InP}} = 5.8688\ \AA$, $a_{\text{AlAs}} = 5.660\ \AA$, $a_{\text{AlSb}} = 6.1355\ \AA$ respectively. Using Vegard's law,

$$a_{\text{InP}} = x a_{\text{AlAs}} + (1-x) a_{\text{AlSb}}. \tag{5}$$

Therefore, the fraction of As in the lattice matched mirror layer is

$$x = \frac{a_{\text{InP}} - a_{\text{AlSb}}}{a_{\text{AlAs}} - a_{AlSb}} = 0.56. \tag{6}$$

End Example

Example 1.3

An 80 $\AA$ wide quantum well composed of InGaAsP, lattice matched to InP with the bandgap corresponding to 1.55 µm (1.55Q), is surrounded by a InGaAsP barrier lattice matched to InP, with the bandgap wavelength of 1.3 µm (1.3Q).

Problem: Determine the energy and wavelength for photons generated in recombination between the ground states of the quantum well at room temperature

Solution : In order to solve this problem, we utilize the tools from the Appendix 1. We will determine the energy levels of this quantum well using Eq. (A1.14). First, we need to compute the energy of the ground state for the quantum well with infinitely high walls. Then, we need to determine the quantum numbers taking into account that the quantum well has finite walls, using Eq. (A1.17).

As mentioned in the problem statement, both the quantum well and the barrier are lattice matched to InP. From Table **??**, $E_{\text{barrier}} = 0.954$ eV, and $E_{\text{well}} = 0.800$ eV. In this material system, only 40 % of the band offset occurs in the conduction band. Therefore, the quantum well barrier height in the conduction band is given by $V_{0\text{C}} = 0.4\,(E_{\text{barrier}} - E_{\text{well}}) = 61.6$ meV and $V_{0\text{V}} = 0.6\,(E_{\text{barrier}} - E_{\text{well}}) = 92.4$ meV in the valence band.

From Eq. (A1.14), the ground state energy for a quantum well with infinite walls, $E_{1\text{c}}^{\infty}$ is given by

$$E_{1\text{c}}^{\infty} = 3.76\frac{m_0}{m}\left(\frac{100\AA}{l}\right)^2 \text{meV} = 3.76\frac{1}{0.045}\left(\frac{100}{80}\right)^2 \text{meV} = 130.55 \text{ meV} \tag{7}$$

where $m = m_\text{c}$ was taken from Table **??**. Similarly, for the valence band, $E_{1\text{v}}^{\infty} = 15.88$ meV,with $m = m_{\text{HH}}$. Now, we can calculate $n_{\max}$for both quantum wells using Eq. A(1.17), $n_{\text{maxc}} = \sqrt{\frac{V_{0\text{c}}}{E_{1\text{c}}^{\infty}}} = 0.69$, and $n_{\text{maxv}} = \sqrt{\frac{V_{0\text{v}}}{E_{1\text{v}}^{\infty}}} = 2.41$. The normalized variable $n_{\max}$, when rounded up to the nearest integer, yields the largest number of bound states possible. Either by reading the chart in Fig. A1.4 or using Eq. (A1.18), we can calculate the lowest quantum numbers for both cases:

$$n_{1\text{c}} = \frac{2}{\pi}\arctan\left[n_{\text{maxc}}\left(1 + 0.6^{n_{\text{maxc}}+1}\right)\right] = 0.49 \tag{8}$$

$$n_{1\text{v}} = \frac{2}{\pi}\arctan\left[n_{\text{maxv}}\left(1 + 0.6^{n_{\text{maxv}}+1}\right)\right] = 0.78 \tag{9}$$

Thus, $E_{1\text{c}} = n_{1\text{c}}^2 E_{1\text{c}}^{\infty} = 31.35$ meV and $E_{1\text{v}} = n_{1\text{v}}^2 E_{1\text{v}}^{\infty} =$ 9.66meV. Finally, the photon energy is given by

$$E_{\text{photon}} = E_{\text{well}} + E_{1\text{c}} + E_{1\text{v}} = 841.01 \text{ meV}. \tag{10}$$

This energy corresponds to the wavelength

$$\lambda = \frac{1.23985eV - \text{µm}}{0.84101 \text{ eV}} = 1.47 \text{ µm}. \tag{11}$$

End Example

Example 2.1

An active-passive cleaved laser chip operating at 1.55 µm consists of a multiple quantum well InP/InGaAsP active section, whose length is 500 µm , and internal loss $\alpha_{ia} = 15$ cm^{-1}, and a passive section whose length is 300 µm and internal loss is $\alpha_{ip} = 10$ cm^{-1}. The active region contains 4–3 nm wide strained InGaAs quantum wells.

Problem: (1) Determine the threshold modal gain of this laser. (2) Determine the mode spacing for this laser.

Solution : In order to calculate the threshold modal gain, we will use the expression from Eq. **??**. For this, we need to determine the average loss of the cavity, $\langle\alpha_i\rangle$, and the mirror loss, α_m. From Eq. **??**,

$$\langle\alpha_i\rangle = \frac{(\alpha_{ia}L_a + \alpha_{ip}L_p)}{L_a + L_p} = \frac{23 \cdot 10^{-4} \cdot 500 + 10 \cdot 10^{-4} \cdot 300}{(500 + 300)10^{-4}\text{cm}} = 18.125 \text{ cm}^{-1}.$$

From Eq. **??**, using the fact that we are dealing with an InP based laser with cleaved facets, where $R \sim 0.32$,

$$\alpha_m = \frac{1}{L_a + L_p} \ln\left(\frac{1}{R}\right) = \frac{1}{800 \cdot 10^{-4}\text{cm}} \ln\left(\frac{1}{0.32}\right) = 14.243 \text{ cm}^{-1}$$

Then, the threshold modal gain is given by Eq. **??**,

$$\Gamma g_{th} = \langle\alpha_i\rangle + \alpha_m = 32.368 \text{ cm}^{-1}$$

Cavity mode spacing can be determined from Eq. **??**, using the fact that $n_{ga} = n_{gp} = 3.8$ for this InGaAsP based laser, and the lasing wavelength of 1.55µm,

$$\partial\lambda = \frac{\lambda^2}{2(\bar{n}_{ga}L_a + \bar{n}_{gp}L_p)} = \frac{1.55^2}{2(3.8 \cdot 500 + 3.8 \cdot 300)}\mu\text{m} = 0.395 \text{ nm}.$$

End Example

Example 2.2

The active material from Example 2.1 is used to fabricate a 500 µm long and 50 µm wide broad area laser. The material gain for the quantum wells can be approximated using Eq. **??**, with $g_{0N} = 1207.29$ cm^{-1} and $N_{tr} = 1.2284 \cdot 10^{18}$ cm^{-3}. The transverse confinement factor Γ_1 is 1% per well.

Problem: (1) Calculate the threshold modal gain for this laser (2) Determine the threshold current for this laser (3) Determine the threshold current for a laser of same dimensions but with 7 quantum wells, assuming that the internal modal loss α_{ia} remains the same

Solution : In order to calculate the threshold current, we first need to calculate the threshold carrier density using the Eq. **??**. Since we are dealing with a long wavelength InP/InGaAsP material system, both spontaneous, Eq. **??** and non-radiative, Eq. **??** threshold current components need to be included. To compute the threshold carrier density, we need to determine the threshold modal gain of this all active laser,

$$\Gamma g_{th} = \langle \alpha_{ia} \rangle + \frac{1}{L} \ln \left(\frac{1}{R} \right) = 37.79 \text{ cm}^{-1}.$$

The transverse confinement factor is determined by the number of quantum wells,

$$\Gamma = N_w \cdot \Gamma_1 = 4 \cdot 0.01 = 0.04.$$

Using Eq. **??**, threshold carrier density is

$$N_{th} = N_{tr} e^{\frac{g_{th}}{g_{0N}}} = N_{tr} e^{\frac{37.79}{0.04 \cdot 1207.29}} = 2.6865 \cdot 10^{18} \text{ cm}^{-3}.$$

Now we can compute the threshold current, using the volume of the active region $V = L \cdot W \cdot N_w \cdot a = 500 \cdot 50 \cdot 4 \cdot 0.003 \cdot 10^{-12} \text{cm}^{-3}$,

$$I_{th} = \frac{qVN_{th}}{\eta_i \tau} \cong \frac{qV}{\eta_i} (BN_{th}^2 + CN_{th}^3) = 47.94 \text{ mA}$$

where $B = 0.3 \cdot 10^{-10} \text{cm}^3/s$ and $C = 3 \cdot 10^{-29} \text{cm}^6/s$. If we now have a laser with 7 quantum wells, the confinement factor will be changed,

$$\Gamma_2 = N_w \cdot \Gamma_1 = 0.07.$$

leading to a changed threshold gain $g_{th2} = 539.84 \text{cm}^{-1}$, threshold carrier density $N_{th2} = 1.9211 \cdot 10^{18}$ cm^{-3}, volume $V = 500 \cdot 50 \cdot 7 \cdot 0.003 \cdot 10^{-12}$ cm^{-3}, and the threshold current of

$$I_{th} = \frac{qV}{\eta_i} (BN_{th}^2 + CN_{th}^3) = 34.00 \text{ mA}.$$

End Example

Example 2.3

A cleaved facet, active 3 µm wide and 500 µm ridge laser is created from the laser structure from Example 2.1. This ridge laser is biased 30mAabove threshold and directly modulated by applying a small signal sine wave current to its active section.

Problem: 1) Determine the resonance peak frequency $f_R = \frac{\omega_R}{2\pi}$ of this laser, assuming injection efficiency η_i of 80%, and assuming no change in the internal losses.

Solution : To calculate the resonance frequency, we will use the Eq. **??**. Therefore, we need to compute the differential gain a at threshold, since the carrier density is clamped at the threshold carrier density. This, we first need to determine the threshold carrier density. Since the internal losses are unchanged, the threshold modal gain for this laser is the same as that for the laser in Example 2.2, $\Gamma g_{th} = 37.79$ cm^{-1}, leading to the same threshold carrier density as calculated in Example 2.2,

$$N_{th} = N_{tr} e^{\frac{g_{th}}{g_{0N}}} = N_{tr} e^{\frac{37.79}{0.04 \cdot 1207.29}} = 2.6865 \cdot 10^{18} \text{ cm}^{-3}.$$

Differential gain a can be computed using expression Eq. **??**,

$$a = \frac{\partial g}{\partial N}|_{N=N_{th}} = \frac{g_{0N}}{N_{th}} = \frac{1207.29 \text{cm}^{-1}}{2.6865 \cdot 10^{18} \text{cm}^{-3}} = 4.49 \cdot 10^{-16} \text{ cm}^2.$$

The resonance frequency is given by $f_R = \frac{\omega_R}{2\pi}$, where ω_R is defined in Eq. **??**,

$$f_R = \frac{1}{2\pi} \left[\frac{\Gamma v_g a}{qV} \eta_i (I - I_{th}) \right]^{1/2} = \left[\frac{\Gamma_1 v_g a}{qV_1} \eta_i (I - I_{th}) \right]^{1/2}.$$

V_1 is the volume of a single quantum well, $V_1 = 3 \cdot 500 \cdot 0.003 \cdot 10^{-12} \text{cm}^{-3} = 4.5 \cdot 10^{-12}$ cm^{-3}, and the group velocity for InGaAsP laser is $v_g = \frac{c}{3.8} = 0.7894 \cdot 10^{10}$ cm/s. Therefore, the resonance frequency is

$$f_R = \frac{1}{2\pi} \left[\frac{0.01 \cdot 0.7894 \cdot 10^{10} \cdot 4.49 \cdot 10^{-16} \text{cm}^2}{1.6 \cdot 10^{-19} \cdot 4.5 \cdot 10^{-12}} 0.8(0.3) \frac{1}{\text{s}^2} \right]^{1/2} = 17.30 \, G\text{Hz}.$$

End Example

Example 2.4

To characterize the active material from Example 2.1, a 50 µm wide and 500 µm long broad area laser is cleaved from the wafer. Its pulsed threshold current is 47.94 mA , and differential efficiency from both facets 48.24% This laser is then recleaved into two 250 µm long lasers, having a pulsed threshold current of 59.44 mA , and differential efficiency of 60.19 %.
Problem: (1) What is the injection efficiency η_i (2) What is the average internal modal loss? (3) Determine J_{tr} and g_0 in the gain vs current density characteristic for each quantum well, assuming $J = J_{tr}e^{\frac{g}{g_0}}$
Solution : To calculate the injection efficiency η_i and the internal modal loss $\langle\alpha_i\rangle$, we will use the Eq. **??**. Once the modal loss is known, and knowing the threshold current densities in the active region, we can construct the modal gain Γg versus current J density curve. Since the confinement factor is known, we can determine the basic material gain g versus current density J for this active material using the equation. From Eq. **??**,

$$\langle\alpha_i\rangle = \frac{\eta_d' - \eta_d}{L\eta_d - L'\eta_d'} \ln\left(\frac{1}{R}\right) = \frac{(0.6019 - 0.4824)\ln\left(\frac{1}{0.32}\right)}{0.05 \cdot 0.4824 - 0.025 \cdot 0.6019}) = 15 \text{ cm}^{-1}$$

and

$$\eta_i = \eta_d\eta_d' \frac{L - L'}{L\eta_d - L'\eta_d'} = 0.6019 \cdot 0.4824 \frac{0.025\text{cm}^{-1}}{0.0091} = 0.80$$

For the gain versus current density characteristic, we utilize two data points from two different lasers:

$$J_{th1} = \frac{\eta_i I_{th1}}{w \cdot L_1} = \frac{0.8 \cdot 47.94\text{mA}}{500 \cdot 50 \cdot 10^{-8}\text{cm}^2} = 153.41 \text{ A/cm}^2 \tag{12}$$

$$J_{th2} = \frac{\eta_i I_{th2}}{w \cdot L_2} = \frac{0.8 \cdot 59.44\text{mA}}{250 \cdot 50 \cdot 10^{-8}\text{cm}^2} = 380.42 \text{ A/cm}^2 \tag{13}$$

Threshold modal gain Γg_{th1} for the 500 µm cavity was calculated in Example 2.2 to be 37.79cm^{-1}. Similarly, for the 250 µm long cavity, the threshold modal gain is $\Gamma g_{th1} = 60.58\text{cm}^{-1}$, where $\Gamma = 0.04$ as discussed in the Example 2.2. From Eq. **??**,

$$g_0 = \frac{g_{th1} - g_{th2}}{ln\frac{J_{th1}}{J_{th2}}} = \frac{944.71 - 1514.43\text{cm}^{-1}}{ln\frac{153.41}{380.42}} = 627\text{cm}^{-1} \tag{14}$$

$$J_{tr} = \frac{J_{th1}}{\exp\frac{g_{th1}}{g_0}} = \frac{153.41\text{A/cm}^2}{\exp\frac{944.71}{627}} = 34 \text{ A/cm}^2 \tag{15}$$

End Example

Example 2.5

Another batch of lasers similar to those from Example 2.3 is made, but this time, the laser contacts exhibit large series resistance, and thus significant amount of heating under CW operation. Consider a 250 µm long, 3 µm wide all-active ridge laser, which can be modeled by a 50 Ω series resistance and an ideal diode with an ideality factor of 3. The InP substrate is 100 µm thick and 500 µm wide, and it is bonded to a good heat sink. The characteristic temperature for threshold current is $T_0 = 25$ K and that for differential efficiency is $T_\eta = 110$ K. Pulsed threshold current is 15 mA and differential efficiency 48.24%.

Problem: (1) What is the thermal impedance (2) What is the new CW threshold current (3) At a bias of 50 mA, what is the power out and the temperature rise of the active region?

Solution : Thermal impedance can be calculated using Eq. **??**,

$$Z_T = \frac{\ln(4h/w)}{\pi \xi l} = \frac{\ln \frac{4 \cdot 100}{3}}{\pi \cdot 0.6 \cdot 25010^{-4}} = 103.78 \text{ °C/W}$$

In order to find the new threshold current, we will need to iterate, given that the temperature increase depends on the new threshold current, which in turn is determined by the temperature increase. At threshold, the output power from the laser can be neglected, therefore, the dissipated power is equal to the input power,

$$P_d = P_{in} = I_{th}^2 \cdot R_s + I \cdot V_d.$$

where V_d is the ideal diode voltage, and is approximately 0.88V for InGaAsP/InP. Assuming that the threshold current increase due to heating is 1mA, $I_{th} = 16$ mA, the dissipated power and temperature increase are

$$P_d = (0.016)^2 \cdot 50 + (0.016)(0.88)m\text{W} = 26.88\ m\text{W}$$
$$\Delta T = P_d Z_T = 0.02688 \cdot 103.78\text{°C} = 2.79 \text{ °C}$$

To check, we plug in the value for ΔT to calculate the threshold current based on known $T_0 = 25$ K,

$$I_{th}^{'}(\Delta T = 2.79\text{°C}) = 15\text{mA} \cdot \exp \frac{2.79}{25} = 16.77 \text{ mA}.$$

Thus, we conclude that we have underestimated the heating effects, and we use $I_{th}^{'}$ to calculate the dissipated power and repeat the process. After a couple of iterations, we end up with the final value for I_{th},

$$I_{th} = 16.9 \text{ mA}.$$

To calculate the output power for the bias current of $I = 50$ mA, we do the following: assuming that the output power is negligible, we calculate the dissipated power, the temperature increase, and then the increase in the threshold temperature and the decrease in the differential efficiency. At that point, we can compute the output power. To iterate, we reduce the dissipated power by the output power value, and repeat the process. After a couple of steps, the process converges.

$$P_d \approxeq P_{in} = (0.05)^2 \cdot 50 + (0.05)(0.88)\text{mW} = 169.00 \text{ mW}$$
$$\Delta T = P_d Z_T = 0.169 \cdot 103.78\text{°C} = 17.54 \text{ °C}$$

The threshold current and the differential efficiency with this much temperature increase are given by

$$I_{th}^{'} = 15\text{mA} \cdot \exp \frac{17.54}{25} = 30.25 \text{ mA}$$
$$\eta_d^{'} = 0.4824 \cdot \exp \frac{-17.54}{110} = 0.4113.$$

The output power is given by

$$P_o = \frac{h\nu}{q} \eta_d^{'} (I - Ith^{'}) = 0.8 \cdot 0.4113(50 - 30.25)\text{mW} = 6.49 \text{ mW}.$$

Reducing the dissipated power by P_o yields $\Delta T = 16.87$ °C and $P_o = 6.80$ mW. Repeating the process yields the final values of

$$\Delta T = 16.83 \text{ °C}$$
$$P_o = 6.82 \text{ mW}$$

The L-I characteristic of this laser is illustrated in Figure **??**.

End Example

Example 3.1

A dielectric transmission line with an air interface is shown in Fig. **??**

Problem: Write **T** matrix parameters for this two port.

Solution : To illustrate the usefulness of **T** matrix formalism, we will solve this problem by multiplying the T matrices corresponding to the building blocks of this structure. For the dielectric transmission line, the T matrix is given by

$$T_1 = \begin{bmatrix} e^{j\phi_1} & 0 \\ 0 & e^{j\phi_1} \end{bmatrix},$$

where $\phi_1 = \beta_1 L_1$. For the dielectric/air interface, we have

$$T_2 = \frac{1}{t_{12}} \begin{bmatrix} 1 & r_{12} \\ r_{12} & 1 \end{bmatrix},$$

where $r_{12} = \frac{n_1 - n_2}{n_1 + n_2}$, and $r_{12}^2 + t_{12}^2 = 1$. Finally, the air segment can be described by the same matrix as the one for the dielectric segment, $T_3 = T_1$, except that $\phi_2 = \beta_1 L_1$ in this case.

To get the full T matrix of the system, we need to multiply through the matrices corresponding to the individual segments,

$$T = T_1 \cdot T_2 \cdot T_3 = \begin{bmatrix} e^{j\phi_1} & 0 \\ 0 & e^{j\phi_1} \end{bmatrix} \cdot \frac{1}{t_{12}} \begin{bmatrix} 1 & r_{12} \\ r_{12} & 1 \end{bmatrix} \cdot \begin{bmatrix} e^{j\phi_2} & 0 \\ 0 & e^{j\phi_2} \end{bmatrix} = \frac{1}{t_{12}} \begin{bmatrix} e^{j(\phi_1+\phi_2)} & r_{12}e^{j(\phi_1-\phi_2)} \\ r_{12}e^{j(\phi_2-\phi_1)} & e^{-j(\phi_1+\phi_2)} \end{bmatrix}.$$

The T matrix can be converted to an S matrix using the relationships between corresponding elements described in Table **??**. This exercise will be useful when we start dealing with periodic grating sections and DFB lasers later in this Chapter.

End Example

Example 3.2

A 980 nm bottom-emitting VCSEL must be designed. The average internal losses and the internal efficiency are 20 cm^{-1} and 0.9, respectively. The mirrors are composed of quarter wave stacks of AlAs/GaAs. The top mirror needs to have a power reflection of 99.6% including air interface, and the bottom (output) mirror needs to have a power reflection of 98.4%. The cavity between the DBR mirrors has to be one optical wavelength in the material thick, and it consists of 3 InGaAs quantum wells, each 8 nm thick separated by 2–8 nm thick GaAs barriers, and clad on each side by another 8 nm thick layer of GaAs. The rest of the cavity is composed of AlGaAs with 20% Al. The mirrors begin with an AlAs quarter wave layer next to the cavity. The quantum wells are centered between the mirrors for best confinement factor.

Problem: (a) Determine the thickness of the two AlGaAs spacer layers (b) Determine the number of the AlAs/GaAs periods in the top and bottom DBR mirrors (c) What is the differential efficiency measured out of the bottom (output) DBR mirror?

Solution : From the statement of the problem, we know that the cavity length is equal to one optical wavelength in the material, $\Sigma n_i l_i = \lambda$. From Table **??**, we have $n_{AlAs} = 2.95, n_{GaAs} = 3.52, n_{AlGaAs(0.2)} = 3.39$ and $n_{InGaAs} = 3.60$. Therefore,

$$3(3.60)8\text{nm} + 4(3.52)8\text{nm} + 2(3.39)s = 980\text{nm} => s = 115.2 \text{ nm}$$

For the bottom mirror, the mirror ends with the semiconductor substrate interface. Since the mirror begins with an AlAs layer, and ends with GaAs, there has to be an odd number of half periods in the mirror. Using Eq. **??**, we have

$$r_g = \frac{1 - \left(\frac{n_1}{n_2}^{2m}\right)}{1 + \left(\frac{n_1}{n_2}^{2m}\right)} => \frac{n_1}{n_2}^{2m} = \frac{1 - r_g}{1 + r_g}.$$

From here, we can calculate the number of periods, m, as

$$m = \frac{\ln\left(\frac{1-r_g}{1+r_g}\right)}{2\ln\frac{n1}{n2}} = \frac{\ln\left(\frac{1-\sqrt{0.984}}{1+\sqrt{0.984}}\right)}{2\ln\left(\frac{2.95}{3.52}\right)} = 15.6$$

The closest half integer is $m = 15.5$. For the top mirror, we apply the same procedure, except that this time we have an air interface at the end, and again an odd number of half periods of the mirror,

$$r_g = \frac{1 - \left(\frac{n_1}{n_2}^{2m}\right)\frac{1}{n_2}}{1 + \left(\frac{n_1}{n_2}^{2m}\right)\frac{1}{n_2}} => m = \frac{\ln n_2 \frac{1-r_g}{1+r_g}}{2\ln\frac{n_1}{n_2}}$$

Plugging in the parameter values, we have

$$m = \frac{\ln 3.52 \cdot \frac{1-\sqrt{0.996}}{1+\sqrt{0.996}}}{2\ln\frac{2.95}{3.52}} = 15.98$$

Since m must be a half-integer, we need to select either 15.5 or 16.5. As we are designing a bottom emitter, we will select $m = 16.5$. To compute the differential efficiency through the bottom mirror, given by Eq. **??**, since we already know the injection efficiency, internal loss and mirror loss, we need to determine the cavity length L_{DBR} and the fraction of the output power F_2. This problem lends itself to using the effective DBR mirror model.

$$L_{DBR} = L_{cavity} + 2L_{eff} = 230\text{nm} + 56\text{nm} + 2\frac{980\text{nm}}{40.57} = 1146 \text{ nm}$$

where, we have used the approximation of Eq. **??** to compute L_{eff} for a large index contrast value. To compute F_2, we use Equations **??** and **??**,

$$F_2 = \frac{(1 - |r_g'|^2)e^{-\alpha_i L_{eff}}}{(1 - |r_g'|^2) + \frac{|r_g'|}{r_1}(1 - r_1^2)} = 0.8.$$

Differential efficiency through the back mirror is then given by (Eq. **??**),

$$\eta_d = F_2\eta_i\frac{T_m}{T_m + A_i} = 0.9 \cdot 0.8\frac{\ln\frac{1}{\sqrt{0.9960.984}}}{114610^{-4}20 + \ln\frac{1}{\sqrt{0.9960.984}}} = 0.581.$$

End Example

Example 3.3

A InGaAsP/InP tunable 1.55 μm DBR laser, as illustrated in Figure **??** is constructed. The length of the gain, phase and DBR sections are 300 μm , 100 μm and 100 μm respectively. Average transverse-lateral internal losses in the gain and phase sections are 15 cm^{-1} and 5 cm^{-1} respectively and the injection efficiency is 0.8. The DBR power reflectivity is $|r_g|^2 = 0.2$ and its power transmission is $|t_g|^2 = 0.7$. The other end is a simple cleaved facet. Assume the modal index of $\bar{n} = 3.4$ for the entire structure.

Problem: (1) Determine differential efficiencies out of each of two ends? (2)Determine the mode spacing, neglecting the change in DBR penetration depth with wavelength (3) If the modal index of the DBR section is tuned by 0.01, by how much is the new lasing mode shifted from the original lasing wavelength?

Solution : Since the DBR laser in this problem is described using reflection and transmission scattering parameters, we will use case (a) from Section **??** to solve it. To determine the differential efficiencies from both ends, we first need to calculate the fractions of the power coming out of both ends, F_1 and F_2. Assuming the cleaved facet introduces no losses, $|t_1|^2 = 1 - |r_1|^2 = 0.68$. From Eq. **??**,

$$F_1 = \frac{t_1^2}{(1-r_1^2) + \frac{r_1}{|r_g|}(1-|r_g|^2)} = \frac{0.68}{0.68 + \sqrt{\frac{0.32}{0.2}}(1-0.2)} = 0.4019 \tag{16}$$

$$F_2 = \frac{t_g^2}{(1-r_g^2) + \frac{|r_g|}{r_1}(1-|r_1|^2)} = \frac{0.7}{(1-0.2) + \sqrt{\frac{0.2}{0.32}}(1-.32)} = 0.5233 \tag{17}$$

The mirror loss is given by Eq. **??**,

$$\alpha_m = \frac{1}{L_a + L_p} \ln\left[\frac{1}{r_1|r_g|}\right] = \frac{1}{0.04\text{cm}} \ln \frac{1}{\sqrt{0.320.2}} = 34.36 \text{ cm}^{-1}$$

Average passive loss, $\langle\alpha_i\rangle$ is given by

$$\langle\alpha_i\rangle_{a+p} = \frac{\langle\alpha_{ia}\rangle L_a + \langle\alpha_{ip}\rangle L_p}{L_a + L_p} = \frac{300 \cdot 15\text{cm}^{-1} + 100 \cdot 5\text{cm}^{-1}}{400} = 12.5 \text{ cm}^{-1}$$

Differential efficiencies are now given from Eq. **??**,

$$\eta_{d1} = F_1\eta_i \frac{\alpha_m}{\langle\alpha_i\rangle_{a+p} + \alpha_m} = 0.4019\frac{34.36}{34.36 + 12.5}0.8 = 0.236 \tag{18}$$

$$\eta_{d2} = F_2\eta_i \frac{\alpha_m}{\langle\alpha_i\rangle_{a+p} + \alpha_m} = 0.5233\frac{34.36}{34.36 + 12.5}0.8 = 0.307 \tag{19}$$

To calculate the mode spacing, we need to take into account the effective length of the DBR mirror. The mode spacing is given by Eq. **??**, and for a weakly reflecting grating, we assume $L_{eff} \approx \frac{1}{2}L_{DBR} = 50$ nm.

$$\delta\lambda = \frac{\lambda^2}{2(\bar{n}_{ga}L_a + \bar{n}_{gp}L_p + \bar{n}_{gp}L_{eff})} = \frac{(1550\text{nm})^2}{2(3.8)(300 + 100 + 50\text{μm})} = 0.702\text{nm}$$

When the DBR mirror section is tuned, two effects come into play. First, the grating Bragg wavelength will change, due to the changed grating index. Then, the cavity mode comb position will change as well, because the effective optical cavity length is now different. For the modal index change of $\Delta\bar{n}_{DBR}$, the center wavelength of the grating moves in direct proportion to the index according to Eq. **??**,

$$\Delta\lambda_g = \lambda_g \frac{\Delta\bar{n}_{DBR}}{\bar{n}_{DBR}} = 1550\frac{0.01}{3.4}\text{nm} = 4.56 \text{ nm}.$$

At the same time, the cavity modes will shift, and we can use the Eq. **??** to determine the amount of shift,

$$\Delta\lambda_m = \frac{\Delta\bar{n}_{DBR}L_{eff}}{\bar{n}_{ga}L_a + \bar{n}_{gp}L_p + \bar{n}_{gDBR}L_{eff}} = \Delta\lambda_g \frac{L_{eff}}{L_a + L_p + L_{eff}} = 4.56\frac{50}{450}\text{nm} = 0.507 \text{ nm}$$

Finally, the new lasing mode will be the cavity mode that is the closest to the new Bragg wavelength. To compute it, we need to compute by how many cavity modes the Bragg wavelength shifted.

$$\Delta\lambda_{lasing} = \Delta\lambda_m + \|\frac{\Delta\lambda_g - \Delta\lambda_m}{\delta\lambda}\|\delta\lambda = 0.507\text{nm} + 6 \cdot 0.702\text{nm} = 4.719 \text{ nm}.$$

End Example

Example 3.4

An InGaAsP/InP 1550 nm quarter wave shifted multiple quantum well buried heterostructure DFB laser was fabricated from a structure whose confinement factor is $\Gamma = 0.06$. The injection efficiency for this structure was determined to be $\eta_i = 75\%$, and the internal loss $\alpha_{ia} = 10$ cm^{-1}. The grating coupling constant is $\kappa = 20$ cm^{-1}, the average effective index of the waveguide is 3.21, and the laser length is 500 µm . The laser facets are AR coated.

Problem: (1) Determine the threshold modal gain of this laser (2) Determine the differential efficiency for this laser (3) Determine the lasing wavelength of this laser (4) Determine the threshold modal gain, differential efficiency and the lasing wavelength for a HR-AR standard DFB laser with the same parameters, assuming optimal HR mirror phase.

Solution : To solve this problem, we need to use Figure **??** (b), which provides us with normalized solutions for the quarter wave shifted DFB mirror loss versus wavelength detuning, in function of the grating length and coupling coefficient. From the problem formulation, $\kappa L_g = 500 \cdot 20 \cdot 10^{-4} = 1.0$. Using the chart for $\kappa L_g = 1.0$,

$$A = \left(\Gamma g_{th} - \langle\alpha_i\rangle\right) L_g = 3.1 => \Gamma g_{th} = \frac{3.1}{L_g} + \langle\alpha_i\rangle = \frac{3.1}{500 \cdot 10^{-4}}\text{cm}^{-1} + 10\text{cm}^{-1} = 72\ \text{cm}^{-1}.$$

For a DFB laser, the differential efficiency is given by

$$\eta_d = \eta_i \frac{A}{\Gamma g_{th} L_g} = 0.75 \frac{3.1}{72 \cdot 0.05} = 0.646,$$

where half of the power would be emitted from each end of the grating, yielding a single sided differential efficiency of $\eta_d = 0.323$. In a quarter-wave shifted DFB laser, the detuning of the lasing wavelength from the Bragg wavelength is zero (as seen from the chart in Figure **??** (b)), so the lasing wavelength is 1550 nm.

For the HR-AR coated DFB laser, we now need to use the chart from Figure **??** (b). We follow the same procedure. Using the chart for $\kappa L_g = 1.0$,

$$A = \left(\Gamma g_{th} - \langle\alpha_i\rangle\right) L_g = 1.2 => \Gamma g_{th} = \frac{1.2}{L_g} + \langle\alpha_i\rangle = \frac{1.2}{500 \cdot 10^{-4}}\text{cm}^{-1} + 10\text{cm}^{-1} = 34\ \text{cm}^{-1}.$$

$$\eta_d = \eta_i \frac{A}{\Gamma g_{th} L_g} = 0.75 \frac{1.2}{34 \cdot 0.05} = 0.529,$$

where in this case, nearly all of the power is coming out of the AR coated facet.

Using the normalized plot of threshold modal gain and wavelength from Fig. **??**, the normalized detuning for the lasing mode is $\delta L_g = 0.35$, therefore the lasing wavelength is

$$\beta = \frac{2\pi}{\lambda}\bar{n} = \frac{2\pi}{\lambda_0}\bar{n} + \frac{\delta L_g}{L_g} => \lambda = \frac{2\pi\bar{n}}{\frac{2\pi}{\lambda_0}\bar{n} + \frac{\delta L_g}{L_g}} = \frac{2\pi}{0.013 + 0.0000007}\text{nm} = 1551.087\ \text{nm}$$

End Example

Example 3.5

Two DFB lasers from Example 3.4, quarter wave shifted and HR-AR coated are operating at 4 times their respective threshold currents. Assume that the value of $\beta_{sp} = 1.25 \cdot 10^{-5}$ and $\eta_r = 0.8$.

Problem: Determine the side mode suppression ratio for both laser types

Solution : To determine the side more suppression ratio, we need to calculate the differences between modal gains and mirror losses for the two adjacent DFB modes, and use the following equation (which neglects the gain change Δg, between the two modes,

$$\text{MSR} = 10 \log \frac{F_1(\lambda_0)\alpha_m(\lambda_0)}{F_1(\lambda_1)\alpha_m(\lambda_1)} \left[\frac{\Delta\alpha}{\delta_G} + 1 \right]$$

In the case of a quarter wave shifted DFB laser, exactly half of the output power will come out of each facet, and this will not be wavelength dependent. Therefore, $F_1(\lambda_0) = F_1(\lambda_1) = 0.5$. From Fig. **??** (b), for the mode +1 and $\kappa L_g = 1.0$, the normalized mirror loss is

$$A_{+1} = (\Gamma g_{th} - \langle\alpha_i\rangle) \, L_g = 4.4$$

From Example 3.3, we have the value for this same parameter for the fundamental mode, $A_0 = 3.1$. We compute δ_G for the fundamental mode, using the Eq. **??**,

$$\delta_G = (\alpha_i + \alpha_m)\beta_{sp}\eta_r \frac{I_{th}}{(I - I_{th})} = (10 + \frac{3.1}{500 \cdot 10^{-4}})\text{cm}^{-1}10^{-5}\frac{I_{th}}{4I_{th} - I_{th}} = 72 \cdot 10^{-5} \cdot \frac{1}{3}\text{cm}^{-1} == 24 \cdot 10^{-5}\ \text{cm}^{-1}.$$

Finally, the MSR is given by

$$\text{MSR} = 10 \log \frac{1 \cdot A_0/L_g}{1 \cdot A_1/L_g} \left[\frac{\frac{A_1 - A_0}{L_g}}{\delta_G} + 1 \right] = 10 \log \frac{3.1}{4.4} \left[\frac{26}{24 \cdot 10^{-5}} + 1 \right] = 48.82\ \text{dB}$$

We note that this is a very large value. For the HR-AR coated laser, we can assume that all the output power will be coming out of one facet, and that there is no wavelength dependence in this behavior. Therefore, $F_1(\lambda_0) = F_1(\lambda_1) = 1.0$. From Fig. **??** (b), and Example 3.3, we have $A_{+1} = (\Gamma g_{th} - \langle\alpha_i\rangle) \, L_g = 1.8$ and $A_0 = 1.2$. This yields a δ_G of

$$\delta_G = (\alpha_i + \alpha_m)\beta_{sp}\eta_r \frac{I_{th}}{(I - I_{th})} = (10 + \frac{1.2}{500 \cdot 10^{-4}})\text{cm}^{-1}10^{-5}\frac{I_{th}}{4I_{th} - I_{th}} = 34 \cdot 10^{-5} \cdot \frac{1}{3}\text{cm}^{-1} == 11.33 \cdot 10^{-5}\ \text{cm}^{-1}.$$

Therefore, the MSR can be calculated as

$$\text{MSR} = 10 \log \frac{1 \cdot A_0/L_g}{1 \cdot A_1/L_g} \left[\frac{\frac{A_1 - A_0}{L_g}}{\delta_G} + 1 \right] = 10 \log \frac{1.2}{1.8} \left[\frac{12}{11.33 \cdot 10^{-5}} + 1 \right] = 48.49\ \text{dB}.$$

End Example