Chapter 5

Dynamic Effects

5.1 INTRODUCTION

5.1.1 Mode Locking

Because the axial modes of a laser cavity satisfy the same axial boundary conditions giving regularly spaced modal frequencies according to Eq. **??**, it is also possible to align then in time such that they will all add in phase at some point along the cavity and be generally out of phase elsewhere. The result as time proceeds is a pulsed output with pulses whose width and repetition rate will be determined by the length and other design aspects of the laser cavity and the active region. Figure XXX illustrates this phenomenon at some instant in time. This type of laser may be of interest for various applications that require optical pulses, such as different medical applications, laser based distance and speed measurements (LADAR and range finding), as well as optical regeneration. To illustrate how a number continuously lasing modes can produce output light in the form of pulses, we can look at a simple case of a cavity with two equal amplitude adjacent longitudinal modes. Using the time and space-varying electric field representation from Eq. **??**, and looking at the total electric field variation in time for constant z (we can use $z = 0$ without the loss of generality, the two mode fields can be expressed as

$$\mathscr{E}_1 = \hat{\mathbf{e}}_y E_0 U(x,y) e^{j(\omega t)} \tag{5.1}$$

$$\mathscr{E}_2 = \hat{\mathbf{e}}_y E_0 U(x,y) e^{j((\omega - \Delta\omega)t)} \tag{5.2}$$

where $\Delta\omega$ is the difference between the two longitudinal cavity modes, and in diode lasers, $\Delta\omega << \omega$. Adding the two modes to obtain the total electric field yields

$$\mathscr{E} = \mathscr{E}_1 + \mathscr{E}_2 = 2\hat{\mathbf{e}}_y E_0 U(x,y) e^{j(\frac{(2\omega+\Delta\omega)t)}{2}} e^{j(\frac{(\Delta\omega)t)}{2}}.$$

This result corresponds to a fast oscillation at frequency $\approx \omega$ modulated by a slowly varying envelope, with frequency $\frac{\Delta\omega}{2}$, . Since the optical power is proportional the amplitude squared of the electric field, two simultaneously lasing modes give rise to pulse-like optical intensity at a given cross section of the laser.

In a practical laser cavity, the number N of cavity modes that can lase simultaneously will be determined by the gain bandwidth of the laser active reion. If we now extend the two-mode approach to this case, the total electric field will be given by

$$\mathscr{E}(t) = \Sigma E_m e^{j(\omega_m t + \phi_m(t))}$$

where E_m is the amplitude, ω_m is the angular frequency, and $\phi_m(t)$ is the time dependent phase of the mth lasing mode. The difference in angular frequency between adjacent cavity modes is given by

$$\Delta\omega = \omega_m - \omega_{m-1} = \frac{2\pi c}{2n_g L},$$

where L is the cavity length, and n_g is the group velocity. We note that the resultant electric field is periodic in time, with a period defined by $T = \frac{2\pi}{\Delta\omega}$. This will be important in the next subsection, when calculating the pulse repetition rate.

The intensity of the light is proportional to $|\mathscr{E}(t)|^2 = \mathscr{E}(t)\mathscr{E}^*(t)$, where $\mathscr{E}^*(t)$ is the complex conjugate of $\mathscr{E}^(t)$. The expression for the intensity contains a number of terms of the form $E_m E_n e^{j(\omega_m - \omega_n)t + (\phi_m(t) - \phi_n(t))}$. If the phase difference term, $\phi_m(t) - \phi_n(t)$, is constant in time, we say that the modes are locked in phase. The practical implication of this case is that the temporal intensity of the ligth at any point in the laser can be described as a sum of a number of phasor vectors, all rotating at angular velocities which are integer multiples of $\Delta\omega$. By fixing the phase differences at zero, and for simplicity, assuming equal amplitudes of the modes, the resultant electric field can be described as

$$\mathscr{E}(t) = E_0 e^{j\omega_0 t} \sum_{m=-N/2}^{N/2} e^{jm\Delta\omega t} = E_0 e^{j\omega_0 t} \frac{\sin(\frac{N\Delta\omega t}{2})}{\frac{\Delta\omega t}{2}}.$$

Here, for simplicity, we have assumed that there is an even number N of modes supported, and that the laser gain peak is at the angular frequency of ω_0. Similarly to the two mode case, Eq.5.1.1, we have the light intensity that is a product of two factors, one varying rapidly with the average frequency ω_0, and the other varying much more slowly, with the frequency of $\Delta\omega$. A graphic respresentation of the Eq. 5.1.1 is shown in Figure XXX. At time $t = 0$, all of the phasor vectors are along the real axis, and the optical pulse is at its peak power. With the time increasing, the optical phasors rotate in the complex plane, reducing the pulse power, until the vector with the highest value of angular velocity, $\frac{N}{2}\Delta\omega$ reaches the negative real axis. At that point in time, all the vectors are evenly distributed, and the resulting amplitude of the electril field is zero. This point in time corresponds to the end of the laser pulse. From this analysis, we can determine the width of the pulse by noting that

$$\frac{N}{2}\Delta\omega\Delta t_p = \pi$$

from where we have that

$$\Delta t_p = \frac{2\pi}{N\delta\omega} = \frac{1}{\Delta\nu}.$$

This result is expected, as it is consistent with the Fourier thansform relationship between the bandwidth and the pulse width. This is the minimum possible value of the pulse width , and it is thus called transform limited.

After the end of the pulse, the phasors will continue to rotate, spanning the whole complex plane, and the resulting output field will be low. However, after the vector corresponding to the $m = 1$ (slowest) mode has rotated over the full circle and it concides with the real positive axis, the other vectors will have rotated an integer number of full circles, and they will be lined up with the phasor corresponding to the first mode. Therefore, the period corresponding to the pulse creation can be calculated based on the angular velocity of the $m = 1$ mode as

$$T = \frac{2\pi}{\Delta\omega} = \frac{2L\bar{n}_g}{c}.$$

This is the same period that was established previously, from examining the equation. Intuitively, it means that the pulse repetition rate is determined by the cavity parameters of the laser.

The output power from a multi-mode laser is equal to the sum of powers of each individual mode. The power contained in each of these modes is the same, whether or not the modes are locked in phase. If for illustrative purpuses, we assume that each mode has the same electric field amplitude E_0, the total output power when the modes are not locked would be proportional to $N \cdot |E_0|^2$. On the other hand, in the mode locked configuration, at the pulse peak, all the mode amplitudes add in phase, leading to the peak power proportional to $N^2 \cdot |E_0|^2$. Since the peak power in the mode locked laser is related to the average power $\langle P \rangle$ as

$$P = N \langle P \rangle .$$

a large number of oscillating modes can greatly enhance the pulse power in a mode locked laser. Mode locking does not change the average power of the laser beam, but rather redistributes this energy in time.

5.1.2 Mode Locking Techniques

One simple technique that can be used for mode locking a laser is to introduce a time variable loss inside the cavity. This type of mode locking is called active mode locking. If this loss is modulated in time at a frequency equal to the mode separation, it will lead to coupling of the two adjacent cavity modes. If the coupling is sufficiently strong, the phase of the different modes will be locked, resulting in mode locking.

In time domain, with a sinusoidal loss modulation applied, the time between loss minima is $T = 2n_gL/c$, the round-trip time for a pulse in the cavity. Therefore, the light that arrives at the variable loss medium when the loss is a minimum has a lower round-trip loss than light arriving at other times. This leads to the process of self-selection of the modes, where only the particular combination of modes which are locked in phase has the lowest lasing threshold.

Another approach to achieving a mode-locked combination of modes spontaneously in the cavity, without the need for a user-supplied modulation, is called passive mode locking. This is generally accomplished by inserting a saturable absorber inside the cavity. The absorption coefficient of the saturable absorber decreases with increasing light intensity, which favors the development of high intensity pulses. The gain for the passively mode locked laser is set just below threshold, so that CW lasing will only occur if the light intensity is high enough to decrease the absorption loss in the saturable absorber. This, in turn, can occur if the modes lock together in phase to create short pulses, since the peak power in each pulse is very high compared with the equivalent CW power. The pulse length is determined by the recovery speed of the saturable absorber material.