Example 4.1

In a semiconductor optical amplifier, the TE and the TM mode gains are found to be the same for a particular wavelength of incident light with sufficient current applied. Considering only C-HH1 transition in a 8 nmwide unstrained GaAs quantum well surrounded by the 20 % Al-AlGaAs barriers, having the lowest transition wavelength of 840 nm.

Problem: What in-plane k-vector and wavelength would satisfy the equal TE and TM gain condition?

Solution : To solve this problem, we need to utilize the material from Appendix 10, and we need to recall that the polarization dependence in the gain of the quantum wells comes from the matrix element M_T. From Figure A10.XXX, which applies for this material system, at the bottom of the first subband, for $k_t = 0$, all of the gain is going to the TE polarized photons. If we can tailor the value of the in-plane k-vector k_t, we can achieve the conditions under which the values of the matrix elements for TE and TM polarization will be the same. From Figure A10.XXX, this condition is fulfilled for $k_t = 0.03\text{Å}^{-1}$, in which case

$$\left|\frac{M_T}{M}\right|^2_{TE} = \left|\frac{M_T}{M}\right|^2_{TM} = 0.21. \tag{1}$$

The transition energy is equal to the bandgap energy, plus kinetic energies of electrons and holes, as expressed by Eq. **??**. Therefore,

$$\Delta E_t = \frac{\hbar^2 k_t^2}{2m_c} + \frac{\hbar^2 k_t^2}{2m_v} = \frac{\hbar^2 k_t^2}{2m_r}. \tag{2}$$

Using values from Table XXX, we can compute the reduced mass as $m_r = \dfrac{m_c m_v}{m_c + m_v} = \dfrac{0.067 \cdot 0.38}{0.067 + 0.38} m_0 = 0.057 \cdot m_0$. From there, $\Delta E_t = 5.98 \cdot 10^{-2}\text{eV}$. Finally, the wavelength can be obtained from the relationship

$$\frac{\Delta\lambda}{\lambda} = -\frac{\Delta E}{E_g}, \tag{3}$$

where E_g corresponds to the transition wavelength of $\lambda = 840\text{nm}$. From here, we have

$$\Delta\lambda = -\frac{\lambda}{E}\Delta E = -\frac{840\text{nm}}{1.24/0.840\text{eV}} \cdot 5.98 \cdot 10^{-2}\text{eV} = 34\text{nm} \tag{4}$$

and $\lambda = 840 - 34\text{nm} = 806\text{nm}$.

End Example

Example 4.2

An optical probe beam is transmitted through an AR-coated GaAs epi-wafer normal to the surface to measure the absorption properties of an active region that lies in the plane of the wafer. The active region contains a single strained InGaAs quantum well, 8 nm in thickness within a GaAs/AlGaAs waveguide, with the confinement factor of 0.015. The probe beam has a wavelength of 970 nm, and the lowest energy level in the quantum well provides an absorption edge at 980 nm. It is determined that the one-pass absorption of the probe beam through the unpumped quantum well is 1.5 %. Also, the threshold modal gain of a 3 µm wide, 500 µm long ridge laser made in this material is 29.8 cm^{-1}.

Problem: (1) What is the maximum material gain at 970 nm for a very strongly pumped active region? (2) What is the Fermi function difference $(f_2 - f_1)$ at the lasing threshold?

Solution : From the transmission measurements, we have that the single pass absorption is 1.5 %. That is,

$$\frac{P_{out}}{P_{in}} = e^{g_{max} d},$$

where d is the thickness of the well. From here, maximum gain is

$$g_{max} = \frac{0.015}{8 \cdot 10^{-7}\text{cm}} = 1.875 \cdot 10^4 \text{cm}^{-1}.$$

To calculate the Fermi function difference $(f_2 - f_1)$, we utilize the relationship for gain **??**,

$$g = g_{max}(f_2 - f_1).$$

Therefore, we need to calculate the threshold gain, which can be obtained from the parameters given in the problem,

$$g_{th} = \frac{\Gamma g}{\Gamma} = \frac{29.8}{0.015} = 1986.67\text{cm}^{-1}. \tag{5}$$

Finally,

$$(f_2 - f_1) = \frac{g_{th}}{g_{max}} = \frac{1986.67}{18750} = 0.106.$$

End Example

Example 4.3

For the ridge lasers from Example XXX, we would like to know the ratio of the spontaneous emission power to the output power at 40 mA bias. Assume that the value of the population inversion factor $n_{sp} = 1.5$.

Problem: (1) Calculate the spontaneous emission power into the mode above threshold (2) Calculate the optical mode density for this laser (3) What is the total spontaneous emission power within a 1 nm bandwidth near the lasing wavelength?

Solution : The spontaneous emission power into a mode can be calculated from the known spontaneous emission rate, active region confinement factor Γ and photon energy. The spontaneous emission rate into a mode is given by Eq. **??**. From here, we have

$$P_{sp} = R_{sp} \cdot (h\nu) \cdot V = \frac{\Gamma g_{th} v_g n_{sp}}{V} \cdot (h\nu) \cdot V = (29.8\text{cm}^{-1})(\frac{3}{4.5} \cdot 10^{10}\frac{\text{cm}}{\text{s}})(1.5)(\frac{1.24\text{eV} \cdot \mu\text{m}}{0.97\mu\text{m}})(1.6 \cdot 10^{-19})\frac{\text{J}}{\text{eV}} = 61.0n\text{W}. \quad (6)$$

To determine the total spontaneous emission power in a given wavelength range, we need know the total spontaneous emission rate over all modes that exist in that wavelength range, as well as the photon energy and the active region volume.

The total spontaneous emission per unit energy and per unit volume is given by Eq. **??** as $R_{sp}^{21} = \frac{1}{h}\rho_0(\nu_{21}) \cdot v_g n_{sp}\bar{g}_{21}$. To compute it, we need to compute the mode density ρ_0 and the average gain $\bar{g}_{21}$. From Appendix 4, the expression for the density of optical modes is given by Eq. **??**,

$$\rho_0(\nu) = \frac{8\pi}{c^3}n^2 n_g \nu^2 => \frac{1}{h}\rho_0(\nu) = \frac{8\pi}{(3 \cdot 10^{10}\text{cm/s})^3}\frac{(3.6)^2(4.5)(\frac{1.24\text{eV}\mu\text{m}}{0.97\mu\text{m}})^2}{(\frac{6.626 \cdot 10^{-34}\text{Js}}{1.6 \cdot 10^{-19}\text{J/eV}})^3} = 1.25 \cdot 10^{15}\text{eV}^{-1}\text{cm}^{-3}. \quad (7)$$

The average gain is given by

$$\bar{g}_{21} = \frac{1}{3}(2g_21^{TE} + g_{21}^{T}M) = \frac{2}{3}g_21^{TE} = \frac{2}{3}1986.67\text{cm}^{-1} = 1324\text{cm}^{-1}. \quad (8)$$

Now, we can calculate the total spontaneous emission rate per unit volume as

$$R_{sp}^{21} = (\frac{1}{h}\rho_0(\nu_{21})) \cdot (v_g)(n_{sp})(\bar{g}_{21}) = (1.25 \cdot 10^{15}\text{eV}^{-1}\text{cm}^{-3})(\frac{3}{4.5} \cdot 10^{10}\frac{\text{cm}}{\text{s}})(1.5)(1324\text{cm}^{-1}) = 1.65 \cdot 10^{28}\text{eV}^{-1}\text{cm}^{-3}\text{s}^{-1}. \quad (9)$$

In order to calculate the total power in the 1 nm bandwidth, we need to convert the bandwidth into energy, and calculate the active region volume,

$$\Delta E = \Delta\lambda\frac{dE}{d\lambda} = \Delta\lambda(-\frac{E_p}{\lambda_p}) = 1\text{nm}(-\frac{1.24\text{eV}\mu\text{m}}{(0.97\mu\text{m})2}) = 1.32 \cdot 10^{-3}\text{eV} \quad (10)$$

$$V_a = w \cdot d \cdot L = (3\mu\text{m})(8\text{nm})(500\mu\text{m}) = 12000 \cdot 10^{-15}\text{cm}^3. \quad (11)$$

Finally, the total spontaneous power emitted in the 1 nm wavelength range around the lasing wavelength is

$$P_{sptotal} = R_sp^{21} \cdot \Delta E \cdot h\nu \cdot V_a = (1.25 \cdot 10^{15}\text{eV}^{-1}\text{cm}^{-3})(1.32 \cdot 10^{-3}\text{eV})(\frac{1.24\text{eV}\mu\text{m}}{0.97\mu\text{m}})(12000 \cdot 10^{-15}\text{cm}^{-3}) = 53.5\mu\text{W}. \quad (12)$$

End Example

Example 4.4

The surface recombination velocity can be estimated using the simple "broad-area" (i.e., infinite stripe width) threshold carrier density, however, in reality the carrier density profile will vary over the cross section of the active region, particularly when the active width is narrow. In this problem, the effects of a finite diffusion constant for carriers in the active region will be examined. Assume that the carrier densities in the active region are high enough that any differences in the diffusion profiles of electrons and holes will set up an electric field which will pull the two densities to nearly the same profile. In this *ambipolar* diffusion limit, the hole diffusion rate is enhanced by a factor of ~ 2 by the forward pull of the electrons, and the electron diffusion rate is limited to approximately twice the normal hole diffusion rate by the backward pull of the holes.

The overall effect is that we can assume the electron and hole densities are equal everywhere in the active region and are characterized by a single ambipolar diffusion constant, D_{np}. The lateral profile of carriers is then governed by the simple diffusion equation:

$$D_{np}\frac{d^2N(x)}{dx^2} = -\frac{I(x)}{qV} + \frac{N(x)}{\tau_{np}}. \tag{13}$$

The carrier lifetime is in general a function of N, however, to obtain analytic solutions, we can evaluate the lifetime at the broad-area threshold value, $\tau_{np}|_{th} = qL_zN_{th}/J_{th}$. The problem we wish to solve is the carrier density profile across the width of the active region in the in-plane laser depicted in Fig. **??**. For this case, we can define two distance regions: one beneath the contact within w where we assume a uniform current injection profile, and the region outside of w where there is no current injection. Mathematically, with $x = 0$ defined as the center of the stripe, we have $I(x) = I_0$ for $x < w/2$, and $I(x) = 0$ for $x > w/2$. In fabricating the laser we can either leave the active region in place outside of the stripe, or we can remove it by etching through the active region outside of the contact area. The first case leads to carrier outdiffusion, while the second case leads to surface recombination. We would like to compare these two cases.

Problem: (1) With the active region in place away from the contact, carriers are free to diffuse outside the stripe width. Solve for the concentration of the carriers $N(x)$ in and out of the stripe assuming the carrier density and its derivative (i.e., the diffusion current) are constant across the $x = w/2$ boundary. Solve for the carrier profile in this case. (2) With the active region etched away, the carriers recombine at the surface. Solve for the concentration of the carriers $N(x)$ under the stripe assuming the diffusion current (defined by the slope of the carrier density) is equal to the surface recombination current, $D_{np}dN/dx = -v_sN$, at the $x = w/2$ boundary. Place your result in terms of the diffusion equivalent surface recombination velocity, $v_{sD} = \sqrt{D_{np}/\tau_{np}}$. Solve for the carrier profile in this case.

Solution : From the carrier diffusion equation, **??**, we can express the carrier concentration as

$$\frac{d^2N(x)}{dx^2} - \frac{N(x)}{D_{np}\tau_{np}} = -\frac{I(x)}{qVD_{np}}. \tag{14}$$

If we define $x = 0$ as the lateral center of the laser stripe, we then have that

$$I(x) = \begin{cases} I_0 & |x| \le |\frac{W}{2}| \\ 0 & |x| > |\frac{W}{2}| \end{cases} \tag{15}$$

If we define $L = \sqrt{D_{np}\tau_{np}}$ and $G = \frac{I(x)}{qVD_{np}}$, then the carrier diffusion equation can be rewritten as

$$\frac{d^2N(x)}{dx^2} - \frac{N(x)}{L^2} = -G.$$

This is a special simplified form of the second order linear partial differential equation, whose solutions can be expressed analythically [Kreyszig].

For $|x| < \left|\frac{W}{2}\right|$, withing the stripe width, the carrier diffusion equation has a solution of

$$N(x) = A\cosh\left(\frac{x}{L}\right) + GL^2,$$

since the solution needs to be symmetric about $x = 0$, due to the statement of the problem. A will be determined by matching the boundary conditions.

For $|x| > \left|\frac{W}{2}\right|$, withing the stripe width, the carrier diffusion equation has a solution of

$$N(x) = Be^{\frac{-|x|}{L}},$$

where B will be determined by matching the boundary conditions. The other mathematically possible solution, $e^{\frac{|x|}{L}}$, is not physical, since the carrier density would go to infinity with $x- > \infty$.

Case 1 - active region left outside the stripe

In this case, the boundary conditions at $x = \frac{w}{a}$ are that $N(x)$ is continuous, and $\frac{dN(x)}{dx}$ is continuous. Applying these boundary conditions, we have

$$A = \frac{-1}{2}GL^2e^{-w/2L} \tag{16}$$

$$B = -A\left(e^{w/L} - 1\right) \tag{17}$$

$$= \frac{GL^2}{2}\left(e^{w/2L} - e^{-w/2L}.\right) \tag{18}$$

$$\tag{19}$$

Therefore, the carrier density profile is given by

$$N(x) = \begin{cases} GL^2\left[1 + \frac{-1}{2}\left(e^{(x-w/2)/L} + \left(e^{(-x-w/2)/L}\right)\right)\right] & |x| \leq \left|\frac{W}{2}\right| \\ \frac{GL^2}{2}\left(e^{(w/2-|x|)/L} - \left(e^{(-|x|-w/2)/L}\right)\right) & |x| > \left|\frac{W}{2}\right| \end{cases} \tag{20}$$

Case 2 - active region etched outside the stripe

In this case, there will be no carriers outside of the active strip. The solution for the carrier density equation inside the strip is the same as previously derived, however, there will be a different boundary condition to satisfy at the surface. Basically, the diffusion current at the boundary needs to be equal to the surface recombination current, $-qv_sN = qD_{np}\frac{dN}{dx}$. If we define surface recombination velocity as $v_{sD} = \sqrt{\frac{D_{np}}{\tau_{np}}}$, we have

$$N(x) = GL^2\left[1 - \frac{e^{x/L} + e^{-x/L}}{\left(1 + \frac{v_{sD}}{v_s}e^{\frac{w}{2L}}\right) + \left(1 - \frac{v_{sD}}{v_s}e^{\frac{-w}{2L}}\right)}\right].$$

End Example

Example 4.5

In a compressive strained, 1550 nm quantum well material, the Auger coefficient was measured to be $C = 6 \cdot 10^{-29}\text{cm}^6/\text{s}$ at 300K. The Auger threshold energy can be assumed to be 10% higher than the bandgap energy corresponding to 1.55 nm lasing wavelength.

Problem: If the temperature were increased to 340K, what is the value of the new Auger coefficient C?

Solution : (2) From the problem statement, we can establish the relationship between Auger coefficients at different temperatures,

$$C(340\text{K}) = C_0 \cdot e^{-\dfrac{0.1 \cdot E_g}{kT}}.$$

.

For this active material, the bandgap energy is $E_g = 800m\text{eV}$. At 300K,

$$C(300\text{K}) = 6 \cdot 10^{-29}\text{cm}^6/\text{s} = C_0 \cdot e^{-\frac{80}{26}},$$

where $kT = 26m\text{eV}$ at room temperature. Finally,

$$C(340\text{K}) = C(300\text{K}) \cdot e^{\frac{300}{340}} = 1.436 \cdot C(300\text{K}) = 8.617 \cdot 10^{-29}\text{cm}^6/\text{s} \tag{21}$$

End Example

Example 4.6

The GaAs/AlGaAs quantum well active material from Table 4.4 is part of a 5 quantum well stack based material used to fabricate an 800 µm long and 3 µm wide ridge laser. The transverse confinement factor Γ_1 is 5.5%. From parameter extraction, the average internal losses are determined to be $\alpha_i = 14\text{cm}^{-1}$., and the injection efficiency is $\eta_i = 0.75$.

Problem: Determine the threshold current for this laser and compare the contributions from spontaneous and non-radiative threshold current components

Solution : The threshold modal gain of this all active laser is given by

$$\Gamma g_{th} = \langle\alpha_i\rangle + \frac{1}{L}\ln\left(\frac{1}{R}\right) = 14\text{ cm}^{-1} + \frac{10^4}{800}\ln\left(\frac{1}{0.32}\right)\text{ cm}^{-1} = 28.24\text{ cm}^{-1}.$$

To calculate the threshold current, we first need to calculate the threshold carrier density using the Eq. **??**. Since we are dealing with a GaAs based material system, we only need to take into account the spontaneous threshold current component, given by Eq. **??**. However, we need to compare the two current components, and thus need to calculate the non-radiative threshold current as well, given by Eq. **??**. Using the two parameter fit from Table 4.4, and Eq. **??**, the threshold carrier density is

$$N_{th} = N_{tr}e^{\frac{g_{th}}{g_{0N}}} = 2.6e^{\frac{28.24}{0.055\cdot 2400}} \cdot 10^{18}; \text{cm}^{-3} = 3.22 \cdot 10^{18}\text{ cm}^{-3}.$$

The volume of the active region consists of 5 8 nm wide quantum wells, bound by the laser facets and the ridge width,

$$V = L \cdot W \cdot N_w \cdot a = 800 \cdot 3 \cdot 5 \cdot 0.008 \cdot 10^{-12}\text{cm}^{-3} = 96 \cdot 10^{-12}\text{cm}^{-3}.$$

Finally, the threshold current is given by

$$I_{th} = \frac{qV}{\eta_i}(BN_{th}^2 + CN_{th}^3) = \frac{1.602 \cdot 10^{-19} \cdot 96 \cdot 10^{-12}\text{cm}^{-3}}{0.75}\left(10.37 \cdot 10^{26} + 1.334 \cdot 10^{26}\right)\ \text{A} = 24\ \text{mA}$$

where $B = 1 \cdot 10^{-10}\text{cm}^3/s$ and $C = 4 \cdot 10^{-30}\text{cm}^6/s$.

The contribution from the Auger non-radiative current is $\frac{1.334}{11.704} = 11.4\%$.

End Example

Example 4.7

Table 4.5 gives two parameter fit data for the material gain of 1% compressively strained 3 nm thick 1.55 µm quantum well on Indium Phosphide that neglect Auger recombination. Now, we would like to include an Auger coefficient, determined in example 4.5.

Problem: Determine the new J_{tr} and g_0 that would best model the gain curve for the gains between 500 and 2500cm^{-1}.

Solution : To solve this problem, we need to compute g_{02} and J_{tr2} of the new logarithmic relationship between the gain and current density. To accomplish this, we need to calculate the current densities for two different values of gain. The new current density is given by $J_2 = J_1 + J_A$, where $J_A = \dfrac{q \cdot C \cdot N^3 \cdot V}{area} = q \cdot d \cdot C \cdot N^3$. From Table XXX, we have that $g_{01} = 2600\text{cm}^{-1}$ and $J_{tr1} = 13\text{A/cm}^2$, and the quantum well width $d = 3\text{nm}$. Since we also know the carrier density versus gain dependence for this material from Table XXX, $g = g_{0N} \ln \frac{N}{N_{tr}}$, and $g_{0N} = 4000\text{cm}^{-1}$ and $N_{tr} = 3.3 \cdot 10^{18}\text{cm}^{-3}$, we are in a position to calculate the total current density. At transparency, $g = 0$, we have

$$J_{tr2} = J_{tr1} + J_{Atr} = 13\text{A/cm}^2 + q \cdot d \cdot CN_{tr}^3 = (13 + 103)\text{A/cm}^2. \tag{22}$$

At $J_1 = e \cdot J_{tr1}$, we have

$$g = g_{01} \ln \frac{e \cdot J_{tr}}{J_{tr}} = g_{01} = 2600\text{cm}^{-1}, \quad N = N_{tr} e^{\frac{g}{g_{0N}}} = 3.33 \cdot 10^{18} e^{\overline{4000}}\text{cm}^{-3} = 6.32 \cdot 10^{18}\text{cm}^{-3}. \tag{23}$$

with 2600 printed above the $\overline{4000}$.

From here,

$$J_A = q \cdot d \cdot C \cdot N^3 = (1.6 \cdot 10^{-19}\text{C})(3 \cdot 10^{-7}\text{cm})(6 \cdot 10^{-29}\text{cm}^6/\text{s})(6.32 \cdot 10^{18}\text{cm}^{-3}) = 727\text{A/cm}^2$$

and finally

$$J_2 = e \cdot J_{tr1} + J_A = 762\text{A/cm}^2.$$

Now, g_{02} can be calculated from the transparency current,

$$g_{02} = \frac{g_{01}}{\ln \dfrac{J_2}{J_{tr2}}} = \frac{2600\text{cm}^{-1}}{\ln 762116} = 1381\text{cm}^{-1}.$$

End Example