5.14:

The 3dB point is defined as the frequency at which

$$|P(\omega)| = \frac{1}{\sqrt{2}}|P(\omega = 0)|$$

or

$$|H(\omega)| = \frac{1}{\sqrt{2}}$$

---

a) From Eq. 5.46,

$$\frac{1}{\sqrt{2}} = |H(\omega)| = \frac{\omega_R^2}{\sqrt{\left[\omega_R^2 - \omega^2\right]^2 + (\omega\gamma)^2}}$$

The solution for problem 5.12 gives values for ω_R and γ. Using these values, we can find ω_{3dB}:

$$\omega_{3dB}^4 + (-2\omega_R^2 + \gamma^2)\omega_{3dB}^2 - \omega_R^4 = 0$$

$$\omega_{3dB} = \sqrt{\frac{-(-2\omega_R^2 + \gamma^2) \pm \sqrt{(-2\omega_R^2 + \gamma^2)^2 + 4\omega_R^4}}{2}}$$

error from 5.12

$22.4 \cdot 10^9$

14.5 GHz·rad   28.8 GHz·rad

	$I = 2I_{th}$	$I = 5I_{th}$
ω_R	~~9.66×10^9~~ rad/s	38.64×10^9 rad/s
γ	1.9×10^9 s^{-1}	5.9×10^9 s^{-1}
ω_{3dB}	~~[illegible] $\times 10^9$~~ rad/s	~~59.8×10^9~~ rad/s
f_{3dB}	~~2.37~~ GHz	~~9.52~~ GHz

$44.7 \cdot 10^9$ Hz·Rad
7.4 GHz

3.56 GHz

(Note: Eq. 5.54 can also be used to get the same results (±1%) since $\gamma^2 \ll \omega_R^2$.)

---

b) Comparing Eq. 5.163 to Eq. 5.45, we can see that carrier transport effects can be included in our model by the insertion of a factor $\frac{1}{1+j\omega\tau_s}$. (Neglecting leakage, $\chi = 1$.) So, proceed as in part (a).

$$\frac{1}{\sqrt{2}} = |H(\omega)| = \left|\frac{1}{1 + j\omega\tau_s}\right| \frac{\omega_R^2}{\sqrt{\left[\omega_R^2 - \omega^2\right]^2 + (\omega\gamma)^2}}$$

We saw in part (a) that we could have neglected γ. So, we can simplify our expression to

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \omega_{3dB}^2\tau_s^2}} \frac{\omega_R^2}{\sqrt{\left[\omega_R^2 - \omega_{3dB}^2\right]^2}}$$

$$2\omega_R^4 = (1 + \omega_{3dB}^2\tau_s^2)(\omega_R^4 - 2\omega_{3dB}^2\omega_R^2 + \omega_{3dB}^4)$$

$\tau_s = 50$ ps

Solve numerically to get   20.4 rad GHz   3.25 GHz

$I = 2I_{th} \rightarrow \omega_{3dB} =$ ~~14.2×10^9~~ rad/s and $f_{3dB} =$ ~~2.?6~~ GHz

$I = 5I_{th} \rightarrow \omega_{3dB} =$ ~~48.0×10^9~~ rad/s and $f_{3dB} =$ ~~7.64~~ GHz

37.2 GHz·rad   5.92 GHz

c) *To include carrier leakage, we use the parameter τ_e which is incorporated by altering the values of* ω_R and γ according to Eq. 5.164 and 5.161:

$$\omega_{tR}^2 = \omega_R^2/\chi$$

$$\gamma_t = \frac{\gamma_{NN}}{\chi} + \gamma_{PP}$$

$$\chi \approx 1 + \frac{\tau_s}{\tau_e} = 1 + \frac{50 \text{ ps}}{200 \text{ ps}} = 1.25 \qquad (5.166)$$

From problem 5.12,

$I = 2I_{th} \rightarrow \quad \gamma_{NN} = 1.095 \times 10^9 \text{ s}^{-1}, \quad \gamma_{PP} = 0.819 \times 10^9 \text{ s}^{-1}$, and $\quad \gamma_t = 1.70 \times 10^9 \text{ s}^{-1}$

$I = 5I_{th} \rightarrow \quad \gamma_{NN} = 2.479 \times 10^9 \text{ s}^{-1}, \quad \gamma_{PP} = 3.411 \times 10^9 \text{ s}^{-1}$, and $\quad \gamma_t = 5.39 \times 10^9 \text{ s}^{-1}$

$$\frac{1}{\sqrt{2}} = \left|\frac{1}{1 + j\omega_{3dB}\tau_s}\right| \frac{\omega_{tR}^2}{\sqrt{\left[\omega_{tR}^2 - \omega_{3dB}^2\right]^2 + (\omega_{3dB}\gamma_t)^2}} \qquad \text{from Eq.(5.163)}$$

γ_t terms are negligible in comparison to ω_{tR} terms. Proceed as in part (b) to get

$$2\omega_{tR}^4 = (1 + \omega_{3dB}^2\tau_s^2)(\omega_{tR}^4 - 2\omega_{3dB}^2\omega_{tR}^2 + \omega_{3dB}^4)$$

Solve numerically to get

~~$I = 2I_{th} \rightarrow \omega_{3dB} = 11.5 \times 10^9$ rad/s and $f_{3dB} = 1.83$ GHz~~ 18.56 GHz·rad 2.94 GHz

~~$I = 5I_{th} \rightarrow \omega_{3dB} = 39.6 \times 10^9$ rad/s and $f_{3dB} = 6.30$ GHz~~ 33.78 GHz·rad 5.38 GHz

5.15:

Using normalized fields for convenience such that

$$< E(t)E(t)^* >= P_1$$

we can write the autocorrelation function in Eq. 5.144 as

$$< E(t)E(t-\tau)^* >= P_1\ e^{j\omega\tau}\ e^{-|\tau|/\tau_{coh}} \tag{5.144}$$

Now, if the power in the delayed leg is attenuated by α, such that $P_2 = \alpha P_1$, we can write the power at the detector as

$$\begin{aligned} P_{det} &=< \left[E(t) + \sqrt{\alpha}E(t-\tau)\right]\ \left[E(t) + \sqrt{\alpha}E(t-\tau)\right]^* > \\ &=< E(t)E(t)^* > +\alpha < E(t-\tau)E(t-\tau)^* > +\sqrt{\alpha} < E(t)E(t-\tau)^* > +\sqrt{\alpha} < E(t-\tau)E(t)^* > \\ &= P_1 + \alpha P_1 + \sqrt{\alpha}P_1\ e^{j\omega\tau}\ e^{-|\tau|/\tau_{coh}} + \sqrt{\alpha}P_1\ e^{-j\omega\tau}\ e^{-|\tau|/\tau_{coh}} \\ &= P_1 + P_2 + 2\sqrt{P_1P_2}\cos(\omega\tau)\ e^{-|\tau|/\tau_{coh}} \end{aligned}$$

Then, fringes are visible as the value of $\cos(\omega\tau)$ is varied:

$$P_{max} = P_1 + P_2 + 2\sqrt{P_1P_2}\ e^{-|\tau|/\tau_{coh}}$$

$$P_{min} = P_1 + P_2 - 2\sqrt{P_1P_2}\ e^{-|\tau|/\tau_{coh}}$$

$$P_{max} - P_{min} = 4\sqrt{P_1P_2}\ e^{-|\tau|/\tau_{coh}}$$

$$P_{max} + P_{min} = 2(P_1 + P_2)$$

Therefore,

$$e^{-|\tau|/\tau_{coh}} = \frac{P_1 + P_2}{2\sqrt{P_1P_2}}\ \frac{P_{max} - P_{min}}{P_{max} + P_{min}} \tag{5.145}$$

The problem asks for the coherence time for the case when $\tau = 3$ ns:

$$e^{-|\tau|/\tau_{coh}} = 0.2$$

$$\tau_{coh} = \frac{-3}{\ln(0.2)}\ \text{ns} = 1.864\ \text{ns}$$

The linewidth is then given by Eq. 5.149:

$$\Delta\nu_{FW} = \frac{1}{\pi\tau_{coh}} = 171\ \text{MHz} \tag{5.149}$$

16:

From Fig. 5.26, estimate μ_2 near $P_{02} = 2.5$ mW:

$$\mu_2 \equiv \frac{\Delta P_{p-p}}{2P} \approx \frac{0.2 \text{ mW}}{2\times 2.5 \text{ mW}} = 0.04$$

Using the effective mirror model with VCSELs, for all practical purposes, we can assume lossless mirrors he transmission is reduced by $e^{-\alpha L_{eff}}$ according to the discussion surrounding Eq. 3.64. However, $L_{eff} < 0.005$ represents a negligible reduction in transmission for VCSELs.)

Eq. 5.175 is derived assuming lossless mirrors and can be used to extract κ_f from μ_2. If all of the light coupled out of mirror 2, then $F_1 = 0$ and since $F_1 + F_2 = 1$ for lossless mirrors, we have shown that $= 1$. The problem also says to assume that we are well above threshold; so we can neglect the third term defining μ_2. With $F_2 = 1$, we can write Eq. 5.175 as:

$$\mu_2 = 2\kappa_f \tau_p \left(\frac{\eta_i}{\eta_d} - 1\right)$$

Solve for κ_f:

$$\begin{aligned}\kappa_f &= \frac{\mu_2}{2\tau_p\left(\frac{\eta_i}{\eta_d} - 1\right)} \\ &= \frac{0.04}{2(2.20 \text{ ps})\left(\frac{0.8}{0.494} - 1\right)} \\ &= 1.468 \times 10^{10} \text{ s}^{-1}\end{aligned}$$

$$\kappa_f = \frac{t_2^2\sqrt{f_{ext}}}{r_2\tau_L}$$

Solve for f_{ext}:

$$f_{ext} = \left(\frac{\kappa_f r_2 \tau_L}{t_2^2}\right)^2$$

From Table 5.1,

$r_2 = \sqrt{R_2} = 0.995$

$r_1 = 1$

$t_2^2 = 1 - 0.995^2 = 0.00998$ assuming lossless mirrors

$\tau_L = \frac{2L}{v_g} = \frac{2(10^{-4} \text{ cm})}{\frac{3}{4.2}\times 10^{10} \text{ cm/s}} = 2.8 \times 10^{-14}$ s

$$\begin{aligned}f_{ext} &= \left(\frac{(1.468\times 10^{10} \text{ s}^{-1})(0.995)(2.8\times 10^{-14} \text{ s})}{0.00998}\right)^2 \\ &= 1.68\times 10^{-3}\end{aligned}$$

$$\begin{aligned}f_{ext}(\text{dB}) &= 10\log(f_{ext}) \\ &= -27.74 \text{ dB}\end{aligned}$$

5.17:

From Fig. 5.26 and Eq. 5.175, $\cos(2\beta L_p)$ goes through one complete period for a change in current of 0.4 mA. Assuming that the change of β with current is approximately linear, we can write

$$\frac{d(2\beta L_p)}{dI} = \pm\frac{2\pi}{0.4\text{ mA}}$$

$$\beta \equiv \frac{2\pi\overline{n}}{\lambda_0}$$

Neglecting the change in index due to current and temperature, we can write the first equation above as

$$2L_p(2\pi\overline{n})\frac{d\frac{1}{\lambda_0}}{dI} = \pm\frac{2\pi}{0.4\text{ mA}}$$

which can be rearranged:

$$L_p = \pm\frac{1}{(0.4\text{ mA})\ \overline{n}\ 2\ \frac{d\left(\frac{1}{\lambda_0}\right)}{dI}}$$

and simplified

$$L_p = \pm\frac{-\lambda_0^2}{(0.8\text{ mA})\overline{n}\left(\frac{d\lambda_0}{dI}\right)}$$

All variables are known except $\frac{d\lambda_0}{dI}$. Assuming that the wavelength change with current is due solely to the change in temperature, we can write

$$\frac{d\lambda_0}{dI} = \frac{d\lambda_0}{dT}\frac{dT}{dP_d}\frac{dP_d}{dI}$$

where T and P_d denote temperature and dissipated power, respectively.

$$\frac{d\lambda_0}{dT} = 0.08\frac{\text{nm}}{^\circ\text{C}} \qquad \text{given in problem}$$

$$\frac{dT}{dP_d} = 3\frac{^\circ\text{C}}{\text{mW}} \qquad \text{given in problem}$$

$$P_d = P_{in} - P_{out} = IV - P_0$$

$$\frac{dP_d}{dI} = \frac{dP_{in}}{dI} - \frac{dP_{out}}{dI} = 3V - \frac{2.5\text{ mW} - 0\text{ mW}}{6.5\text{ mA} - 1.75\text{ mA}} = 2.47\frac{\text{mW}}{\text{mA}}$$

$$\frac{d\lambda_0}{dI} = 0.59\frac{\text{nm}}{\text{mA}}$$

Then, plugging back into the equation for L_p, we have

$$L_p = \frac{(980\text{ nm})^2}{(0.8\text{ mA})(4.2)\left(0.59\frac{\text{nm}}{\text{mA}}\right)}$$

$$L_p = \text{substrate thickness} = 480\mu\text{m}$$

5.18:

a)

$$\kappa_f = \frac{v_g t_2^2 \sqrt{f_{ext}}}{2Lr_2} \qquad \text{(p. 247)}$$

$$\kappa_f = \frac{v_g(1 - r_2^2)(r_3 t_c)}{2Lr_2}$$

$\overline{n}_g$ is not specified by the problem. Assume that laser is made in the InGaAsP material system and the mode has $\overline{n}_g = 4.0$ as mentioned on p. 40.

Assume a facet reflectivity $r_2^2 = 0.32$.

$$\kappa_f = \frac{\frac{3.0\times10^{10}\ \text{cm/s}}{4.0}(1 - 0.32)(\sqrt{0.04}\sqrt{0.25})}{2(0.03\ \text{cm})\sqrt{0.32}} = 1.50 \times 10^{10}\ \text{s}^{-1}$$

$$\tau_{ext} = \frac{2n_{fiber}L_p}{c} = \frac{2(1.45)L_p}{3 \times 10^{10}\ \text{cm/s}} = (0.967 \times 10^{-5}\ \text{s/km})L_p \qquad \text{(p. 250)}$$

$$C = \kappa_f \tau_{ext}\sqrt{1+\alpha^2} = (1.45 \times 10^5\ \text{km}^{-1})L_p\sqrt{1+\alpha^2} \qquad (5.180)$$

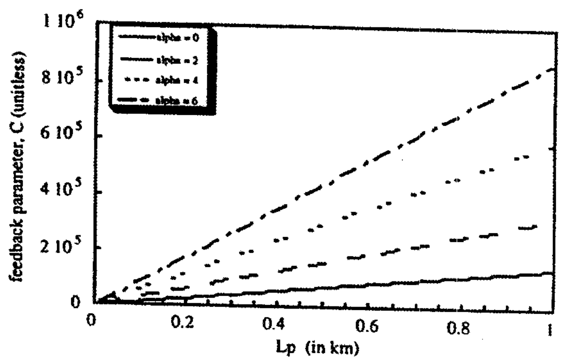

Figure 5.18a. Feedback coefficient, C, vs. fiber length, L_p.

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b)

$$\frac{\Delta\nu}{\Delta\nu_0} = \frac{1}{(1 + C\ \cos(2\beta L_p + \phi_\alpha))^2} \qquad (5.181)$$

Since $\cos(2\beta L_p + \phi_\alpha)$ can be adjusted to any value between -1 and +1, we can break this equation into 2 cases:

Case 1: $C < 1$:

minimum $\frac{\Delta\nu}{\Delta\nu_0} = \frac{1}{(1+C)^2}$

maximum $\frac{\Delta\nu}{\Delta\nu_0} = \frac{1}{(1-C)^2}$

Case 2: $C \geq 1$:

minimum $\frac{\Delta\nu}{\Delta\nu_0} = \frac{1}{(1+C)^2}$

maximum $\frac{\Delta\nu}{\Delta\nu_0} = \infty$

For $C \geq 1$, $\cos(2\beta L_p + \phi_\alpha)$ can adjusted to be equal to $1/C$, which yields an infinite bandwidth. In this case, the laser has split into two lasing modes, which implies that that $\Delta\nu$ is meaningless for this case.

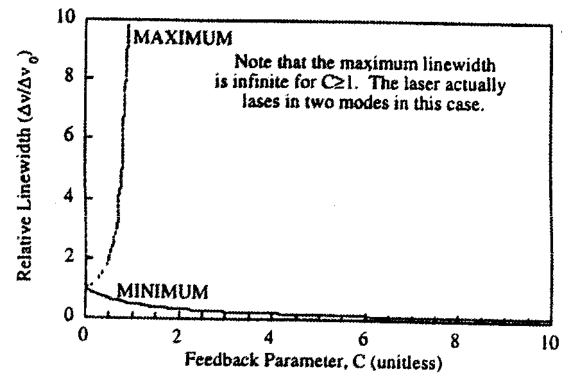

Figure 5.18b. Laser linewidth vs. feedback coefficient, C.