5.3:

Use Eq. 5.38 as a starting place. (This neglects intermodal gain compression.)

$$\frac{d}{dt}\begin{bmatrix} dN \\ dN_{P1} \\ dN_{P2} \end{bmatrix} = \begin{bmatrix} -\gamma_{NN} & -\gamma_{NP_1} & -\gamma_{NP_2} \\ \gamma_{P_1N} & -\gamma_{P_1P_1} & 0 \\ \gamma_{P_2N} & 0 & -\gamma_{P_2P_2} \end{bmatrix} \begin{bmatrix} dN \\ dN_{P1} \\ dN_{P2} \end{bmatrix} + \frac{\eta_i}{qV}\begin{bmatrix} dI \\ 0 \\ 0 \end{bmatrix}$$

Proceeding as in the text, we write the equivalent of Eq. 5.40:

$$\begin{bmatrix} \gamma_{NN}+j\omega & \gamma_{NP_1} & \gamma_{NP_2} \\ -\gamma_{P_1N} & \gamma_{P_1P_1}+j\omega & 0 \\ -\gamma_{P_2N} & 0 & \gamma_{P_2P_2}+j\omega \end{bmatrix} \begin{bmatrix} dN \\ dN_{P1} \\ dN_{P2} \end{bmatrix} = \frac{\eta_i I_1}{qV}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Define Δ:

$$\begin{aligned}\Delta &= \begin{vmatrix} \gamma_{NN}+j\omega & \gamma_{NP_1} & \gamma_{NP_2} \\ -\gamma_{P_1N} & \gamma_{P_1P_1}+j\omega & 0 \\ -\gamma_{P_2N} & 0 & \gamma_{P_2P_2}+j\omega \end{vmatrix} \\ &= (\gamma_{NN}+j\omega)(\gamma_{P_1P_1}+j\omega)(\gamma_{P_2P_2}+j\omega)+\gamma_{NP_1}\gamma_{P_1N}(\gamma_{P_2P_2}+j\omega)+\gamma_{NP_2}\gamma_{P_2N}(\gamma_{P_1P_1}+j\omega)\end{aligned}$$

By Cramer's Rule,

$$N_1 = \frac{\eta_i I_1}{qV}\frac{1}{\Delta}\begin{vmatrix} 1 & \gamma_{NP_1} & \gamma_{NP_2} \\ 0 & \gamma_{P_1P_1}+j\omega & 0 \\ 0 & 0 & \gamma_{P_2P_2}+j\omega \end{vmatrix} = \frac{\eta_i I_1}{qV}\frac{1}{\Delta}(\gamma_{P_1P_1}+j\omega)(\gamma_{P_2P_2}+j\omega)$$

$$N_{P1} = \frac{\eta_i I_1}{qV}\frac{1}{\Delta}\begin{vmatrix} \gamma_{NN}+j\omega & 1 & \gamma_{NP_2} \\ -\gamma_{P_1N} & 0 & 0 \\ -\gamma_{P_2N} & 0 & \gamma_{P_2P_2}+j\omega \end{vmatrix} = \frac{\eta_i I_1}{qV}\frac{1}{\Delta}(\gamma_{P_1N})(\gamma_{P_2P_2}+j\omega)$$

$$N_{P2} = \frac{\eta_i I_1}{qV}\frac{1}{\Delta}\begin{vmatrix} \gamma_{NN}+j\omega & \gamma_{NP_1} & 1 \\ -\gamma_{P_1N} & \gamma_{P_1P_1}+j\omega & 0 \\ -\gamma_{P_2N} & 0 & 0 \end{vmatrix} = \frac{\eta_i I_1}{qV}\frac{1}{\Delta}(\gamma_{P_2N})(\gamma_{P_1P_1}+j\omega)$$

The response of the total photon density is given by

$$\begin{aligned}N_P &= N_{P1}+N_{P2} \\ &= \frac{\eta_i I_1}{qV}\frac{1}{\Delta}(\gamma_{P_1N})(\gamma_{P_2P_2}+j\omega)+\frac{\eta_i I_1}{qV}\frac{1}{\Delta}(\gamma_{P_2N})(\gamma_{P_1P_1}+j\omega) \\ &= \frac{\eta_i I_1}{qV}\frac{1}{\Delta}\left((\gamma_{P_1N})(\gamma_{P_2P_2}+j\omega)+(\gamma_{P_2N})(\gamma_{P_1P_1}+j\omega)\right) \\ &= \frac{\eta_i I_1}{qV}\frac{\left((\gamma_{P_1N})(\gamma_{P_2P_2}+j\omega)+(\gamma_{P_2N})(\gamma_{P_1P_1}+j\omega)\right)}{(\gamma_{NN}+j\omega)(\gamma_{P_1P_1}+j\omega)(\gamma_{P_2P_2}+j\omega)+\gamma_{NP_1}\gamma_{P_1N}(\gamma_{P_2P_2}+j\omega)+\gamma_{NP_2}\gamma_{P_2N}(\gamma_{P_1P_1}+j\omega)}\end{aligned}$$

The total photon density resembles the modulation response of the first mode only if the coupling between the carrier population, N, and photon population of the second mode, N_{P2}, is negligible. (i.e. we require $\gamma_{P_2N}\approx 0$ and $\gamma_{NP_2}\approx 0$).

Similarly, the total photon density resembles the second mode if coupling between the carrier population and the first mode is negligible.

To make this answer more intuitive, we can rearrange the complicated expression for N_P to write the transfer function as one might expect, with $\omega_R^2 \approx \omega_{R1}^2 + \omega_{R2}^2$.

$$\begin{aligned}
N_P &= N_{P1} + N_{P2} \\
&= \frac{\eta_{P_1N}}{(\gamma_{NN} + j\omega)(\gamma_{P_1P_1} + j\omega) + \gamma_{NP_1}\gamma_{P_1N} + \gamma_{NP_2}\gamma_{P_2N}\frac{\gamma_{P_1P_1}+j\omega}{\gamma_{P_2P_2}+j\omega}} \\
&\quad + \frac{\eta_{P_2N}}{(\gamma_{NN} + j\omega)(\gamma_{P_2P_2} + j\omega) + \gamma_{NP_2}\gamma_{P_2N} + \gamma_{NP_1}\gamma_{P_1N}\frac{\gamma_{P_2P_2}+j\omega}{\gamma_{P_1P_1}+j\omega}} \\
&= \frac{\gamma_{P_1N}}{\omega_{R1}^2 - \omega^2 + j\omega\gamma_1 + \gamma_{NP_2}\gamma_{P_2N}\kappa} + \frac{\gamma_{P_2N}}{\omega_{R2}^2 - \omega^2 + j\omega\gamma_2 + \frac{\gamma_{NP_1}\gamma_{P_1N}}{\kappa}} \\
&\approx \frac{\gamma_{P_1N}}{\omega_{R1}^2 - \omega^2 + j\omega\gamma_1 + \omega_{R2}^2\kappa} + \frac{\gamma_{P_2N}}{\omega_{R2}^2 - \omega^2 + j\omega\gamma_2 + \frac{\omega_{R1}^2}{\kappa}} \\
&\approx \frac{\gamma_{P_1N}}{\omega_{R1}^2 + \omega_{R2}^2 - \omega^2 + j\omega\gamma_1} + \frac{\gamma_{P_2N}}{\omega_{R2}^2 + \omega_{R1}^2 - \omega^2 + j\omega\gamma_2}
\end{aligned}$$

where we have used the following substitutions:

$\omega_{Ri}^2 = \gamma_{NP_i}\gamma_{P_iN} + \gamma_{NN}\gamma_{P_iP_i}$ for $i = 1,2$

$\gamma_i = \gamma_{NN}\gamma_{P_iP_i}$ for $i = 1,2$

$\kappa \equiv \frac{\gamma_{P_1P_1}+j\omega}{\gamma_{P_2P_2}+j\omega} \approx 1.$ This approximation is valid for $\gamma_{P_1P_1} \approx \gamma_{P_2P_2}$ (i.e. roughly equal powers in each mode) or if $\omega \gg \gamma_{P_1P_1}$ and if $\omega \gg \gamma_{P_2P_2}$. For typical numbers, this is true for frequencies larger than a few hundred MHz.

$\omega_{Ri} \approx \gamma_{NP_i}\gamma_{P_iN}$ This approximation assumes that $\gamma_{P_iN}\gamma_{NP_i} \gg \gamma_{NN}\gamma_{P_iP_i}$, which is true for values from Table 5.1.

Thus the response is made up of two $H(\omega)$ functions with different damping, but the same ω_R, where $\omega_R^2 = \omega_{R1}^2 + \omega_{R2}^2$. So for ω_R, we can use $N_p = N_{p1} + N_{p2}$ as if the two modes were one combined mode.

5.6:

Evaluate ω_R using Eqs. 5.49 and 5.51. Then compare the values obtained by each method.

Eq. 5.49:

$$\omega_R^2 = \frac{v_g a N_p}{\tau_p} + \left[\frac{\Gamma v_g a_p N_p}{\tau_{\Delta N}} + \frac{\Gamma R'_{sp}}{N_p \tau_{\Delta N}}\right]\left(1 - \frac{\tau_{\Delta N}}{\tau'_{\Delta N}}\right) + \frac{1}{\tau'_{\Delta N}\tau_p}$$

Assume that values of a, a_p, $\tau_{\Delta N}$, $\tau'_{\Delta N}$, and R'_{sp} don't change signficantly from their threshold values

$$\omega_R^2 = \frac{(3/4.2 \times 10^{10}\ \mathrm{cm/s})(5.34 \times 10^{-16}\ \mathrm{cm^2})(2.43 \times 10^{14}\ \mathrm{cm^{-3}})}{(2.77 \times 10^{-12}\ \mathrm{s})} +$$
$$\left[\frac{0.032(3/4.2 \times 10^{10}\ \mathrm{cm/s})(2.37 \times 10^{-14}\ \mathrm{cm^2})(2.43 \times 10^{14}\ \mathrm{cm^{-3}})}{(1.57 \times 10^{-9}\ \mathrm{s})} + \frac{0.032(1.02 \times 10^{23}\ \mathrm{cm^{-3}/s})}{(2.43x10^{14}\ \mathrm{cm^{-3}})(1.57x10^{-9}\ \mathrm{s})}\right]$$
$$\left(1 - \frac{(1.57 \times 10^{-9}\ \mathrm{s})}{(44.3 \times 10^{-6}\ \mathrm{s})}\right) + \frac{1}{(44.3 \times 10^{-6}\ \mathrm{s})(2.77 \times 10^{-12}\ \mathrm{s})}$$

$$\omega_R^2 = 3.346 \times 10^{20} + \left[8.384 \times 10^{17} + 8.555 \times 10^{15}\right](1) + 8.149 \times 10^{15}$$

$$\omega_R^2 = 3.355 \times 10^{20}\left(\frac{rad}{s}\right)^2$$

Eq. 5.51:

$$\omega_R^2 \approx \frac{v_g a N_p}{\tau_p}$$
$$= \frac{(3/4.2 \times 10^{10}\ \mathrm{cm/s})(5.34 \times 10^{-16}\ \mathrm{cm^2})(2.43 \times 10^{14}\ \mathrm{cm^{-3}})}{(2.77 \times 10^{-12}\ \mathrm{s})}$$
$$= 3.346 \times 10^{20}\left(\frac{rad}{s}\right)^2$$

$$\omega_{R,5.49} = 1.832 \times 10^{10}\frac{rad}{s}$$

$$\omega_{R,5.51} = 1.829 \times 10^{10}\frac{rad}{s}$$

$$\Delta\omega_R = \omega_{R,5.49} - \omega_{R,5.51} = 2.3 \times 10^{7}\frac{rad}{s}$$

(5.7)

Given in Problem
L = 300um
w = 3um
t = 3 x 80A
dI = (15)(2*I_{th}) = 0.2I_{th}
λ_o = 0.98um
$\langle\alpha_i\rangle$ = 5cm^{-1}

From table 5.1 we get:
group velocity: v_g
gain parameters: N_{tr}, N_s, g_o, and ε
recombination parameters: A,B, and C
resonance parameters: γ_{NN}, γ_{PP}, and ω_R

Find threshold gain, carrier concentration, and current at steady state:

Threshold gain:

$$g_{th} = \frac{\langle\alpha_i\rangle - \frac{1}{L}\ln\frac{1}{R}}{\Gamma} = \frac{5cm^{-1} + \frac{1}{0.03cm}\ln\frac{1}{0.32}}{0.06} = 718cm^{-1}$$

Threshold carrier concentration:
From table 5.1 the gain parameters for the laser let us write

$$g = \frac{g_o}{1+\varepsilon N_P}\ln\left(\frac{N+N_s}{N_{tr}+N_s}\right)$$

Neglecting gain compression ($\varepsilon N_P << 1$) can calculate the carrier concentration N_{th} as

$$N_{th} = 2.48 \times 10^{18}\ cm^{-3}$$

Threshold current:

$$A = 0$$

$$B = 0.8e-10 cm^3/s$$

$$C = 3.5e-30 cm^6/s$$

$$I_{th} = \frac{qV}{\eta_i}(AN_{th} + BN_{th}^2 + CN_{th}^3)$$

$$= \frac{(1.6e-19)(300e-4)(3e-4)(80e-8)(3)}{0.8}\left[(0.8e-10)(2.48e18)^2 + (3.5e-30)(2.48e-18)^3\right]$$

$$= 2.356mA$$

Find the steady state value for N_P for $I = 2.0I_{th}$

$$N_P = \frac{\eta_i (I - I_{th})}{q g_{th} v_g V} = 1.067x10^{14} cm^{-3}$$

The difference between N_P for $I = 2.0I_{th}$ and $I = 2.2I_{th}$ is

$$\Delta N_P = \frac{\eta_i (\Delta I)}{q g_{th} v_g V} = 2.13x10^{13} cm^{-3}$$

$$dN_P(t) = dN_P(t = \infty)\left[1 - e^{-\gamma t/2}\cos(\omega_{osc} t)\right]$$

$$\omega_{osc} = \omega_R \sqrt{1 - (\gamma / 2\omega_R)^2}$$

Use values in table 5.1 to solve:

$$\gamma = \gamma_{NN} + \gamma_{PP} = (1.56 + 1.32)x10^9 = 2.88x10^9\ s^{-1}$$

<u>(this γ has been calculated using the values in the table that are valid only for $P_o = 1mW$, it will be more accurate to recalculate these values using the correct N_P)</u>

$\omega_{osc} = 1.819 \text{ x } 10^{10}$ rad/s

$dN_P(t) = (2.13 \text{ x } 10^{13}\ cm^{-3})[1 - e^{-(2.88e9)t/2} \cos((1.819x10^{10} \text{ rad/s})t)]$

Now calculate the power from a snigle facet:

$$P_o(t) = \frac{F_1 v_g \alpha_m (N_P + dN_P(t)) h\nu V}{\Gamma}$$

$$= 1.056mW + (9.91x10^{-15} x dN_P(t))\, mW$$

See attached matlab plot