5.8:

Evaluate the chirp using values from Fig. 5.6 and Table 5.1.

$$\Delta\nu = \frac{\alpha v_g \Gamma a \Delta N}{4\pi} \tag{5.75}$$

From Table 5.1, we find most of the parameters we need:

$v_g = \frac{c}{\bar{n}_g} = (3/4.2) \times 10^{10}$ cm/s

$\Gamma = \Gamma_{xy}\Gamma_z = 0.032 \times 1 = 0.032$

$a = 5.34 \times 10^{-16}$ cm^2

$\alpha = 5$ (given in problem statement)

ΔN can be approximated using values extracted from Fig. 5.6(a). To do this, first find ω_R and γ_R using the peaks of the carrier density in Fig 5.6(a).

$$\Delta N = \Delta N(t) = \Delta N_0\ e^{-\gamma t} \sin(\omega_R t) \tag{5.63}$$

$$\omega_R \approx \frac{2\pi}{t_2 - t_1} = \frac{2\pi}{(0.55 - 0.1)ns} = 1.40 \times 10^{10}\ s^{-1}$$

where t_2 and t_1 are the peaks of the first two oscillations in the carrier density shown in Fig. 5.6(a).

Since at the peaks, $cos(\omega_R t) \approx 1$, we have

$$\Delta N(t_1) = \Delta N_0\ e^{-\gamma t_1} \approx (3.784 - 3.774) \times 10^{18}\ \text{cm}^{-3} = 10. \times 10^{15}\ \text{cm}^{-3}$$

$$\Delta N(t_2) = \Delta N_0\ e^{-\gamma t_2} \approx (3.781 - 3.774) \times 10^{18}\ \text{cm}^{-3} = 7. \times 10^{15}\ \text{cm}^{-3}$$

Solving for γ and N_0,

$$\gamma \approx 7.93 \times 10^8\ s^{-1}$$

$$\Delta N_0 \approx 7.58 \times 10^{15}\ \text{cm}^{-3}$$

Use the analytic expression for ΔN in the equation for chirp:

$$\Delta N(t) = \Delta N_0\ e^{-\gamma t} sin(\omega_R t) = (7.58 \times 10^{15}\ \text{cm}^{-3})\ e^{(-7.93\times10^8\ s^{-1})t} sin((1.40 \times 10^{10}\ s^{-1})t)$$

Use these values for parameters in Eq. 5.75 (see above) to find an analytical expression for $\Delta\nu$:

$$\Delta\nu = (3.68 \times 10^8\ s^{-1})\ e^{(-7.93\times10^8\ s^{-1})t} \sin((1.40 \times 10^{10}\ s^{-1})t)$$

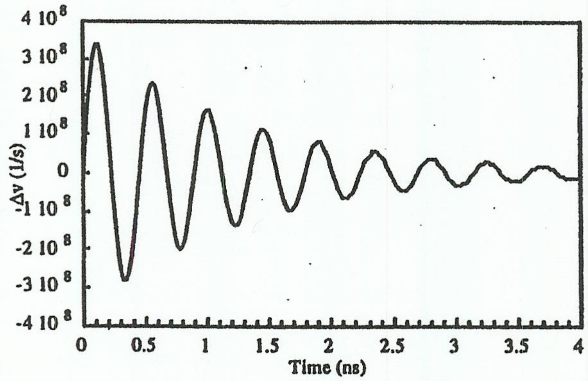

Figure 5.8. Chirp response of laser of Fig 5.6(a).

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5.9:

To find the peak frequency, maximize the magnitude of the transfer function in Eq. 5.46 by minimizing the magnitude of the denominator (or, equivalently, the square of the magnitude of the denominator):

$$\frac{d}{d\omega}\left|(\omega_R^2 - \omega^2 + j\omega\gamma)\right|^2 = 0$$

$$\frac{d}{d\omega}\left[(\omega_R^2 - \omega^2 + j\omega\gamma)\ (\omega_R^2 - \omega^2 - j\omega\gamma)\right] = 0$$

$$\frac{d}{d\omega}\left[\omega_R^4 + \omega^4 - 2\omega^2\omega_R^2 + \omega^2\gamma^2\right] = 0$$

$$4\omega^3 - 4\omega\omega_R^2 + 2\omega\gamma^2 = 0$$

$$4\omega(\omega^2 - (\omega_R^2 - \frac{1}{2}\gamma^2)) = 0$$

The roots of this equation are

$$\omega = 0, \pm\sqrt{\omega_R^2 - \frac{1}{2}\gamma^2}$$

To have a minimum, the second derivative must be positive:

$$\frac{d}{d\omega}\left[4\omega^3 - 4\omega\omega_R^2 + 2\omega\gamma^2\right] > 0$$

$$3\omega^2 - \omega_R^2 + \frac{1}{2}\gamma^2 > 0$$

So the magnitude of the transfer function reaches its maximum value for

$\omega = 0$ for $\omega_R^2 \leq \frac{1}{2}\gamma^2$

$\omega = \sqrt{\omega_R^2 - \frac{1}{2}\gamma^2}$ for $\omega_R^2 > \frac{1}{2}\gamma^2$

From Problem 5.8,

$\omega = 1.40 \times 10^{10}\ s^{-1}$

$\gamma = 7.93 \times 10^{8}\ s^{-1}$

So $\omega_R^2 > \frac{1}{2}\gamma^2$

$$\omega_{peak} = \sqrt{\omega_R^2 - \frac{1}{2}\gamma^2} = 1.3988 \times 10^{10}\ s^{-1}$$

$$\omega_R - \omega_{peak} = 1.2 \times 10^{7}\ s^{-1}$$

$$f_{peak} = \frac{\omega_{peak}}{2\pi} = 2.23GHz$$

$$f_R - f_{peak} = \frac{(\omega_R - \omega_{peak})}{2\pi} = 1.8MHz$$