5.12:

Use Eq. 5.131 to calculate the RIN of the VCSEL:

$$\frac{RIN}{\Delta f} = \frac{2h\nu}{P_0}\left[\frac{a_1 + a_2\omega^2}{\omega_R^4}|H(\omega)|^2 + 1\right] \tag{5.131}$$

We do not have values for P_0, $H(\omega)$, a_1, and a_2. So we must write these variables using known quantities (i.e. those given in Table 5.1):

$$H(\omega) = \frac{\omega_R^2}{\omega_R^2 - \omega^2 + j\omega\gamma} \tag{5.46}$$

To find the output power, P_0, assume linearity over the current range of interest (it will be shown later that gain compression is negligible in these cases):

$$P_0 = F_1\eta_d(I - I_{th})\frac{h\nu}{q}$$

The definitions for a_1 and a_2 are given by Eq. 5.130:

$$a_1 = \frac{8\pi(\Delta\nu)_{ST}P_0}{h\nu}\frac{1}{\tau_{\Delta N}^2} + \eta_0\omega_R^4\left[\frac{\eta_i(I + I_{th})}{I_{st}} - 1\right]$$

$$a_2 = \frac{8\pi(\Delta\nu)_{ST}P_0}{h\nu} - 2\eta_0\omega_R^2\frac{\Gamma a_p}{a}$$

These can be rewritten using the definitions below these equations on p. 229 and by replacing output power in these equations using $P_0 = \frac{N_p F_1 v_g \alpha_m h\nu V}{\Gamma}$:

$$a_1 = \frac{2R'_{sp}F_1 v_g\alpha_m V}{\tau_{\Delta N}^2} + \eta_0\omega_R^4\frac{2I_{th}}{I - I_{th}}$$

$$a_2 = 2R'_{sp}F_1 v_g\alpha_m V - 2\eta_0\omega_R^2\frac{\Gamma a_p}{a}$$

Now, use values from Table 5.1 to solve Eq. 5.131:

$R'_{sp} = 2.09 \times 10^{23}$ cm^{-3} s^{-1} Spontaneous emission is clamped at threshold. So the threshold value can be used above threshold.

$F_1 = 0.9$

$v_g = \frac{3}{4.2} \times 10^{10}$ cm/s

$\alpha_m = 43.6$ cm^{-1}

$V = 2.4 \times 10^{-12}$ cm^{-3}

$\tau_{\Delta N} = 1.52$ ns Carrier density is clamped at threshold. So τ_{Δ_N} is also clamped. See Eq. 5.27.

$\eta_0 = 0.617$

$\omega_R = (3.423\text{ GHz})2\pi\frac{I - I_{th}}{(2.32\text{ mA} - I_{th})}$ This can be obtained using Eq. 5.51 and the fact that $N_p \propto I$.

$I = 2I_{th} \rightarrow \omega_R = 9.66 \times 10^9 rad/s$

$I = 5I_{th} \rightarrow \omega_R = 38.64 \times 10^9 rad/s$

NOTE: The solution has an error here.
omega_r^2 is proportional to Np and I, not omega_r. Therefore:
I = 2Ith --> omega_r = 1.441e10 rad/s
I = 5th --> omega_r = 2.883e10 rad/s

$\Gamma = 0.0382$

To find whether gain compression can be neglected, we must calculate an estimate of the photon density at the desired current levels. Using linear extrapolation from the current level and photon density at 1 mW output power (Table 5.1), we find the photon density at $I = 2I_{th}$ and $I = 5I_{th}$:

$\frac{I_1 - I_{th}}{I_2 - I_{th}} = \frac{N_{p1}}{N_{p2}}$

$N_p = (2.80 \times 10^{14}\ \mathrm{cm}^{-3}) \frac{I - 0.719\ \mathrm{mA}}{2.32\ \mathrm{mA} - 0.719\ \mathrm{mA}}$

$I = 2I_{th} \rightarrow N_p = 1.2 \times 10^{14}\ \mathrm{cm}^{-3}$

$I = 5I_{th} \rightarrow N_p = 5.0 \times 10^{14}\ \mathrm{cm}^{-3}$

$\epsilon = 1.5 \times 10^{-17}\ \mathrm{cm}^3$ The low value means that $\epsilon N_p \ll 1$ and gain compression can be neglected. (More accurate values of N_p can be calculated using the rate equations, if necessary.)

$a_p = \frac{a_{p0}}{1+\epsilon N_p} \approx a_{p0} = 2.50 \times 10^{-14}\ \mathrm{cm}^2$ (5.32)

$a = \frac{a_0}{1+\epsilon N_p} \approx a_0 = 5.10 \times 10^{-11}\ \mathrm{cm}^2$ (5.31)

$\gamma = \gamma_{NN} + \gamma_{PP}$

γ changes due to the change in N_p according to Eq. 5.36:

$\gamma_{NN} = 1/\tau_{\Delta N} + v_g a N_p$

$\gamma_{PP} = \Gamma v_g a_p N_p$

$I = 2I_{th} \rightarrow$ $\gamma_{NN} = 1.095 \times 10^9\ \mathrm{s}^{-1}$, $\gamma_{PP} = 0.819 \times 10^9\ \mathrm{s}^{-1}$, and $\gamma = 1.9 \times 10^9\ \mathrm{s}^{-1}$

$I = 5I_{th} \rightarrow$ $\gamma_{NN} = 2.479 \times 10^9\ \mathrm{s}^{-1}$, $\gamma_{PP} = 3.411 \times 10^9\ \mathrm{s}^{-1}$, and $\gamma = 5.9 \times 10^9\ \mathrm{s}^{-1}$

Plugging these values in, we can solve for the variables in the original equation:

variable	$I = 2I_{th}$	$I = 5I_{th}$
ω_R	$9.66 \times 10^9\ rad/s$	$38.64 \times 10^9\ rad/s$
$\lvert H(\omega)\rvert$	$1.00015 \approx 1$	$1.000007 \approx 1$
P_0	0.40 mW	1.62 mW
a_1(first term)	$1.217 \times 10^{41}\ \mathrm{s}^{-4}$	$1.217 \times 10^{41}\ \mathrm{s}^{-4}$
a_1(second term)	$0.107 \times 10^{41}\ \mathrm{s}^{-4}$	$6.877 \times 10^{41}\ \mathrm{s}^{-4}$
a_2(first term)	$2.812 \times 10^{23}\ \mathrm{s}^{-2}$	$2.812 \times 10^{23}\ \mathrm{s}^{-2}$
a_2(second term)	$-0.0022 \times 10^{23}\ \mathrm{s}^{-2}$	$-0.0345 \times 10^{23}\ \mathrm{s}^{-2}$

Using the values in this table, we can calculate the RIN for this VCSEL:

$$I = 2I_{th} \rightarrow \frac{RIN}{\Delta f} = 1.67 \times 10^{-14}\ \mathrm{s} = -138\ dB/Hz$$

$$I = 5I_{th} \rightarrow \frac{RIN}{\Delta f} = 3.41 \times 10^{-16}\ \mathrm{s} = -155\ dB/Hz$$

Simplifying approximations could have been made along the way. By comparing the values of the second terms of a_1 and a_2 to the first terms of these variables, we can judge the validity of using a low power approximation for the $I = 2I_{th}$ case. A low power approximation (omitting the second term of each of these variables) would have given an error in a_1 of only 10% and an error in a_2 of $\ll$ 1%. Since a_1 dominates Eq. 5.131 in this case, this leads to roughly a 10% error in the calculation of the RIN. If this error could be tolerated, then the low power approximation would be satisfactory in this case.

The high power (low frequency) approximation omits the $a_2\omega^2$ term and sets the transfer function to 1 in Eq. 5.131. (This approximation is summarized by Eq. 5.133.) This approximation is valid for both the $I = 2I_{th}$ case and the $I = 5I_{th}$ case. For $I = 2I_{th}$, $a_2\omega^2 \approx$ (2.%)a_1. For $I = 5I_{th}$, $a_2\omega^2 \approx$ (0.3%)a_1. Therefore, the error in each case is relatively small when this approximation is used.