turbed upper half-band the Wannier function located at the atom at position $2n\pi$ may be written

$$a'(w-2n\pi)=(2/\pi)[a(w-2n\pi-\pi)+a(w-2n\pi+\pi)] \\ -(2/3\pi)[a(w-2n\pi-3\pi)+a(w-2n\pi+3\pi)]\cdots.$$

We now wish to consider the sum $\sum(n)\exp(2\pi ing)\times a'(w-2n\pi)$, found by locating these Wannier functions on the atoms at positions $2n\pi$. When we insert $a'(w-2n\pi)$, as defined above, in this sum, we may rearrange the resulting double sum, so that it may be written

$$\sum(n)\exp[2\pi i(n+\tfrac{1}{2})g]a[w-2(n+\tfrac{1}{2})\pi] \\ \times[(2/\pi)(\exp\pi ig+\exp-\pi ig) \\ -(2/3\pi)(\exp 3\pi ig+\exp-3\pi ig)\cdots].$$

From Eqs. (46) and (47), however, we know that the function $(2/\pi)(\exp\pi ig+\exp-\pi ig)\cdots$ equals unity when g is between $-\frac{1}{2}$ and $\frac{1}{2}$, which includes the whole first unit cell of our lattice of double periodicity. Thus the wave function is

$$\sum(n)\exp[2\pi i(n+\tfrac{1}{2})g]a[w-2(n+\tfrac{1}{2})\pi],$$

just as if we had the original Wannier functions located at the points $\pm\pi$, $\pm 3\pi$, etc. Thus we verify the statement made in the text and show a simple example of a case where the identical wave functions can be expressed in terms of two types of Wannier functions, one, $a[w-2(n+\frac{1}{2})\pi]$, concentrated on a given atom, the other, $a'(w-2n\pi)$, extended over many atoms.

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Statistics of the Recombinations of Holes and Electrons

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The statistics of the recombination of holes and electrons in semiconductors is analyzed on the basis of a model in which the recombination occurs through the mechanism of trapping. A trap is assumed to have an energy level in the energy gap so that its charge may have either of two values differing by one electronic charge. The dependence of lifetime of injected carriers upon initial conductivity and upon injected carrier density is discussed.

SECTION 1. INTRODUCTION

In connection with studies of transistor physics, the recombination of holes and electrons plays an important role. The lifetime of injected carriers in germanium has been found to be a structure sensitive property of the material. This suggests that the recombination process takes place through the medium of imperfections of some sort in the germanium crystal.[1] It is the purpose of this paper to investigate the mathematical consequences of a particular type of imperfection. We shall accordingly suppose that the crystal contains a density N_t of traps which contribute to the recombination process.

In Fig. 1 we illustrate the way in which holes and electrons may be recombined through the traps. The figure illustrates a trap which may exist in either of two states differing by one electronic unit of charge, being either negative or neutral; similar treatments may be applied to other possibilities, such as neutral or positive, or cases in which the charge changes between -1 and -2 units. If the trap is neutral it may capture an electron from the conduction band. The energy loss of the electron is then converted into heat or light or both depending upon the nature of the trapping process. It may also capture an electron from the valence band represented by part (d) of the figure, in which case it acquires a negative charge and leaves a hole in the valence band. Parts (b) and (c) represent the emission of an electron and the capture of a hole.

The effects which we shall consider in this study arise from the statistics of the processes shown in Fig. 1, and the limitation in rate is assumed to be due to the availability of electrons and holes to enter the traps. We shall neglect another possible limiting factor in the recombination process, the time of readjustment of the electron in the trap once it is trapped: Thus, the electron in part (a) of the figure might be trapped in an excited state in the trap and require some time before falling to the ground state. While the electron was in the excited state, the ability of the trap to emit an electron or to capture a hole would be different from the conditions represented in part (b) and (c) of the figure and there would, therefore, be a time lag before the trap reached its normal state. We shall assume that the readjustment time for a trapped electron is negligible compared to time required on the average for the trap to emit the electron or to capture a hole.

[1] See W. Shockley, *Electrons and Holes in Semiconductors* (D. van Nostrand Company, Inc., New York, 1950), p. 347. The methods of measuring lifetime and its role in transistor electronics are also discussed in this reference. This process of recombination has also been discussed by R. N. Hall, Phys. Rev. **83**, 228 (1951) and **87**, 387 (1952).

SECTION 2. THE BASIC FORMULATION FOR THE PROBLEM

In Table I we define most of the symbols used in the analysis.

Since the processes involved are governed by Fermi-Dirac statistics, we introduce the symbol f to represent the probability that a quantum state be occupied. f is a function of the energy level E of the quantum state and of the Fermi level F:

$$f=1/[1+\exp\{(E-F)/kT\}]. \qquad (2.1)$$

We also introduce the symbol f_p to represent the probability that the state is empty or, in other words, occupied by a hole. (We use the symbol p for holes to be consistent with the notation for p-type semiconductors in which the carriers are positive; we similarly use the symbol n for electrons to be consistent with n-type.) The relation between f_p and f is

$$f_p=1-f=f\exp[(E-F)/kT]. \qquad (2.2)$$

We shall first consider the electron capture process. The probability of electron capture will be dependent upon the initial quantum state of the electron. If we consider a unit volume of material, there will be a total number

$$N(E)dE \qquad (2.3)$$

of quantum states in the energy range dE. We shall denote by $c_n(E)$ the average probability per unit time that an electron in the range dE be captured by an empty trap. If the speed of the electron is v and the cross section for capture by a trap is A, then we have

$$c_n(E)=\text{average of } vA \text{ for states of energy } E. \qquad (2.4)$$

If the number of trapping centers per unit volume is N_t, then the rate of capture will evidently be

$$f_{pt}N_tc_n(E)f(E)N(E)dE, \qquad (2.5)$$

where f_{pt} represents the probability that a trap is empty and thus capable of capturing an electron. $f(E)$ is the fraction of states of energy E that are occupied by electrons.

The probability that an electron be emitted from the trap into the band of energies in range dE will be proportional to the number of electrons in the traps times the probability that the states in range dE are empty. It can, therefore, be represented by the equation

$$f_tN_te_nf_p(E)N(E)dE, \qquad (2.6)$$

where e_n is the emission constant corresponding to c_n.

We shall next make the assumption that the electrons in the conduction band are in thermal equilibrium among themselves. That is, we shall assume that the factors f and f_p are given by a Fermi-Dirac function of the form (2.1) or (2.2) with a suitable value of the Fermi level. This quasi-Fermi level,* or q.f.l., is denoted by F_n. The fraction of the traps which are occupied may also be described by a q.f.l. F_t for the traps. If the system is in thermal equilibrium then, of course, the relationship,

$$F_n=F_t \text{ for thermal equilibrium,} \qquad (2.7)$$

must apply. For this case the principle of detail balancing requires that the rate of capture and the rate of emission of electrons must be equal. We shall next apply this consideration to determine the relationship between e_n and c_n.

The net rate of capture (i.e., capture minus emission) for the energy interval dE may be written in the form

$$dU_{cn}=[f_{pt}f(E)-(e_n/c_n)f_tf_p(E)]N_tc_n(E)N(E)dE. \qquad (2.8)$$

For thermal equilibrium the quantity in the square brackets must be zero and this leads to the result

$$e_n/c_n=\exp[(E_t-E)/kT]. \qquad (2.9)$$

Inserting (2.9) in (2.8) we find that the square bracket depends only on F_n and F_t. The total rate of electron capture U_{cn} is then obtained by integrating over dE:

$$U_{cn}=[1-\exp\{(F_t-F_n)/kT\}]f_{pt}N_t \times\int_{\epsilon_c}^{\infty} f(E)N(E)c_n(E)dE, \qquad (2.10)$$

where the integration extends from the bottom of the conduction band to all higher levels. When the system is in equilibrium the square bracket vanishes. On the other hand, if F_n is greater than F_t, then there is a higher density of electrons in the conduction band than is in keeping with the state of the traps and the exponential term is less than unity so that there is a net rate of capture.

An entirely similar expression may be derived for U_{cp}, the net rate of hole capture.

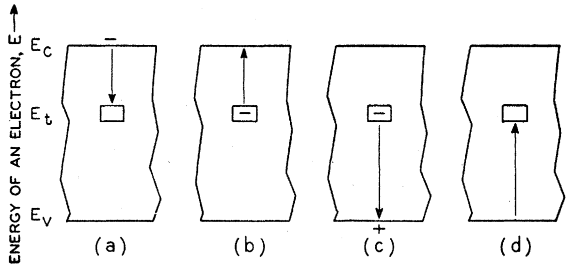

FIG. 1. The basic processes involved in recombination by trapping: (a) electron capture, (b) electron emission, (c) hole capture, (d) hole emission.

SECTION 3. APPLICATION TO THE CASE OF NONDEGENERATE SEMICONDUCTORS

We shall now adapt the expressions discussed above to the case of a semiconductor in which the electron and hole distributions are nondegenerate. For this case expression (2.10) derived in the preceding section may

* The quasi-Fermi level has been referred to as the chemical potential and as the "imref."

be rewritten in the form

$$U_{cn}=[1-\exp(F_t-F_n)/kT]f_{pt}nC_n, \qquad (3.1)$$

where the new symbols are defined by the equations

$$n=N_c\exp(F_n-E_c)/kT, \qquad (3.2)$$

$$N_c=\int_{E_c}^{\infty}[\exp(E_c-E)/kT]N(E)dE, \qquad (3.3)$$

$$C_n=N_t\langle c_n\rangle, \qquad (3.4)$$

$$\langle c_n\rangle=\int_{E_c}^{\infty}[\exp(E_c-E)/kT]c_n(E)N(E)dE\div N_c. \qquad (3.5)$$

The quantity $\langle c_n\rangle$ is the average value of c_n over the states in the conduction band. It has the dimensions of cm^3/sec as may be seen from (2.4) and from the definition of $c_n(E)$. Due to the motion of the electrons in respect to the center, the center in effect sweeps out a volume $\langle c_n\rangle$ of space in unit time. Since N_t has the dimensions cm^{-3}, $C_n=N_t\langle c_n\rangle$ has the dimensions of sec^{-1}; it represents the fraction of space swept out per unit time. Thus C_n is simply the probability per unit time than an electron in the conduction band will be captured for the case in which the traps are all empty and, consequently, in a position to capture electrons. By a similar procedure, averaging over the valence band, we may define C_p, the probability per unit time that a hole will be captured if the traps are filled with electrons so that they are in a condition to capture holes.

For the case of nondegenerate statistics f_p is nearly unity for the states in the conduction band, and consequently the rate of emission (given in (2.6) before integration over dE) is a function of f_t alone. That the rate of emission is independent of F_n follows in (3.1) from the fact that the dependences upon F_n in the exponential and in n itself cancel. Thus, we find that

$$f_{pt}n\exp(F_t-F_n)/kT=f_tN_c\exp(E_t-E_c)/kT\equiv f_tn_1, \qquad (3.6)$$

where

$$n_1=N_c\exp(E_t-E_c)/kT \qquad (3.7)$$

is the number of electrons in the conduction band for the case in which the Fermi level falls at E_t.

Expressing the net rate of capture in terms of n_1, we obtain

$$U_{cn}=C_nf_{pt}n-C_nf_tn_1. \qquad (3.8)$$

An entirely similar treatment may be carried out for holes leading to the equation

$$U_{cp}=C_pf_tp-C_pf_{pt}p_1. \qquad (3.9)$$

SECTION 4. RATE OF RECOMBINATION FOR STEADY-STATE CONDITIONS

In this section we shall evaluate the rate of recombination for nonequilibrium conditions. We shall suppose that hole-electron pairs are being generated at a constant rate U by light or by some form of carrier injection. For steady-state conditions, the net rate of capture of electrons must be equal to that of holes. If the concentrations of holes and electrons are n and p, then the equality of rates leads to

$$C_n(1-f_t)n-C_nf_tn_1=C_pf_tp-C_p(1-f_t)p_1. \qquad (4.1)$$

This equation may be solved for f_t, thus obtaining

$$f_t=(C_nn+C_pp_1)/[C_n(n+n_1)+C_p(p+p_1)], \qquad (4.2)$$

and for f_{pt},

$$f_{pt}=1-f_t=(C_nn_1+C_pp)/[C_n(n+n_1)+C_p(p+p_1)]. \qquad (4.3)$$

When these values are substituted into the rate expressions, the net rate of recombination is obtained:

$$U=C_nC_p(pn-p_1n_1)/[C_n(n+n_1)+C_p(p+p_1)]. \qquad (4.4)$$

In this equation the product p_1n_1 is independent of the energy level E_t of the traps and has the value

$$p_1n_1=N_cN_p\exp(E_v-E_c)/kT$$
$$=N_cN_p\exp(-E_G/kT)=n_i^2, \qquad (4.5)$$

where n_i is the electron or hole concentration in an intrinsic sample, in which n and p are equal. For later use we shall introduce also an energy level corresponding to an intrinsic sample. This energy E_i is the energy at which the Fermi level would lie in an intrinsic sample. Its value is

$$E_i=\tfrac{1}{2}(E_c-E_v)+\tfrac{1}{2}kT\ln(N_v/N_c). \qquad (4.6)$$

In the subsequent development we shall assume that

$$E_t>E_i, \qquad (4.7)$$

so that

$$n_1>n_i>p_1. \qquad (4.8)$$

The case of $E_t<E_i$ can be understood by reversing the roles of holes and electrons.

TABLE I. Symbols.

b	= ratio of electron to hole mobility
n	= density of electrons in conduction band
p	= density of holes in valence band
E_v	= energy of highest valence band level
E_c	= energy of lowest conduction band level
E_G	= energy gap $=E_c-E_v$
E_t	= effective energy level of traps (Appendix B)
F	= Fermi level for thermal equilibrium
F_n	= quasi-Fermi level (q.f.l.) for electrons
F_p	= q.f.l. for holes
F_t	= q.f.l. for traps
n_i	= density of electrons in an intrinsic specimen
N_t	= density of traps
$N(E)$	= density of energy levels per unit energy range
N_c	= effective density of levels for conduction band
N_v	= effective density of levels for valence band
f_t	= fraction of traps occupied by electrons
f_{pt}	= fraction of traps occupied by holes

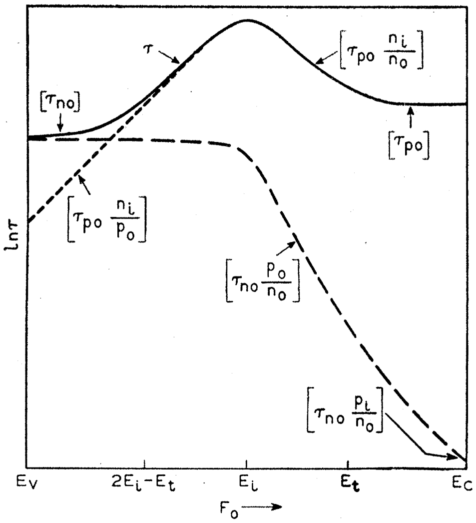

FIG. 2. Dependence of lifetime upon composition of the specimen. (The composition determines F_0, the Fermi level for equilibrium.) The solid curve gives total lifetime τ; the dashed curves give the two terms of which τ is the sum. The expressions in [ ] are approximations valid for the straight segments of the curves.

SECTION 5. EVALUATION OF THE LIFETIME FOR A SIMPLE CASE

In this section we shall consider how the lifetime of carriers depends upon the conductivity of a sample containing a fixed number of traps. We shall deal with the case of low disturbances in carrier density. In the next section we shall consider an extension of the reasoning to large disturbances in carrier densities. As a further simplification, we shall assume, in this section, that the majority carrier density under equilibrium conditions is large compared to the trap density so that we may neglect the change in charge density produced by changing concentrations in the traps. This condition is relaxed and a more general treatment carried out in the appendix. It is shown there that the simplified analysis of the present section is valid if any one of the four quantities n_0, p_0, n_1, p_1 is large compared to N_t. In this section, the deviations of electron and hole densities from their thermal equilibrium values, denoted by n_0 and p_0, must be equal so as to preserve electrical neutrality. If we let δn represent this deviation, then

$$n=n_0+\delta n, \quad p=p_0+\delta n. \tag{5.1}$$

In terms of this deviation and the corresponding rate of recombination U, we may define a lifetime τ by the equation

$$\tau\equiv\delta n/U. \tag{5.2}$$

From Eq. (4.4) we find that τ is given by

$$\begin{aligned}\tau=&(n_0+n_1+\delta n)/(n_0+p_0+\delta n)C_p\\&+(p_0+p_1+\delta n)/(n_0+p_0+\delta n)C_n\\=&\tau_{p0}(n_0+n_1+\delta n)/(n_0+p_0+\delta n)\\&+\tau_{n0}(p_0+p_1+\delta p)/(n_0+p_0+\delta n),\end{aligned} \tag{5.3}$$

where

$$\tau_{p0}\equiv 1/C_p, \quad \tau_{n0}\equiv 1/C_n. \tag{5.4}$$

The quantity τ_{p0} is the lifetime for holes injected into highly n-type specimens. For such specimens, the traps are filled so that the rate at which injected holes are captured and annihilated is simply C_p times the injected carrier density. The quantity τ_{n0} is similarly the lifetime of electrons in a highly p-type sample.

In the next section we shall consider the dependence of τ upon δn for large values of δn. For small values for δn the value of τ is simply

$$\tau=\tau_{p0}(n_0+n_1)/(n_0+p_0)+\tau_{n0}(p_0+p_1)/(n_0+p_0). \tag{5.5}$$

If it is assumed that the capture constants C_n and C_p are relatively insensitive to temperature, then the temperature dependence of τ can be predicted from the relatively simple dependencies of n_1, p_1, n_0, and p_0 upon temperature. Some observations by F. S. Goucher[2] and R. N. Hall[3] appear to be, in general, consistent with these predictions.

In Fig. 2 we represent in a qualitative fashion the dependence of τ upon the composition of the specimen.[4] The two terms in (5.5) of which τ is the sum are also plotted separately on the figure. The composition is represented by F_0 the Fermi level for thermal equilibrium conditions. Nondegenerate specimens correspond to values F_0 lying in the energy gap between E_v and E_c and an intrinsic sample corresponds to $F_0=E_i$. From the plot it is seen that there are four distinct regions to be considered for F_0. The behavior of τ in these regions may be understood by considering the following special cases.

In an n-type sample, where $F_0>E_i$, $n_0\gg p_0$, we have in accordance with approximation (4.8)

$$\begin{aligned}\tau&\doteq\tau_{p0}(1+n_1/n_0)\\&=\tau_{p0}[1+\exp(E_t-F_0)/kT],\end{aligned} \tag{5.6}$$

with a similar relation,

$$\begin{aligned}\tau&=\tau_{n0}+\tau_{p0}n_1/p_0\\&=\tau_{n0}+\tau_{p0}\exp(E_t+F_0-2E_i)/kT,\end{aligned} \tag{5.7}$$

for a p-type sample, $F_0<E_i$, $p_0\gg n_0$.

From (5.6) and (5.7) we see that, as F_0 increases from E_v to E_c, we can distinguish the four regions as follows:

(1) In a sample sufficiently strongly p-type that $F_0<2E_i-E_t$, $p_0\ll n_1$, τ is constant and equal to τ_{n0}. This corresponds to all the traps being empty and the number of holes being large enough that a hole will immediately recombine with every trapped electron.

(2) If the sample is somewhat less strongly p-type so

[2] Personal communication.

[3] R. N. Hall, personal communication.

[4] Data which is, in general, consistent with the trends indicated in Fig. 2 have been reported by R. N. Hall, Phys. Rev. **86**, 600T (1952) and **87**, 387 (1952).

that $F_0 > 2E_i - E_t$, $n_1 \gg p_0$, then τ increases with F_0

$$\tau = \tau_{p0} n_1/p_0 = \tau_{p0} \exp(F_0 - 2E_i + E_t)/kT. \quad (5.8)$$

In this case the traps are still mostly empty but there are not a sufficient number of holes to recombine with each trapped electron before the latter is re-emitted to the conduction band.

(3) In an n-type sample where $F_0 < E_t$, $n_0 \ll n_1$, τ decreases with F_0

$$\tau = \tau_{p0} n_1/n_0 = \tau_{p0} \exp(E_t - F_0)/kT. \quad (5.9)$$

Here the traps are still largely empty and the recombination is limited by the fact that an empty trap cannot capture a hole.

(4) Finally, in a sample sufficiently strongly n-type that $F_0 > E_t$, $n_0 \gg n_1$, the lifetime is again constant, now equal to τ_{p0}. This corresponds to full traps, all set to capture holes, and sufficient electrons that an electron recombines at once with every hole that is trapped.

It is interesting to note that if $\tau_{n0} = \tau_{p0}$, then τ is symmetrical in $F_0 - E_i$ so that the same behavior would arise for traps with energies lying at $E_t - E_i$ below E_i as for those lying at $E_t - E_i$ above E_i. This symmetry follows at once from (5.3) and thus applies to large as well as to small densities.

The exact position and value of the maximum value τ_m of τ will depend on the ratio τ_{n0}/τ_{p0}. Unless this differs by an order of magnitude from unity, the maximum will occur near intrinsic. For the case $\tau_{p0} = \tau_{n0}$, the maximum is seen from (5.5) to correspond to $F_0 = E_i$ so that

$$\begin{aligned}\tau_m &= \tau_{p0}[1 + (n_1 + p_1)/2n_i] \\ &= \tau_{p0}[1 + \cosh(E_t - E_i)/kT].\end{aligned} \quad (5.10)$$

Thus the magnitude of the total possible variation of τ with composition is determined by the absolute difference in the energy level of the traps and the Fermi level for an intrinsic sample. For $E_t = E_i$, τ could vary by a factor of only 2.

SECTION 6. THE DEPENDENCE OF LIFETIME UPON CARRIER DENSITY

For large values of injected carrier densities, it is necessary to retain the δn terms of Eq. (5.3). If we denote by τ_0 the lifetime for vanishingly small values of δn as given by Eq. (5.5), then the value of τ for larger values of δn becomes

$$\tau = \tau_0\{1 + \delta n(\tau_{p0} + \tau_{n0})/[\tau_{p0}(n_0 + n_1) + \tau_{n0}(p_0 + p_1)]\} \div \{1 + \delta n/(n_0 + p_0)\}. \quad (6.1)$$

This expression is of the form

$$\tau = \tau_0(1 + a\delta n)/(1 + c\delta n), \quad (6.2)$$

from which it is seen that τ increases monotonically with δn if $a > c$ and decreases monotonically for $a < c$.

The limiting value for τ as δn approaches infinity is

$$\tau_\infty = \tau_{p0} + \tau_{n0}. \quad (6.3)$$

It is evident that if τ_0 is different from τ_∞, then there is a monotonic variation of τ from one to the other with increasing δn.

The variation δn can be deduced from the change in conductivity $\delta\sigma$ so that a comparison with experiment may be made in a relatively straight forward way. Thus for an n-type sample we have

$$\delta n/(n_0 + p_0) \doteq \delta n/n_0 \doteq \delta\sigma b/\sigma_0(1 + b), \quad (6.4)$$

where σ_0 is the equilibrium conductivity and b is the ratio of electron mobility to hole mobility. From Eq. (6.1) we then obtain

$$\begin{aligned}&\tau[1 + \delta\sigma b/\sigma_0(1 + b)]/\tau_0 \\ &= 1 + \delta n(\tau_{p0} + \tau_{n0})/[\tau_{p0}(n_0 + n_1) + \tau_{n0}(p_0 + p_1)].\end{aligned} \quad (6.5)$$

For a strongly n-type sample, so that n_0 dominates the denominator in (6.5), the right side becomes approximately

$$1 + \delta\sigma b(\tau_{p0} + \tau_{n0})/\tau_{p0}\sigma_0(1 + b). \quad (6.6)$$

This relationship shows that if the left side of (6.5), which involves directly measurable quantities, is plotted as a function of $\delta\sigma$, the result should be a straight line. (It should be noted that if a number of different types of trapping centers are involved, the linear relationship between the left side of (6.5) and $\delta\sigma/\sigma_0$ will in general be modified.)

It should be noted that at high carrier densities the rate of recombination through traps is linear in the carrier density whereas any direct recombination would be quadratic. This is in agreement with the findings of R. N. Hall[5] for p-n junctions operating with high injected densities in the region of recombination.

SECTION 7. INTERPRETATION IN TERMS OF CAPTURE RESISTANCES

The behavior of the lifetime as a function of composition shown in Fig. 2 may be given a somewhat more physical interpretation with the aid of the concept of "recombination resistances." For this purpose we note that the quasi-Fermi levels are analogous to voltages and the U's to currents. We thus introduce

$$R_n = (F_n - F_t)/U_{cn} \doteq kT/f_{pt} n_0 C_n, \quad (7.1)$$

$$R_p = (F_t - F_p)/U_{cp} \doteq kT/f_t p_0 C_p, \quad (7.2)$$

the approximations holding for differences in the F's small compared to kT. For the steady state the recombination currents are equal and we have

$$U(R_n + R_p) = (F_n - F_t) + (F_t - F_p) = F_n - F_p, \quad (7.3)$$

an equation analogous to that for resistances in series.

In Fig. 3, we plot R_n and R_p as functions of F_0. Evidently the recombination rate will be limited by the larger of the two. The sloping lines have a slope of $(1/kT)$. It is seen that if τ_{n0} and τ_{p0} are approximately

[5] R. N. Hall, Phys. Rev. 83, 228 (1951).

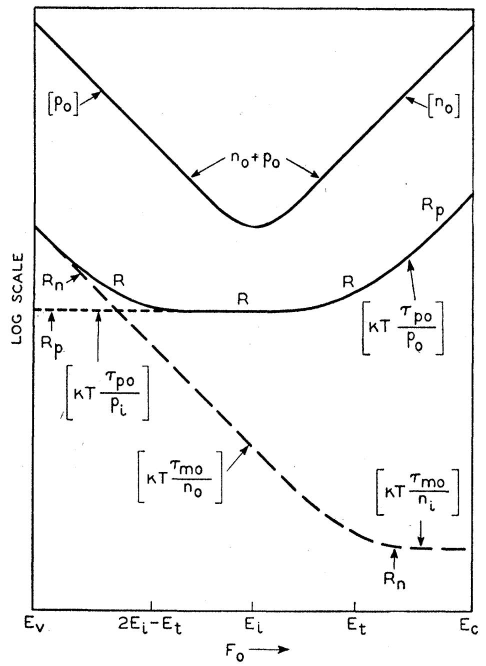

FIG. 3. Variation of recombination resistances with F_0. Solid curve represents total resistance $R=R_p+R_n$; dashed curves give R_p and R_n. Expressions in [ ] are approximations valid for the straight segments of the curves.

equal, recombination is limited by hole trapping for all specimens except those with $p_0>n_1$.

The sloping portions of the $\ln R$ *vs* F_0 plot correspond to constant lifetime. This result may be seen from the relationship between R and τ which is derived as follows:

$$\tau=\delta n/U=\delta nR/(F_n-F_p), \tag{7.4}$$

and

$$\delta(np)=n_i^2\{[\exp(F_n-F_p)/kT]-1\} \doteq n_i^2(F_n-F_p)/kT\doteq(n_0+p_0)\delta n, \tag{7.5}$$

so that

$$\tau=n_i^2R/kT(n_0+p_0). \tag{7.6}$$

On Fig. 3 we have also drawn $\ln(n_0+p_0)$; the straight line portions have slopes of $(1/kT)$, so that they cancel the slopes of $\ln R$ to give the constant lifetime portions of Fig. 2.

The effect of a number of different sorts of traps may be considered on the same basis. For each variety, the recombination is represented by a pair of resistances in series and these series pairs are combined in parallel for the entire system.

SECTION 8. RATE OF GENERATION IN SPACE-CHARGE REGIONS

If the minority carriers are swept out of an n-type specimen, so that

$$\delta n=-p_0 \quad\text{and}\quad p=0, \tag{8.1}$$

then from Eq. (4.4) the net rate of generation is

$$-U=C_nC_pn_i^2/[C_n(n_0+n_1-p_0)+C_pp_1], \tag{8.2}$$

corresponding to a lifetime of

$$\tau=\delta n/U=-p_0/U = \tau_{p0}(n_0+n_1-p_0)/n_0+\tau_{n0}p_1/n_0, \tag{8.3}$$

a result in agreement with (5.3). If the specimen is strongly n-type, p_0 and p_1 will be $\ll n_0$, and τ will be approximately

$$\tau_0\doteq\tau_{p0}(n_0+n_1)/n_0, \tag{8.4}$$

the value for very small disturbances. Hence the net rate of generation of minority carriers is represented by

$$-U=(p_0-p)/\tau_0. \tag{8.5}$$

This is the formula used, for example, in treating the reverse currents generated in the n-region of a p-n junction.

On the basis of the above reasoning, it would at first appear that the maximum rate at which hole-electron pairs could be generated in the n-type material would be given by (8.2), which is approximately equal to p_0/τ_0. Much greater values may occur in some cases, however; in particular, if the space-charge region in a p-n junction biased in the reverse direction penetrates the n-region, then both p and n may be much less than p_1. Under these conditions (4.4) reduces to

$$-U_{\text{sp.ch.}}=C_nC_pn_i^2/(C_nn_1+C_pp_1). \tag{8.6}$$

This corresponds to a "lifetime" defined as

$$\tau_{\text{sp.ch.}}=-p_0/U_{\text{sp.ch.}}=\tau_{p0}(n_1/n_0)+\tau_{p0}p_1/n_0. \tag{8.7}$$

This lifetime will be smaller that τ_0 for an n-type sample with $F_0>E_t$ approximately in the ratio.

$$\tau_{\text{sp.ch.}}/\tau_0=n_1/(n_1+n_0) \tag{8.8}$$

and the rate of generation of hole electron pairs will be greater in about the same ratio.

Hence the reverse current furnished by an element of volume of n-type material having $F_0>E_t$ will be increased by a factor

$$\tau_0/\tau_{\text{sp.ch.}}=\exp(F_0-E_t)kT \tag{8.9}$$

as it enters the space charge region. For $E_i<F_0<E_t$, on the other hand, the change is less than a factor of two.

For p-type samples, the value for the space-charge case

$$\tau_{\text{sp.ch.}}=-n_0/U_{\text{sp.ch.}}=\tau_{p0}n_1/p_0+\tau_{n0}p_1/p_0, \tag{8.10}$$

while

$$\tau_0=\tau_{p0}n_1/p_0+\tau_{n0}(p_0+p_1-n_0)/p_0. \tag{8.11}$$

It is again seen that $\tau_{\text{sp.ch.}}$ is less than τ_0 but the ratio does not become large until p_0 becomes greater than n_1 corresponding to

$$F_0<2E_i-E_t, \tag{8.12}$$

for which

$$\tau_0/\tau_{\text{sp.ch.}} \doteq (\tau_{n0}/\tau_{p0}) \exp(2E_i - E_t - F_0)/kT. \quad (8.13)$$

In strongly n-type or p-type material, the minority carrier density is small and, consequently, the generation of minority carriers is suppressed since the traps are generally in the state corresponding to the sign of the majority carriers. The large increases of hole-electron pair generation given in (8.9) and (8.13) in these cases result from removal of the majority carriers. Under these conditions the traps assume an average state given by (4.2) and (4.3) with

$$f_t = C_p p_1/(C_n n_1 + C_p p_1), \quad (8.14)$$

$$f_{pt} = C_n n_1/(C_n n_1 + C_p p_1), \quad (8.15)$$

and emit electrons and holes at the rates

$$C_n n_1 f_t = C_p p_1 f_{pt} = -U_{\text{sp.ch.}} \quad (8.16)$$

as given in Eq. (8.6).

APPENDIX A

In this appendix, we consider the general case where the density of traps is not small compared with the normal carrier density; δn and δp are then not necessarily equal, the difference being due to the deviation $N_t\delta f_t$ of the number of filled traps from the thermal equilibrium value.

We consider only the case where the disturbances in carrier density are small enough that only first-order terms in δn and δp need be considered. The recombination rates are then linear functions of δn and δp:

$$U_{cn} = A_{nn}\delta n + A_{np}\delta p, \quad U_{cp} = A_{pn}\delta n + A_{pp}\delta p, \quad (A1)$$

where the A's are constants which we shall evaluate [Eq. (A5)].

Equations (A1), together with the continuity equations, provide a set of linear partial differential equations from which the excess carrier densities δp and δn can be found for any set of boundary conditions. In this paper we are primarily concerned with the steady state, $U_{cn} = U_{cp} = U$, rather than with transient conditions. Since in general $\delta n \neq \delta p$, we have two lifetimes,

$$\begin{aligned} \tau_p &= \delta p/U = (A_{pp} - A_{np})/(A_{nn}A_{pp} - A_{np}A_{pn}), \\ \tau_n &= \delta n/U = (A_{nn} - A_{pn})/(A_{nn}A_{pp} - A_{np}A_{pn}). \end{aligned} \quad (A2)$$

In general $\tau_p \neq \tau_n$. For most purposes the lifetime of the minority carrier is of the most practical interest; we shall, therefore, find τ_p for an n-type specimen and τ_n for a p-type.

To begin we find the A's in Eq. (A1). From Eqs. (3.8) and (3.9) of the text, it is seen that small deviations δn, δp, δf_t from the equilibrium values n_0, p_0, f_t give recombination rates

$$\begin{aligned} U_{cn} &= C_n[(1-f_t)\delta n - (n_0+n_1)\delta f_t], \\ U_{cp} &= C_p[f_t\delta p + (p_0+p_1)\delta f_t]. \end{aligned} \quad (A3)$$

The relation between δf_t, δn, and δp comes from the requirement of electrical neutrality

$$\delta p - \delta n = N_t \delta f_t. \quad (A4)$$

Since f_t in (A3) refers to equilibrium, we have

$$f_t = \frac{1}{1+(n_1/n_0)} = 1 - \frac{1}{1+(p_1/p_0)}. \quad (A5)$$

Substituting (A4) and (A5) into (A3) gives (A1) with

$$\begin{aligned} A_{nn} &= C_n\left[\frac{n_1}{n_0+n_1} + \frac{n_0+n_1}{N_t}\right], \\ A_{np} &= -C_n\frac{n_0+n_1}{N_t}, \\ A_{pn} &= -C_p\frac{p_0+p_1}{N_t}, \\ A_{pp} &= C_p\left[\frac{p_1}{p_0+p_1} + \frac{p_0+p_1}{N_t}\right]. \end{aligned} \quad (A6)$$

Substituting (A5) into (A2) gives the lifetimes

$$\begin{aligned} \tau_p &= \frac{\tau_{n0}(p_0+p_1) + \tau_{p0}[n_0+n_1+N_t(1+n_0/n_1)^{-1}]}{n_0+p_0+N_t(1+n_0/n_1)^{-1}(1+n_1/n_0)^{-1}}, \\ \tau_n &= \frac{\tau_{p0}(n_0+n_1) + \tau_{n0}[p_0+p_1+N_t(1+p_0/p_1)^{-1}]}{n_0+p_0+N_t(1+p_0/p_1)^{-1}(1+p_1/p_0)^{-1}}. \end{aligned} \quad (A7)$$

The two expressions (A7) are symmetrical in n and p. It should be noted that $n_0/n_1 = p_1/p_0$, hence the denominators are the same in the two formulas.

For $N_t = 0$, the two equations (A7) both reduce to (5.5) of the text. At the other extreme, $N_t = \infty$, we have

$$\tau_p = \tau_{p0}(1+n_1/n_0), \quad (A8)$$

which is the same as (5.6) of the text for an n-type sample. Thus the lifetime of a hole in n-type material is correctly given by the simple theory except near intrinsic. For $N_t = \infty$, τ_n becomes

$$\tau_n = \tau_{n0}(1+p_1/p_0) = \tau_{n0}[1+\exp(F_0-E_t)/kT]. \quad (A9)$$

Thus for $E_t > E_i$, $\tau_n = \tau_{n0}$ for all p-type samples when N_t is very large compared to both p_0 and n_1. The interpretation of (A8) is that $1+n_1/n$ is simply an alternative way of writing $1/f_t$. Hence $1/\tau_p$ is simply $1/\tau_{p0}$ times the probability that a trap be ready to capture a hole by being occupied by an electron. A similar interpretation applies to (A9).

We next consider the lifetime of the minority carrier for the four distinct regions discussed in Sec. 5 of the text.

In regions (3) and (4), n-type material, we have seen, [Eq. (A8)] that the formula for lifetime of a hole is independent of N_t. (The lifetime itself is, of course,

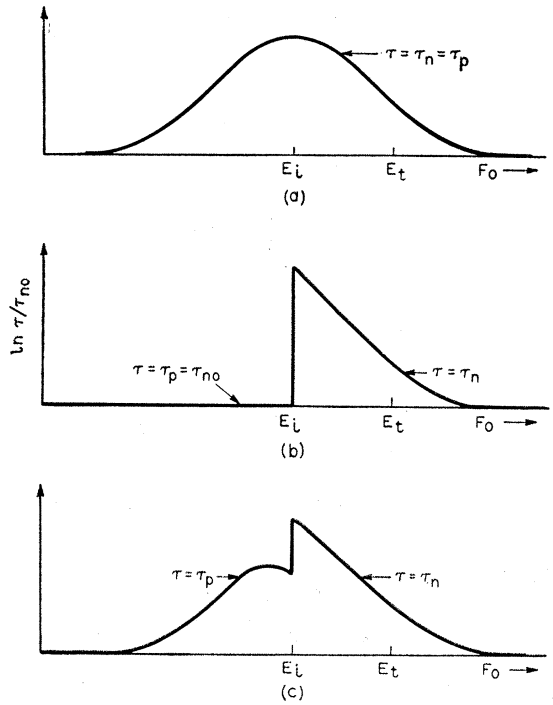

FIG. 4. Variation of lifetime of minority carrier with F_0: (a) $N_t=0$, (b) N_t=infinity, (c) intermediate case.

inversely proportional to N_t since τ_{p0} is inversely proportional to N_t.) Likewise in region (1), $p_0 \gg n_1$, it is seen that $\tau_n=\tau_{n0}$. Hence, the analysis of the text correctly gives the lifetime of the minority carrier, except in region (2). The formula for lifetime of the majority carrier will depend on N_t over a wider range, but this is generally of less interest.

In region (2), $n_1 \gg p_0 \gg n_0$, we have from (A7)

$$\tau_n=\tau_{n0}+\tau_{p0}\frac{n_1}{p_0+(N_t p_1/p_0)}. \tag{A10}$$

There are three distinct sub cases under (A10) corresponding to different ranges of F_0.

(2a) When $n_1<p_0+N_t p_1/p_0$, then $\tau_n=\tau_{n0}$. Since in region (ii), $n_1 \gg p_0$, this requires $n_1<N_t p_1/p_0$. It will be convenient to express N_t in terms of an energy level E^*, such that $n_0=N_t$ when $F_0=E^*$. Also, for convenience, we take E_i as the zero of energy. Then, by definition,

$$N_t=n_i\exp(E^*/kT). \tag{A11}$$

Thus in terms of energies, we have $\tau_n=\tau_{n0}$ when

$$-E^*+2E_t<F_0<0. \tag{A12}$$

This case is only possible when there are enough traps that

$$E^*>2E_t. \tag{A13}$$

(2b) When $n_1>N_t p_1/p_0>p_0$ then $\tau=\tau_{p0}n_1p_0/N_t p_1$. In terms of energies this corresponds to

$$\begin{gathered}\tfrac{1}{2}(E_t-E^*)<F_0<2E_t-E^*,\\ \tau_n=\tau_{p0}\exp[(2E_t-E^*-F_0)/kT].\end{gathered} \tag{A14}$$

In order for this case to be possible the number of traps must be such that

$$E_t<E^*<3E_t. \tag{A15}$$

(2c) When $p_0>N_t p_1/p_0$, $\tau_n=\tau_{p0}n_1/p$, which corresponds to

$$\begin{gathered}-E_t<F_0<\tfrac{1}{2}(E_t-E^*),\\ \tau_n=\tau_{p0}\exp[(E_t+F_0)/kT].\end{gathered} \tag{A16}$$

This case is possible only for

$$E^*<3E_t. \tag{A17}$$

In case (2c), τ_n is independent of E^* and therefore of N_t, and is the same as in the text.

It is seen that if $E^*<E_t$, $N_t<n_1$ then cases (2a) and (2b) are eliminated and, for all values of F_0 both positive and negative, the lifetime of the minority carrier is correctly given by the simplified formula (5.5) of the text. It is also readily verified from Eqs. (A7) that (5.5) of the text correctly gives $\tau_p=\tau_n$ when N_t is small compared with any one of the four quantities n_0, p_0, n_1, p_1.

As in the text the case $E_t<E_i$ can be understood by reversing the role of holes and electrons.

Figure 4 shows the variation of the lifetime of the minority carrier for $E_t>E_i=0$. For convenience τ_{p0} and τ_{n0} are taken equal. Due to the logarithmic functions involved, a considerable difference between τ_{p0} and τ_{n0} would be required to appreciably alter the curves.

APPENDIX B. EFFECT OF DEGENERACY OF THE STATES OF THE TRAPS

The quantity E_t used as the energy level for the traps is in fact an effective energy-level related to the energy-level E_t (true) by the equation

$$E_t=E_t(\text{true})+kT\ln(w_p/w), \tag{B1}$$

where w_p and w are the degeneracies of an empty and full trap, respectively. This result may be derived by noting that the probability factor for the electrons in traps is

$$w^{N_t f_t}w_p^{N_t f_{pt}}N_t!/(N_t f_t)!(N_t f_{pt})!. \tag{B2}$$

When the customary maximizing process for Fermi-Dirac statistics is carried out, the degeneracy factor in (B2) produces the effect of (B1), which may then be used in equations like (2.1) and (2.2). In case excited states in the empty and full traps make a significant contribution, then the ratio w_p/w should be replaced by a ratio of states-sums.