8.1:

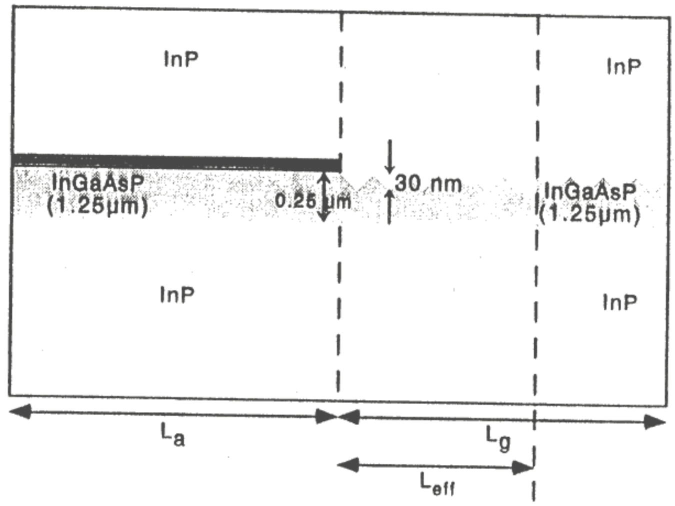

Figure 8.1. Tunable laser

Given:

$\lambda = 1.55\mu$m

$\Delta f = 100$ GHz

4 QW gain region with gain curve as described by Fig. 4.25

waveguide width $= w = 1.5\mu$m

$< \alpha_i > = 10$ cm^{-1}

$\eta_i = 70\%$

If grating is lossless, $R = 0.50$

From Table 1.1,

$$n(\text{InP}) = 3.17$$

Interpolate for InGaAsP by assuming that index varies linearly with index from InP where $n = 3.17$ and $\lambda_{E_g} = 0.918\mu$m to InGaAsP(1.3μm) where $n = 3.40$ and $\lambda_{E_g} = 1.3\mu$m.

$$n(\text{InGaAsP } 1.25\mu\text{m}) \approx 3.37$$

---

a) Follow Example 8.2.2 on pp. 347-348.

For the passive waveguide sections, calculate effective index and mode parameters as in Appendix 3:

$V = \frac{2\pi}{\lambda} d\sqrt{n_g^2 - n_c^2} = \frac{2\pi}{1.55\mu m}(0.25\mu\text{m})\sqrt{3.37^2 - 3.17^2} = 1.159$

$a = 0$

From Fig. A3.2, $b \approx 0.24$.

$\overline{n} = \sqrt{n_c^2 + b(n_g^2 - n_c^2)} = \sqrt{3.17^2 + 0.24(3.37^2 - 3.17^2)} = 3.227$

$k_{gx} = \frac{2\pi}{\lambda}\sqrt{n_g^2 - \overline{n}^2} = 3.94\mu\text{m}^{-1}$

$\gamma_{cx} = \frac{2\pi}{\lambda}\sqrt{\overline{n}^2 - n_c^2} = 2.44\mu\text{m}^{-1}$

To find the grating length, use the fact that $R = 0.50$ without loss:

$$\tanh^2(\kappa L_g) = r_g^2 = 0.50$$

$$L_g = \frac{0.8814}{\kappa}$$

Find κ for a triangular grating:

$$\kappa = \frac{8}{\pi^2} k_0 a \frac{n_g^2 - \overline{n}^2}{2\overline{n} d_{eff}} \tag{6.44}$$

$d_{eff} = d + \frac{2}{\gamma_{cx}} = 0.25\mu\text{m} + \frac{2}{2.44\mu\text{m}^{-1}} = 1.07\mu\text{m}$

$a = \frac{30\text{ nm}}{2} = 1.5 \times 10^{-2}\mu\text{m}$

$\kappa = \frac{8}{\pi^2}\frac{2\pi}{1.55\mu\text{m}}(1.5 \times 10^{-2}\mu\text{m})\frac{3.37^2 - 3.227^2}{2(3.227)(1.07\mu\text{m})} = 0.00675\mu\text{m}^{-1} = 67.5\text{ cm}^{-1}$

$$L_g = \frac{0.8814}{0.00675\mu\text{m}^{-1}} = 130\mu\text{m}$$

Find the effective grating length:

$$L_{eff} = \frac{1}{2\kappa}\tanh(\kappa L_g) = \frac{1}{2(0.00675\mu\text{m}^{-1})}\sqrt{0.5} = 52.4\mu\text{m}$$

Find the total cavity length based on the frequency spacing, $\Delta f = 100$ GHz:

$$\Delta\lambda = \frac{\lambda^2 \Delta f}{c} = \frac{(1.55 \times 10^{-4}\text{ cm})^2\ (100 \times 10^9\text{ s}^{-1})}{3 \times 10^{10}\text{ cm/s}} = 8.0 \times 10^{-8}\text{ cm} = 8.0\text{Å}$$

The mode spacing, $\Delta\lambda$, is given by

$$\Delta\lambda = \frac{\lambda^2}{2 < \overline{n} >_g L_{tot}} \tag{3.39}$$

$< \overline{n} >_g \approx 4$ for InGaAsP/InP lasers (see p. 40 or p. 348). Solve for L_{tot}:

$$L_{tot} = \frac{(1.55 \times 10^{-4}\text{ cm})^2}{2(4.0)(8 \times 10^{-8}\text{ cm})} = 3.75 \times 10^{-2}\text{ cm} = 375\mu\text{m}$$

$$L_a = L_{tot} - L_{eff} = 323\mu\text{m}$$

---

b) To plot the LI curve, we must first find the differential efficiency:

$$\eta_{d1} = F_1 \eta_i \frac{\alpha_m}{< \alpha_i > + \alpha_m}$$

Assume a facet reflectivity of $R \approx 0.32$ and a lossless grating:

$\alpha_m = \frac{1}{L}\ \ln\frac{1}{R} = \frac{1}{375 \times 10^{-4}\text{ cm}} \ln\left(\frac{1}{(\sqrt{0.32 \times 0.50})}\right) = 24.4\text{ cm}^{-1}$

$$F_1 = \frac{t_1^2}{(1 - r_1^2) + \frac{r_1}{r_g}(1 - r_g^2)} = \frac{1 - 0.32}{(1 - 0.32) + \sqrt{\frac{0.32}{0.50}}(1 - 0.5)} = 0.630$$

$$\eta_{d1} = 0.630(0.70)\frac{24.4\text{ cm}^{-1}}{(10\text{ cm}^{-1}) + (24.4\text{ cm}^{-1})} = 31.3\%$$

Find confinement factor, Γ by numerically integrating the intensity profile of the waveguide using the parameters k_x, γ, $\overline{n}$, n, and d from part (a). (See Eq. A5.13)

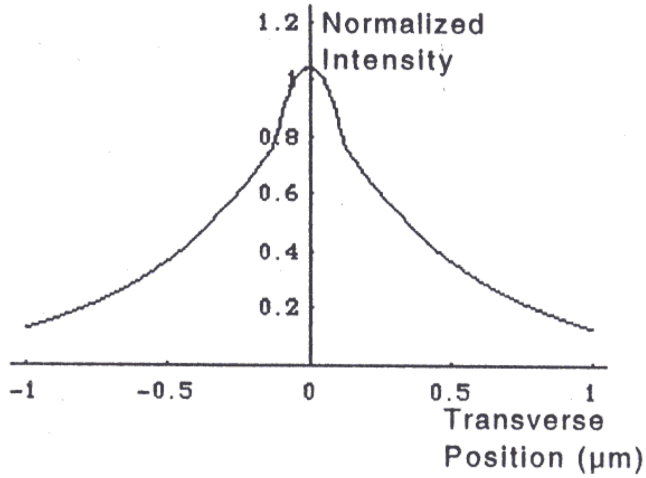

Figure 8.1b. Normalized field intensity profile for use in calculating Γ_x

$$\Gamma_{xy} \approx \Gamma_x = \cancel{2.2\%}\ 4.4\ \%.$$

$$\Gamma_z \approx \frac{L_a}{L_{tot}} = \frac{323\mu\text{m}}{375\mu\text{m}} = 86.1\%$$

Find threshold gain:

$$g_{th} = \frac{\alpha_m + <\alpha_i>}{\Gamma_{xy}\Gamma_z}$$

$$= \frac{(24.4 + 10)\ \text{cm}^{-1}}{(0.044)(0.861)}$$

$$= 1100\ \text{cm}^{-1} \quad \cancel{= 1815\ \text{cm}^{-1}}\ \text{per QW}$$

$$\cancel{\approx 450 cm^{-1}\ \text{per QW}}$$

From Fig. 4.25b, we can calculate the threshold current. To obtain $g_{th} = \cancel{450}\ 1100\ \text{cm}^{-1}$, the threshold current density is $J_{th} = \cancel{(175\ \text{A/cm}^2)}\ 537\text{A/cm}^2 \times (4\ \text{wells}) = \cancel{700\ \text{A/cm}^2}$. $2137\ \text{A/cm}2$

$$I_{th} = \frac{J_{th}L_a w}{\eta_i} = \frac{\cancel{(700\ \text{A/cm}^2)}(323\times10^{-4}\ \text{cm})(1.5\times10^{-4}\ \text{cm})}{0.7} \cancel{= 0.00485\ \text{A} = 4.85\ \text{mA}}$$

$$= 14.8\ \text{mA}$$

The power from the facet is then given by

$$P_1 = \eta_{d1}\frac{h\nu}{q}(I - I_{th})$$

$$\frac{h\nu}{q} = \frac{1.24\mu\text{m W/A}}{1.55\mu\text{m}} = 0.80\ \text{W/A}$$

$$P_1 = (0.25\ \text{mW/mA})(I - 14.8\ \text{mA})$$

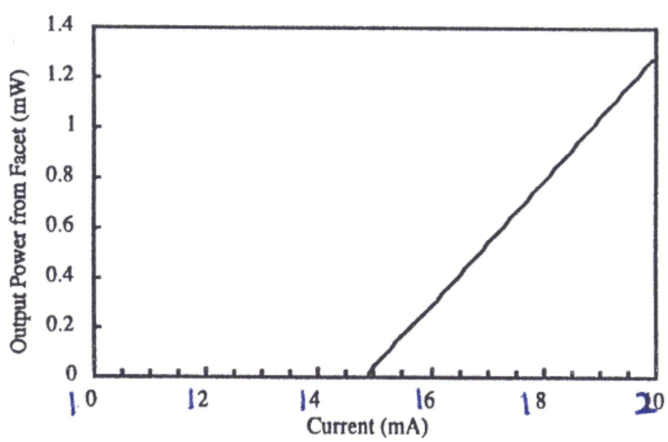

Figure 8.1b. Single facet output power vs. current

c) We only consider the mode hopping in this problem (following Example 8.2.2). The shift in the DBR mirror peak reflectivity is given by

$$\frac{\Delta\lambda_B}{\lambda_B} = \frac{\Delta\overline{n}_g}{\overline{n}_g} \tag{8.1}$$

where $\overline{n}_g$ is the grating effective index.

$$\begin{aligned}\Delta\overline{n}_g &= \frac{d\overline{n}_g}{dN} N \\ &= \left(-\Gamma_{xy} 10^{-20}\ \mathrm{cm}^3\right) N\end{aligned}$$

$$\Gamma_{xy} = \Gamma_x = \frac{1 + 2\gamma d/V^2}{1 + 2/\gamma d} = \frac{1 + 2(2.44\mu\mathrm{m}^{-1})(0.25\mu\mathrm{m})/(1.159)^2}{1 + 2/(2.44\mu\mathrm{m}^{-1})(0.25\mu\mathrm{m})} = 0.446 \tag{A3.14}$$

In example 8.2.2, Auger recombination was neglected, which allowed the derivation of an analytical expression for the carrier density, N, as a function of current, I_g. However, since we are told to include both radiative and Auger recombination, we must solve for N numerically from the steady state carrier rate equation:

$$\frac{I\eta_i}{qV} = BN^2 + CN^3$$

Assume that $\eta_i = 0.70$ for current injection into the waveguide of the grating section.

$B \approx 10^{-10}\ \mathrm{cm}^3/\mathrm{s}$ for InGaAsP alloys from p. 31

$C = 3 \times 10^{-29}\ \mathrm{cm}^6/\mathrm{s}$ for 1.25μm InGaAsP from p. 350.

$V = L_g w d = (130.5\mu\mathrm{m})(1.5\mu\mathrm{m})(0.25\mu\mathrm{m}) = 48.95\mu\mathrm{m}^3 = 4.90\ \mathrm{cm}^{-11}$

Since $c = \lambda f$,

$$\Delta f = -\frac{c}{\lambda^2}\Delta\lambda$$

$$\begin{aligned}\Delta f &= -\frac{c}{\lambda}\,\frac{\left(-\Gamma_{xy} 10^{-20}\ \mathrm{cm}^3\right) N}{\overline{n}_g} \\ &= \frac{3 \times 10^{14}\mu\mathrm{m/s}}{1.55\mu\mathrm{m}}\,\frac{\left(0.44610^{-20}\ \mathrm{cm}^3\right) N}{3.227} \\ &= \left(2.67510^{-7}\ \mathrm{cm}^3/\mathrm{s}\right) N\end{aligned}$$

Solve numerically for carrier density, N. Then solve for shift in Bragg frequency.

The mode spacing of cavity modes is given in the problem to be $\Delta\nu = 100$ GHz. So a mode hop occurs for every 100 GHz shift in the Bragg frequency.

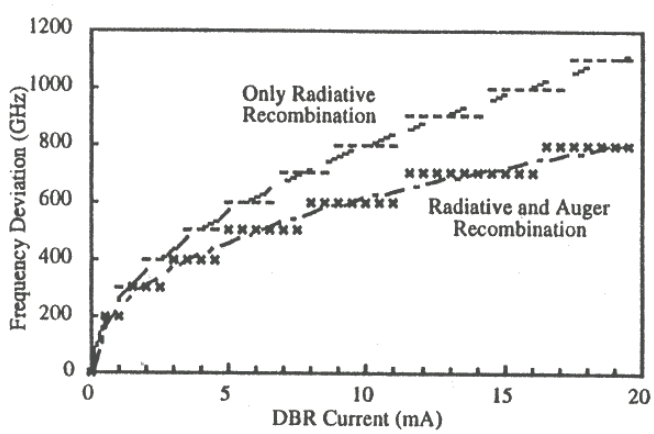

Figure 8.1c. Frequency tuning due to DBR mirror current.

For comparison, the plot of frequency tuning when Auger recombination is neglected is also included in this figure. As we can see, Auger recombination has a large effect on the amount of current required for tuning.

Note: A slight frequency shift occurs on each of the modes due to the shift in cavity modes caused by the change in index. From chapter 3, we can calculate the shift in wavelength of these modes.

Cavity Mode shift:

$$\Delta\lambda_m = \lambda_m \left(\frac{\Delta\overline{n}_{DBR} L_{eff}}{\overline{n}_a L_a + \overline{n}_{DBR} L_{eff}} \right) \tag{3.65}$$

Assuming that $\overline{n}_{DBR} \approx \overline{n}_a$, this reduces to

$$\begin{aligned} \Delta\lambda_m &= \lambda_m \left(\frac{\Delta\overline{n}_{DBR} L_{eff}}{\overline{n}(L_a + L_{eff})} \right) \\ &\approx (1.55\mu\mathrm{m}) \left(\frac{\Delta\overline{n}_{DBR}(52.4\mu\mathrm{m})}{3.227((323\mu\mathrm{m}) + (52.4\mu\mathrm{m}))} \right) \\ &\approx (0.067\mu\mathrm{m})\Delta\overline{n}_g \end{aligned}$$

$$\begin{aligned} \Delta f &= -\frac{c}{\lambda^2}\Delta\lambda \\ &= -\frac{3 \times 10^14\mu\mathrm{m/s}}{1.55\mu\mathrm{m}^2}(0.067\mu\mathrm{m}) \left(-(0.446)10^{-20}\ \mathrm{cm}^3\right) N \\ &= \left(3.7310^{-8}\ \mathrm{cm}^3/\mathrm{s}\right) N \end{aligned}$$

Comparison of this shift to the shift for the Bragg frequency shift shows that the cavity mode shift is only about 14% of the shift for the DBR mirror.

8.2:

a) Find length of amplifier:

In order for the amplifier to have the fastest switching speed, the amplifier should be biased around the knee of the g vs. N curve according to Eq. 8.17. Appendix 17 describes the knee mathematically as the current at which

$$\frac{g}{N} = \frac{dg}{dN}$$

This occurs for $g = g_{0_N}$ assuming a logarithmic dependence $g(N) = g_{0_N} \ln\left(\frac{N}{N_0}\right)$.

$$g_{opt} = g_{0_N} = 1800 \text{ cm}^{-1} \qquad \text{(Table 4.4)}$$

$$J_{opt} = J_{tr}\, e^{\frac{g_{0_N}}{g_{0_J}}} = (81 \text{ A/cm}^2)\, e^{1800/583} = 1.78 \text{ kA/cm}^2$$

The problem states that $<\alpha_i>= 15 \text{ cm}^{-1}$ and $\Gamma = 0.06$.

So, for the ON state,

$$\Gamma g_{ON} - \alpha = (0.06)(1800 \text{ cm}^{-1}) - (15 \text{ cm}^{-1}) = 93 \text{ cm}^{-1}$$

$$P_{out} = P_{in}\, e^{(\Gamma g_{ON} - <\alpha_i>)L}$$

$$\begin{aligned} L_{amp} &= \frac{\ln\left(\frac{P_{out}}{P_{in}}\right)}{\Gamma g_{ON} - <\alpha_i>} \\ &= \frac{\ln\left(\frac{10 \text{ mW}}{3 \text{ mW}}\right)}{93 \text{ cm}^{-1}} \\ &= 129 \mu\text{m} \end{aligned}$$

Find width of amplifier:

An approximate amplifier bandwidth is described by Eq. 8.17:

$$f_c = \frac{1}{2\pi}\left(\frac{1}{\tau} + a' \frac{P_{out}\Gamma_{xy}}{wdh\nu}\right)$$

The terms in this equation that vary with width are a', Γ_{xy}, and $\frac{1}{w}$:

$$a' = \frac{a_0'}{1 + P/P_s}$$

Since $P \gg P_s$ in this case,

$$\begin{aligned} \frac{\Gamma_{xy} a'}{w} &\approx \frac{\Gamma_{xy} a_0' P_s}{Pw} \\ &= \frac{\Gamma_{xy} a_0' wdh\nu}{Pa\Gamma_{xy}\tau w} \\ &= \frac{a_0' dh\nu}{Pa\tau} \end{aligned}$$

The only variable in this final equation that varies with width is a. Due to gain compression, a decreases as w increases. So to make f_c large, we want to make w as large as possible.

Therefore, choose $w = 5\mu$m.

---

b) First find the current density needed in the off state of the amplifier:

$$P_{out} = P_{in}\, e^{(\Gamma g_{OFF} - <\alpha_i>)L}$$

$$\begin{aligned} g_{OFF} &= \frac{1}{\Gamma}\left[\frac{1}{L_{amp}} \ln\left(\frac{P_{out}}{P_{in}}\right) + <\alpha_i>\right] \\ &= \frac{1}{0.06}\left[\frac{1}{0.0129\ \text{cm}} \ln\left(\frac{1\ \text{mW}}{3\ \text{mW}}\right) + 15\ \text{cm}^{-1}\right] \\ &= -1164\ \text{cm}^{-1} \end{aligned}$$

$$\begin{aligned} J_{OFF} &= J_{tr}\, e^{g_{OFF}/g_{0J}} \\ &= (81\ \text{A/cm}^2)\, e^{-1164/583} \\ &= \cancel{596\ \text{A/cm}}{}^2 \quad 11\ \text{A/cm}^2 \quad \text{per well} \end{aligned}$$

Then calculate current densities for the amplifier:

Amplifier ON state:

$$\begin{aligned} I_{ON} &= \cancel{\eta_i} J_{ON} w L_{amp} / \eta_i \\ &= \cancel{(0.7)}(1.78 \times 10^3\ \text{A/cm}^2)(5 \times 10^{-4}\ \text{cm})(129 \times 10^{-4}\ \text{cm}) \times (4\ QWs) / \eta_i \\ &\cancel{= 32.1\ \text{mA}} \\ &= 65.6\ \text{mA} \end{aligned}$$

Amplifier OFF state:

$$\begin{aligned} I_{OFF} &= \eta_i J_{OFF} w L_{amp} \\ &= \cancel{(0.7)}(\cancel{596}\ 11\ \text{A/cm}^2)(5 \times 10^{-4}\ \text{cm})(129 \times 10^{-4}\ \text{cm}) \times (4\ QWs) / 0.7 \\ &\cancel{= 10.8\ \text{mA}} \\ &= \cancel{0.88\ \text{mA}}\ 0.408\ \text{mA} \end{aligned}$$

For the laser, first calculate the mirror loss so that we can find threshold modal gain.

$$\alpha_m = \frac{1}{2L_{eff}} \ln\left(\frac{1}{r_g^2}\right)$$

$\kappa = \frac{\kappa L_g}{L_g} = \frac{0.7}{(400\mu\text{m})/2} = 35\ \text{cm}^{-1}$

$L_{eff} = \frac{1}{2\kappa}\tanh(\kappa L_g) = 86.3\mu\text{m}$ (3.59)

$r_g = \tanh(\kappa L_g) = \tanh(0.7) = 0.604$

$$\begin{aligned} g_{th} &= \frac{1}{\Gamma}(<\alpha_i> + \alpha_m) \\ &= \frac{1}{0.06}(15 + 58.3)\ \text{cm}^{-1} \\ &= 1222\ \text{cm}^{-1} \quad \text{per well} \\ &\cancel{= 306\ \text{cm}^{-1}} \quad \cancel{\text{per QW}} \end{aligned}$$

$$J_{th} = J_{tr}\, e^{\frac{g_{th}}{g_0}} = \cancel{137}\ 659\ \text{A/cm}^2 \quad \text{per QW}$$

$$I_{th} = \eta_i w L_a J_{th} \times (4\ QW) = \cancel{7.7\ \text{mA}}\quad 75.2\ \text{mA}$$

$$\eta_d' = \eta_i F_1 \frac{\alpha_m}{\alpha_m + <\alpha_i>} e^{(L_g - L_{eff})(\Gamma g_{th} - <\alpha_i>)} = \cancel{0.540}\quad 0.278$$

$$P_{out} = \frac{h\nu}{q}\eta_d'(I - I_{th})$$

$$I_{laser} = \frac{P_{out}}{\eta_d' \frac{h\nu}{q}} + I_{th}$$

$$= \frac{3\ \text{mW}}{0.278\ \cancel{0.540}(0.8\ \text{mW/mA})} + (\cancel{7.7\ \text{mA}})\quad 75.2$$

$$\cancel{= 14.6\ \text{m}}\text{A}$$

$$= \quad 91.7\ \text{mA}$$

---

c) Assume that $a' = 10^{-15}\ \text{cm}^2$ as described on p. 359.

Assume that $\tau \approx 3$ ns.

$$f_c = \frac{1}{2\pi}\left(\frac{1}{\tau} + a'\frac{P_{out}\Gamma_{xy}}{wdh\nu}\right)$$
$$= \frac{1}{2\pi}\left(\frac{1}{3\ \text{ns}} + (10^{-15}\ \text{cm}^2)\frac{(0.01\ \text{W})(0.06)}{(5\times10^{-4}\ \text{cm})(4\times70\times10^{-8}\ \text{cm})(0.8\ \text{eV})(1.6\times10^{-19}\ \text{J/ eV})}\right) \qquad (8.17)$$
$$= \frac{1}{2\pi}\left(\frac{1}{3\ \text{ns}} + \frac{1}{0.3\ \text{ns}}\right)$$
$$= 0.6\ \text{GHz}$$

---

d) Increasing the laser bias will produce a higher photon density, which will shorten the carrier lifetime in the amplifier section and therefore increase the amplifier bandwidth.