7.11:

The method used in Appendix 16 can be followed step by step to solve this problem with Mathematica (see below). Note that the eigenvalue problem expressed by Eq. A16.6 has many eigenvalues (and eigenvectors). The fundamental mode can be found by choosing the largest positive eigenvalue. The field profile $U(x, y)$ will be given by the eigenvector associated with this eigenvalue.

```
dx = 0.1;
dy = 0.4;
lambda = 1.55; (* unit of length is micron *)
k0=2 Pi/lambda;
dX = N[dx k0];
dY = N[dy k0];
jmax = Floor[4.8/dy+1];
imax = Floor[2.3/dx+1];
(***********************)
(* Define Index Matrix *)
(***********************)
n = Table[Table[0,{jmax}],{imax}];
For[y=0;j=1,y<=4.8,y=y+dy;j++,
  For[x=0;i=1,x<=0.65,x = x+dx;i++,
    n[[i,j]]=3.34;
    ];
  For[,x<=0.85,x = x+dx;i++,
    n[[i,j]]=3.44;
    ];
  For[,x<=1.95,x = x+dx;i++,
    If[1.4 < y <= 3.4,
      (*then*) n[[i,j]]=3.44,
      (*else*) n[[i,j]]=1.0
      ];
    ];
  For[,x<=2.3,x = x+dx;i++,
    n[[i,j]]=1.0;
    ];
  ];
(**********************)
(*  Define Variables  *)
(**********************)
(* define a_j^i *)
functiona[i_,j_] := n[[i,j]]^2 - 2/dX^2 - 2/dY^2;
a=Array[functiona,imax,jmax];
(* define b *)
b=1/dY^2;
(* define B *)
B=1/dX^2;
(**********************)
(*   Define Matrix A  *)
(**********************)
matA = IdentityMatrix[imax*jmax];
For[i=1,i<=imax,i++,
  For[j=1,j<=jmax,j++,
    matA[[(i-1)*jmax+j,      (i-1)*jmax+j     ]] = a[[i,j]]
    ];
  For[j=1,j<jmax,j++,
    matA[[(i-1)*jmax+j+1,    (i-1)*jmax+j     ]] = b;
    matA[[(i-1)*jmax+j,      (i-1)*jmax+j+1   ]] = b
    ];
  ];
For[i=1,i<imax,i++,
  For[j=1,j<=jmax,j++,
    matA[[(i-1)*jmax+j+jmax,(i-1)*jmax+j      ]] = B;
    matA[[(i-1)*jmax+j,      (i-1)*jmax+j+jmax]] = B
    ];
  ];
```

Once the matrices have been defined, all of the eigenvalues can be found easily:

```
ans = Eigenvalues[matA]
{ -24.4988, -24.4988, -24.1238, -24.1238, -23.8421, -23.8421, -23.6552, -23.6551, -23.4741, -23.4739, -23.2695,
-23.2694, -23.0162, -23.0132, -22.7978, -22.7794, -22.7352, -22.6301, -22.6093, -22.5022, -22.4154, -22.3376,
-22.0981, -21.9388, -21.8675, -21.6847, -21.5377, -21.513, -21.4278, -21.3154, -21.3129, -20.936, -20.9357,
-20.4692, -20.4691, -20.0894, -20.0894, -19.5604, -19.5604, -19.1861, -19.186, -18.7175, -18.7175, -18.3313,
-18.3313, -17.5651, -17.5588, -17.468, -17.2684, -17.1897, -17.1038, -16.8427, -16.7108, -16.6416, -16.4188,
-16.3149, -16.2895, -16.1413, -15.3806, -15.3806, -15.0024, -15.0024, -14.535, -14.5348, -14.165, -14.1535,
-14.1381, -14.0081, -13.9252, -13.8323, -13.6711, -13.5732, -13.5294, -13.372, -13.2995, -13.1707, -13.1072,
-13.1045, -13.0522, -12.9155, -12.9084, -12.8934, -12.788, -12.759, -12.7265, -12.6661, -12.6299, -12.6004,
-12.5111, -12.3196, -12.2589, -12.254, -12.1833, -12.1034, -11.9689, -11.9019, -11.8861, -11.8673, -11.8226,
-11.8095, -11.6231, -11.559, 11.5297, -11.5016, -11.3689, 11.2624, -11.1531, -11.1371, -11.0212, 10.9811, -10.9498,
10.8919, -10.8648, -10.7656, 10.758, -10.6791, -10.6601, -10.6513, -10.6461, 10.5908, 10.5604, 10.4915, -10.49,
10.3901, -10.305, 10.2873, -10.2588, 10.237, 10.2151, -10.2082, 10.1131, -10.0863, 9.99329, 9.9344, -9.93283,
-9.85808, -9.79841, 9.78065, -9.77422, 9.64918, 9.64729, -9.56637, 9.52132, 9.4889, -9.43048, -9.43034, 9.35246,
9.29375, 9.26185, -9.21357, 9.17657, -9.11469, 9.10506, -9.07504, -9.0743, 9.05266, 8.94848, -8.90259, 8.89723,
-8.83531, 8.72152, -8.712, -8.68452, 8.63025, -8.44671, 8.44533, 8.35276, -8.25815, 8.23658, -8.22765, 8.21108,
8.11019, -8.09445, -8.05408, 8.02538, -8.01122, -7.87305, -7.86232, 7.85277, 7.82184, -7.78234, -7.71838, -7.66906,
-7.65262, 7.63883, -7.4264, 7.29001, -7.21238, -7.20193, 7.08208, -7.04245, 7.01975, 6.84667, -6.82881, -6.82473,
-6.67668, 6.61269, 6.55818, 6.45101, -6.41055, -6.31511, -6.3127, 6.2359, -6.14995, 6.1443, 6.00541, -5.95158,
-5.93839, -5.86248, 5.79377, 5.6937, 5.68789, 5.60073, -5.52887, -5.47132, -5.44864, 5.38578, 5.33344, 5.27503,
-5.19514, -5.15276, -5.10375, -5.08788, -5.07358, 4.95147, -4.85774, -4.76566, -4.75093, 4.58766, -4.54749,
-4.34369, 4.31562, -4.30878, -4.23602, -4.04311, 4.0314, -4.02438, -3.94444, -3.92679, -3.86473, 3.78319, -3.67285,
-3.63119, -3.4791, 3.47057, 3.36469, -3.33663, 3.31587, -3.24505, -3.17049, -3.15421, 3.08806, -3.06368, 2.96685,
2.89108, -2.79789, -2.79535, -2.73025, 2.70989, -2.63621, -2.6004, 2.50692, 2.5041, -2.35834, 2.35377, -2.27454,
-2.24418, -2.19137, 2.16139, 2.12976, 2.03676, -2.01751, -2.00978, -1.7704, -1.76911, 1.69062, -1.68933, -1.59863,
-1.55342, -1.40046, -1.40012, 1.32529, -1.31532, -1.2027, -1.1365, -1.08984, -1.08776, 1.05235, -0.955905,
-0.747433, -0.735746, -0.723903, -0.723297, 0.675483, 0.633131, 0.632933, -0.624621, -0.596308, -0.595971,
0.427503, -0.396743, -0.362682, -0.35195, -0.263229, 0.250729, 0.250352, -0.218444, -0.218352, -0.20017, 0.176097,
0.140708, 0.0396406 }
```

The eigenvalue corresponding to the fundamental mode is the largest positive eigenvalue: 11.53. This corresponds to an index for the fundamental mode of

$$\bar{n} = 3.396$$

```
value = ans[[103]];
InputForm[value]
nbar = Sqrt[value];
  (* the modal index of the fundamental mode
  is the sqrt of the largest positive eigenvalue *)
InputForm[nbar]
11.52965762343541035
3.395534953941044375
```

Next we solve for the eigenvectors of the system and pick out the eigenvector associated with the eigenvalue chosen for the fundamental mode. This eigenvector gives us the field profile of the fundamental mode.

```
ans = Eigensystem[matA];

vector = ans[[2,103]];

modefield = Table[Table[0,{jmax}],{imax}];
For[i=1,i<=imax,i++,
  For[j=1,j<=jmax,j++,
    modefield[[i,j]] = vector[[(i-1)*jmax+j]];
    ];
  ];

ListContourPlot[modefield, AspectRatio -> 2.3/4.8];
```

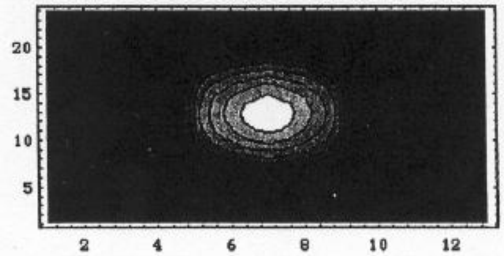

Figure 7.11. A contour plot (to scale) of the field profile for the fundamental mode.

$$\overline{n} = 3.396$$