6.4:

Plot Γg_{th} vs. δ

$\lambda = 1.55\mu$m

$< \alpha_i > = 15$ cm^{-1}

$\kappa = 50$ cm^{-1}

Mode is aligned with DBR at 1.55μm.

Consider $\phi = 0$, 90^o, and 180^o

and $\kappa L_g = 0.5$ and 1

From chapter 2, we know that

$$\Gamma g_{th} = < \alpha_i > + \frac{1}{L_a} \ln \left(\frac{1}{|r_1 r_{eff}|} \right) \tag{2.23}$$

The only unknown on the right side of this equation is r_{eff}:

$$r_{eff} = r_g + \frac{t_g^2 r_2}{1 - r_g r_2} \tag{6.46}$$

$$\tilde{r}_g = -j \frac{\tilde{\kappa}_{-1} \tanh(\tilde{\sigma} L_g)}{\tilde{\sigma} + j\tilde{\delta} \tanh(\tilde{\sigma} L_g)} \tag{6.37}$$

$$\tilde{t}_g = \frac{\tilde{\sigma} \text{sech}(\tilde{\sigma} L_g)}{\tilde{\sigma} + j\tilde{\delta} \tanh(\tilde{\sigma} L_g)} e^{-j\beta_0 L_g} \tag{6.40}$$

where

The phase term of t_g is accounted for in the expression for r_2. So $e^{-j\beta_0 L_g} = 1$.

$r_1 = \sqrt{0.32}$

$r_2 = \sqrt{0.32}\, e^{j\phi_2}$

$\tilde{\sigma} = \sqrt{\kappa^2 - \tilde{\delta}^2}$ (6.32)

$\tilde{\delta} = \tilde{\beta} - \beta_0 = \frac{2\pi\bar{n}}{\lambda} - \frac{2\pi\bar{n}}{\lambda_0} + j\frac{<g>_{xy} - <\alpha_i>}{2}$ (6.33)

$< g >_{xy} = 0$ in DBR

$< \alpha_i > = 15$ cm^{-1}

$\rightarrow \quad \tilde{\delta} = \delta - j\frac{<\alpha_i>}{2}$

$\tilde{\kappa}_{-1} = \kappa = 50$ cm^{-1} since there is no perturbation of the loss or gain.

$L_g = \frac{\kappa L_g}{\kappa}$

All of these equations can be combined to create an expression for threshold modal gain, Γg_{th} in terms of the variables κL_g, ϕ_2, and δ.

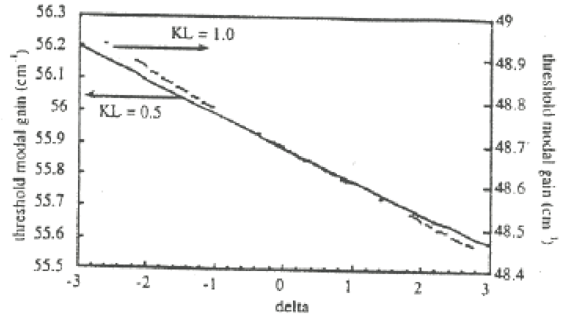

Figure 6.4. Threshold modal gain vs. detuning for $\phi_2 = 0^o$.

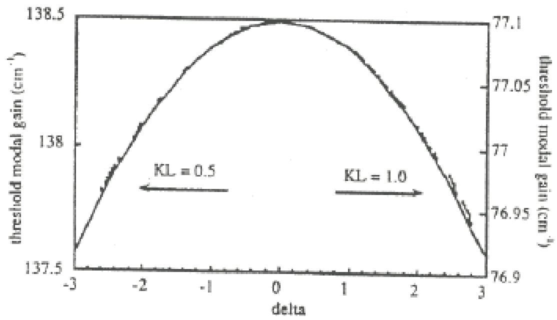

Figure 6.4. Threshold modal gain vs. detuning for $\phi_2 = 90^o$.

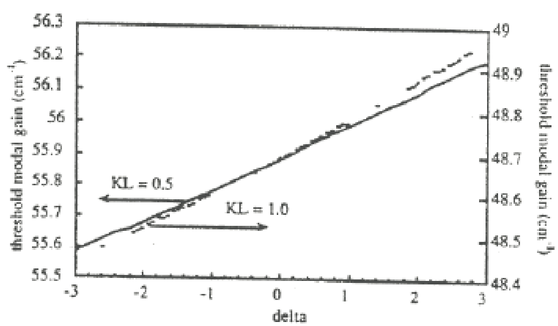

Figure 6.4. Threshold modal gain vs. detuning for $\phi_2 = 180^o$.

6.7:

a) The characteristic equation is

$$\tilde{\sigma}_{th} = -j\tilde{\delta}_{th} \tanh(\tilde{\sigma}_{th} L_g) \tag{6.41}$$

$\tilde{\sigma}_{th} = \sqrt{\kappa^2 - \tilde{\delta}_{th}^2}$

$\kappa = \frac{\kappa L}{L} = \frac{1}{400\mu m} = 25 \text{ cm}^{-1}$

$\tilde{\delta}_{th} = \beta_{th} - \beta_0 + j\frac{<g>_{xy} - <\alpha_i>_{xy}}{2} = \Delta\beta + \frac{j<g>_{xy}}{2}$

$< \alpha_i >= 20 \text{ cm}^{-1}$

Solve the characteristic equation given above for the two unknowns, $\Delta\beta$ and $< g >_{xy}$. Solve numerically:

First mode:

$$\Delta\beta = \pm 65 \text{ cm}^{-1}$$

$$< g >_{xy} = 103 \text{ cm}^{-1}$$

$$\delta L = \Delta\beta L = \pm 2.6$$

Second mode:

$$\Delta\beta = \pm 141 \text{ cm}^{-1}$$

$$< g >_{xy} = 152 \text{ cm}^{-1}$$

$$\delta L = \Delta\beta L = \pm 5.64$$

---

b) For a DFB with a $\frac{\lambda}{4}$ shift, the characteristic equation is (neglecting gain across the short cavity length, $\frac{\lambda}{2\bar{n}}$)

$$\tilde{r}_g^2 = 1 \tag{Problem6.6}$$

$$\tilde{r}_g = \frac{\kappa \tanh\left(\frac{\tilde{\sigma} L}{2}\right)}{\tilde{\sigma} + j\tilde{\delta} \tanh\left(\frac{\tilde{\sigma} L}{2}\right)} \tag{6.37}$$

Solving as in part (a), we get

First mode:

$$\Delta\beta = 0$$

$$< g >_{xy} = 96 \text{ cm}^{-1}$$

$$\delta L = \Delta\beta L = 0$$

Second mode:

$$\Delta\beta = \pm 113 \text{ cm}^{-1}$$

$$< g >_{xy} = 128 \text{ cm}^{-1}$$

$$\delta L = \Delta\beta L = \pm 4.52$$

c) The mode supression ratio is given by

$$MSR = \left(\frac{\Delta\alpha + \Delta g}{\delta} + 1\right) \frac{\alpha_m(\lambda_0)}{\alpha_m(\lambda_1)} \frac{F_1(\lambda_0)}{F_1(\lambda_1)} \qquad (3.73, 3.74)$$

$\frac{F_1(\lambda_0)}{F_1(\lambda_1)} \approx 1$

$\alpha_m = < g >_{xy} - < \alpha_i >$

$\Delta\alpha = \alpha_m(\lambda_1) - \alpha_m(\lambda_0)$

$\Delta g \approx 0$ (since $\Delta\lambda$ is small, material gain is approximately constant over this wavelength range)

$\delta = \eta_r \beta_{sp} < g >_{xy} \frac{I_{th}}{I - I_{th}}$ (3.77)

We can use the threshold value of $< g >_{xy}$ because the gain is controlled by the carrier density, which is clamped at threshold.

Assume that

$\eta_r = 1$

$\beta_{sp} = 10^{-4}$

$I = 2I_{th}$

---

case(a):

$\delta = 10^{-4}(103 \text{ cm}^{-1}) = 0.0103 \text{ cm}^{-1}$

$\alpha_m(\lambda_0) = 103 - 20 = 83 \text{ cm}^{-1}$

$\alpha_m(\lambda_1) = 152 - 20 = 132 \text{ cm}^{-1}$

$\Delta\alpha = 132 - 83 = 49 \text{ cm}^{-1}$

$MSR = 2991$

$$MSR(\text{ dB}) = 10\log(MSR) = 34.8 \text{ dB}$$

---

case(b):

$\delta = 10^{-4}(96 \text{ cm}^{-1}) = 0.0096 \text{ cm}^{-1}$

$\alpha_m(\lambda_0) = 96 - 20 = 76 \text{ cm}^{-1}$

$\alpha_m(\lambda_1) = 128 - 20 = 108 \text{ cm}^{-1}$

$\Delta\alpha = 108 - 76 = 32 \text{ cm}^{-1}$

$MSR = 2346$

$$MSR(\text{ dB}) = 10\log(MSR) = 33.7 \text{ dB}$$

![star] 6.8:

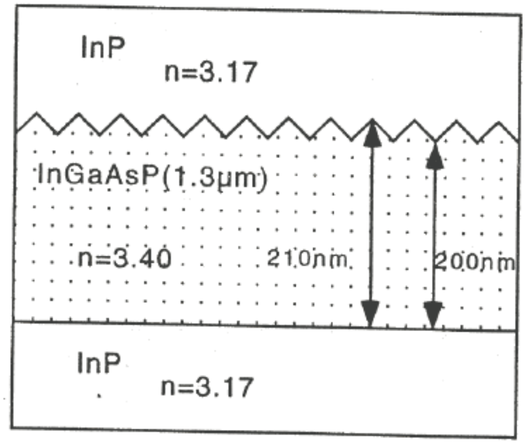

Figure 6.8. Passive grating index profile.

$$\kappa = Gk_0 a \frac{n_g^2 - \overline{n}^2}{2\overline{n} d_{eff}} \qquad (6.44)$$

$d_{eff} = d + \frac{1}{\gamma_{x1}} + \frac{1}{\gamma_{x3}}$ (6.44)

$G = \frac{4}{\pi}$ for a square wave (p.279)

$n_g = 3.40$ (Table 1.1)

$a = 5$ nm

Find waveguide parameters to solve for d_{eff}:

$$V = k_0 d\sqrt{n_2^2 - n_3^2}$$

$$= \frac{2\pi}{1.55\mu\text{m}}(0.205\mu\text{m})\sqrt{3.40^2 - 3.17^2} \qquad (A3.12)$$

$$= 1.02$$

From Fig. A3.2, $b \approx 0.20$.

$$\overline{n} = \sqrt{b(n_2^2 - n_3^2) + n_3^2}$$

$$= \sqrt{0.20(3.40^2 - 3.17^2) + 3.17^2} \qquad (A3.12)$$

$$= 3.217$$

$$\gamma_{x1} = \gamma_{x3} = k_0\sqrt{\overline{n}^2 - n_1^2}$$

$$= \frac{2\pi}{1.55\mu\text{m}}\sqrt{3.217^2 - 3.17^2} \qquad (\text{p. } 279)$$

$$= 2.228\mu\text{m}^{-1}$$

$$d_{eff} = d + \frac{1}{\gamma_{x1}} + \frac{1}{\gamma_{x3}}$$

$$= (0.205\mu\text{m}) + \frac{1}{2.228\mu\text{m}^{-1}} + \frac{1}{2.228\mu\text{m}^{-1}}$$

$$= 1.102\mu\text{m}$$

$$\kappa = 44.cm^{-1}$$

6.10:

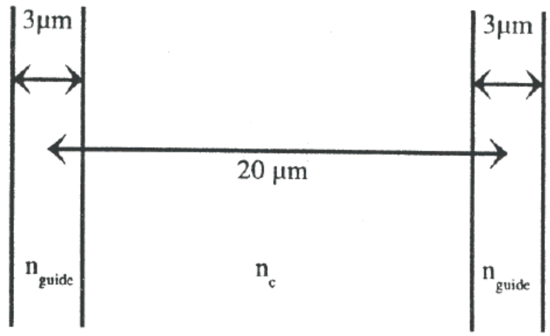

Figure 6.10. Index perturbation profile.

$n_c = 3.50$

$\Delta n = 0.04$

$n_{guide} = 3.54$

$\Gamma_x = 0.4$

$\overline{n} = \Gamma_x n_{guide} + (1 - \Gamma_x) n_c = 3.516$

---

a)

$$\begin{aligned} \kappa_{21} = \kappa_{12} &= \frac{k_0^2}{2\beta} \frac{\int \Delta\epsilon U_1^* U_2 \, dA}{\int |U_1|^2 \, dA} \\ &= \frac{4\pi^2}{\lambda_0} \frac{1}{2} \frac{\lambda_0}{2\pi\overline{n}} (2n_c \Delta n) \frac{\int U_1^* U_2 \, dA}{\int |U_1|^2 \, dA} \\ &= \frac{\pi}{(1.55\mu\text{m})3.516} (2(3.5)(0.04))(0.40 * 10\%) \\ &= 64.6 \text{ cm}^{-1} \end{aligned} \tag{6.61}$$

Note that the overlap integral between the modes of each guide was estimated instead of being calculated exactly. This estimate assumes that the field profile of the mode is roughly constant at 100% and 10% within the primary and coupled guides, respectively.

---

b) To find the length for complete coupling, use Eq. 6.70:

$$\sin^2(\kappa_{12} L_c) = 1 \tag{6.70}$$

$$\kappa_{12} L_c = \frac{\pi}{2}$$

$$L_c = \frac{\pi}{2\kappa_{12}} = \frac{\pi}{2(64.6 \text{ cm}^{-1})} = 0.0243 \text{ cm} \tag{6.71}$$

$$L_c = 243\mu\text{m}$$