In the following questions we wish to design an InP-based 1550 nm sampled-grating DBR (SGDBR) and a ring-resonator mirror (RRM) laser using an analogous vernier tuning mechanism.

Both use an 'offset quantum-well' gain region integrated with a common underlying passive waveguide consisting of 1.3 μm bandgap quaternary material, 350 nm thick, as illustrated in Fig. 8.7 of the Text for the SGDBR. The gain region incorporates 7—7nm thick—InGaAs QWs separated by 7nm thick barriers also of 1.3 μm bandgap Q-material. The material gain within each well can be approximated by $g = 650 \ln(J/60)$ cm^{-1}, where J is the current density per well given in A/cm^2, and the well material index is 3.56. The injection efficiency is 0.7 for both active and tuning sections.

Where needed, assume the waveguides are 3 μm wide with negligible current spreading, and the waveguide modal group indexes in the active and passive sections are 3.9 and 3.8, respectively. Neglect any change in modal index in the grating regions. We also assume a passive modal loss of 5 cm^{-1} in the straight passive sections with an additional 3 cm^{-1} in the curved and coupler regions of the RRM laser, and a modal loss of 15 cm^{-1} in the active sections when lasing. We assume the tuning (phase) sections are tuned by current injection, that the recombination is radiative with $B = 0.5 \times 10^{-10}$ cm^3/s in the Q-material, and that the Q-material index and loss change by $\Delta n = -0.01\,\Delta N$ and $\Delta\alpha = 5$ cm^{-1} ΔN, respectively, for the increase in carrier density ΔN given in units of 10^{18} cm^{-3}.

(20 pts)

1. For the active, passive, and tuning waveguide sections:

(a) What is the effective index of the passive waveguide sections?

(b) What is the effective index of the offset QW active sections?

(c) What is the modal gain vs. terminal current formula for the active sections?

(d) For the 75 μm long tuning sections, what current is required for a modal index change of 0.005, and what is the new modal loss at that current?

(40pts)

2. The SGDBR, illustrated in Fig. 8.7a, has the following dimensions:
--gain length = 500 μm,
--phase tuning length = 75 μm, (neglect space between gain and phase and phase and mirror)
--grating burst length in the mirrors = 5 μm,
--grating coupling coefficient (in the bursts) = 300 cm^{-1},
--back mirror, #2, has 10 bursts with a sampling period = 50 μm,
--front mirror, #1, has 3 bursts with a sampling period = 40 μm

(a) What are the maximum power reflections of the front and back mirrors?
(b) What are the FWHM widths of the power reflection peaks for the front and back mirrors?
(c) What is the wavelength spacing between peaks in the front and back mirrors?
(d) What is the overall FWHM envelope bandwidth of both mirrors (where the peaks in the power reflection spectrum fall to half of the center value?
(e) What is the axial laser mode spacing?
(f) How much current is required on the back mirror to move from one mirror peak alignment to the next (assuming the center peaks are aligned for zero current)?
(g) How much current is required on the phase section to tune from one axial laser mode to the next (assuming a pair of mirror peaks are aligned at some wavelength)?
(h) Assuming the central mirror peaks and an axial cavity mode are aligned at zero current, what currents are required on the back mirror, the front mirror and the phase section to continuously tune the laser output by exactly 1 nm?
(i) What are the laser threshold current and differential efficiency for no tuning currents?
(j) What are the laser threshold current and differential efficiency for tuning currents on both mirrors and the phase section sufficient to change all of the modal indexes by 0.005?

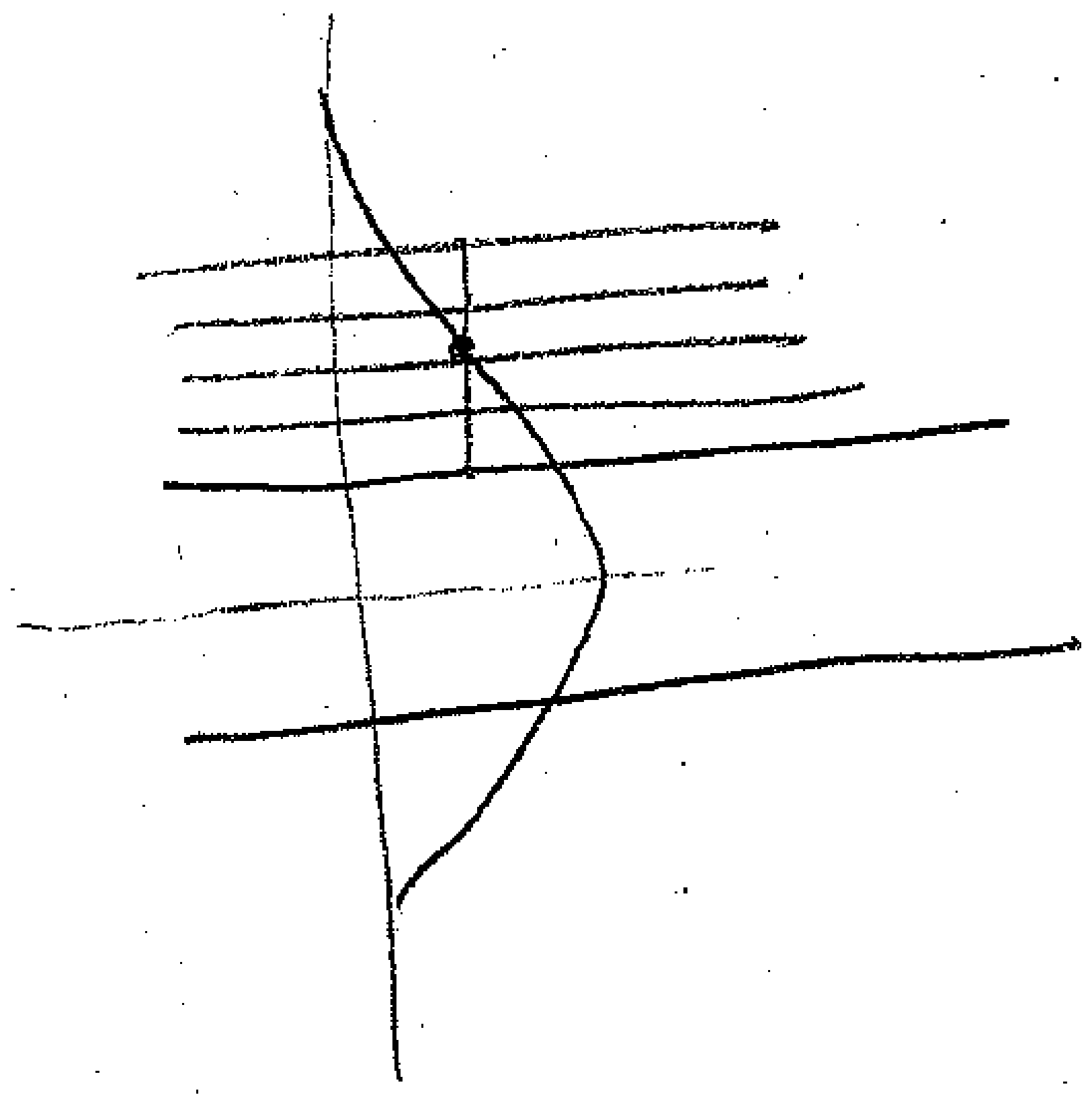
① a)

$$V = \frac{2\pi}{\lambda} d \left(n_2^2 - n_1^2\right)^{1/2} = \frac{2\pi}{1.55\,\mu m} \cdot 0.35\,\mu m \left(3.4^2 - 3.17^2\right)^{1/2} = 1.744$$ ①

$$b = 1 - \frac{\ln\left(1 + \frac{V^2}{2}\right)}{V^2/2} = 0.392$$

$$\bar{n} = \sqrt{b(n_2^2 - n_1^2) + n_1^2} = 3.2621 \quad \text{passive}$$

b) $$\Delta n = \frac{n_A \cdot \Delta n_A}{\bar{n}} \frac{\int_A |U|^2 dA}{\int |U|^2 dA} = \underbrace{\frac{(3.56^2 - 3.17^2)}{2 \cdot 3.2621}}_{0.4023} \cdot \left.\frac{\int_A |U|^2 dA}{\int |U|^2 dA}\right\} \Gamma_A$$



$15 \cdot 7/2 = 7.5 \cdot 7$

$$\Gamma_A = \frac{|U(0)|^2 \cos^2 k_x \frac{d}{2} \cdot e^{-2\gamma_x (7.5 \cdot 7\,nm)} \cdot 7 \cdot 7\,\mu m}{\frac{d_{eff}}{2} |U(0)|^2}$$

$$k_x = \frac{2\pi}{\lambda} \sqrt{3.4^2 - 3.2621^2} = 3.885\,\mu m^{-1}$$

$$\gamma_x = \frac{2\pi}{\lambda} \sqrt{3.2621^2 - 3.17^2} = 3.120\,\mu m^{-1}$$

$$\Rightarrow d_{eff} = 0.35\,\mu m + \frac{2}{3.120\,\mu m} = 0.991\,\mu m$$

$$\Gamma_A = \frac{\cos^2\left(3.885 \cdot \frac{0.35}{2}\right) \cdot e^{-3.12 \cdot 52.5 \cdot 10^{-3} \cdot 2} \cdot 49 \cdot 10^{-3}\,\mu m}{\frac{0.951}{2}\,\mu m} = 0.04309$$

$$\Delta n = 0.4023 \cdot 0.04309 = 0.01734 \qquad \bar{n}_{active} = 3.2621 + 0.01734 = 3.2794$$

c) $$g = 650\,cm^{-1} \cdot \ln \frac{J}{60\,A/cm^2} \qquad J = \frac{\eta_i I}{A \cdot 7} \text{ / per well}$$

~~$\Gamma_{xy} g = \Gamma_A \cdot 650\,cm^{-1}$~~ $\quad \Gamma_{xy} \cdot \Gamma_A = \Gamma \Rightarrow$

$$\Gamma g = \frac{L_A}{L_T} \cdot \Gamma_{xy} \cdot g_0 \ln\left[\frac{I_A \cdot 0.7}{7 \cdot 60 \cdot 3\,\mu m \cdot L_A}\right] = \frac{L_A}{L_T} 28\,cm^{-1} \ln \frac{I_A}{0.18 I_A \left(\frac{A}{cm}\right)}$$

d) $L_{ps} = 75\mu m$ $\quad \Gamma_x(wgd) = \frac{V^2}{2+V^2} = \frac{1.744^2}{2+1.744^2} = 0.6033$ ②

$\Delta\bar{n} = 0.005 \Rightarrow \Delta n = \frac{0.005}{\Gamma_x} = \frac{0.005}{0.6033} = 0.00829 = \frac{0.01\,\Delta N}{10^{18}\,cm^{-3}} \Rightarrow$

$\Rightarrow \Delta N = 0.829 \cdot 10^{18}\,cm^{-3}$

$I = \frac{q \cdot V \cdot B \cdot N^2}{\eta_i} \Rightarrow \Delta N = \sqrt{\frac{\eta_i\, I_{ph}}{q \cdot V \cdot B}}$

$$I_{ph} = \frac{(0.829 \cdot 10^{18}\,cm^{-3})^2 \cdot (1.602 \cdot 10^{-19} c)(3 * 75 * 0.35 \cdot 10^{-12}\,cm^3) \cdot 0.5 \cdot 10^{-10}\,\frac{cm^3}{J}}{0.7}$$

$I_{ph} = 0.618\,mA$ $\quad \Delta\alpha = 5\,cm^{-1} \cdot (0.829) = 4.145\,cm^{-1}$

---

② $L_g = 500\mu m$ $\quad$ BM $10 \times S_0 = 50\mu m$

$L_{ph} = 75\mu m$ $\quad$ FM $3 \times S_0 = 40\mu m$

$L_B = 5\mu m$

$\kappa = 300\,cm^{-1}$

a) $R_B' = \tanh^2 \kappa L_B = \tanh^2(300 \cdot 50 \cdot 10^{-4}) = 0.8193$ $\quad$ lossless

$R_F' = \tanh^2 \kappa L_F = \tanh^2(300 \cdot 15 \cdot 10^{-4}) = 0.1780$

with loss:

$R_B = R_B' \cdot e^{-2\langle\alpha_i\rangle L_{effB}}$ $\quad L_{effB} = \frac{1}{2\kappa_B} \cdot \tanh \kappa L_B$

$\kappa_B = \frac{10 \times 5 \times 300\,cm^{-1}}{\cancel{10 \times 50}\; 9 \times 50 + 10 \times 5} = 30\,cm^{-1}$ $\Rightarrow$ $L_{eff} = 157\mu m$

$R_B = 0.8193 \cdot e^{-2 \cdot 5\,cm^{-1} \cdot 0.0157\,cm} = 0.7046$

$R_F: L_{eff} = \frac{1}{2}(\cancel{1} \cdot 5 + 80) = 42.5\mu m$ $\quad R_F = 0.1706$

b) $$\Delta\lambda_{1/2B} = \frac{\lambda^2}{2n_g 2L_{eff}} = \frac{1.55^2\,\mu m^2}{2\cdot 3.8\cdot 2\cdot 151\,\mu m} = 1.047\,nm$$

$$\Delta\lambda_{1/2F} = \frac{1.55^2\,\mu m^2}{2\cdot 3.8\cdot 85\,\mu m} = 3.719\,nm$$

c) $$\Delta\lambda_{pB} = \frac{1.55^2\,\mu m^2}{2\cdot 3.8\cdot 50\,\mu m} = 6.322\,nm$$

$$\Delta\lambda_{pF} = \frac{1.55^2\,\mu m^2}{2\cdot 3.8\cdot 40\,\mu m} = 7.903\,nm$$

d) $$\Delta\lambda_{envelope} = \frac{1.55\,\mu m^2}{2\cdot 3.8\cdot 5\,\mu m} = 63.22\,nm$$

e) $$\Delta\lambda_m = \frac{(1.55\,\mu m)^2}{2\cdot 3.8\cdot(75+151+412.5)+2\cdot 3.9\cdot 500} = 0.4044\,nm$$

f) back mirror $L_g = 505\,\mu m$

need to move $\Delta\lambda_{PF} - \Delta\lambda_{PB} = 7.503\,nm - 6.322\,nm = 1.581\,nm$

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta\bar{n}_{Bg}}{\bar{n}_g} \Rightarrow \Delta\bar{n}_{Bg} = 0.003876$$

$$I \sim N^2 \qquad I_{Bg} = \left(\frac{0.003876}{0.005}\right)^2 (0.618\,mA)\cdot\frac{505}{75} = 2.5\,mA$$

g) $$\Delta\lambda_m = 0.4044\,nm = \lambda\cdot\frac{\Delta n_p L_p}{\sum n_g L}$$

$$\Delta n_p = \frac{\Delta\lambda}{\lambda}\cdot\frac{\sum n_g L}{L_p} = \frac{0.4044\,nm}{1550\,nm}\cdot\frac{5940.6\,\mu m}{275\,\mu m} = 0.01034$$

$$I_{ph} = \left(\frac{0.01034}{0.005}\right)^2 0.618\,mA = 2.643\,mA$$

h) first, move mirror peaks, then, see how much mode has

$\Delta\lambda_m = 1nm$ $\Delta\lambda_{PB} = \Delta\lambda_{PF} = 1nm$ $\frac{\Delta\lambda}{\lambda} = \frac{\Delta n}{n_g}$ ⑤

$$I_{B6} = \left(\frac{\frac{1nm}{1550nm}\cdot 3.8}{0.005}\right)^2 0.618mA\,\frac{505}{75} = 1.000mA$$

$$I_{F6} = I_{B6}\cdot\frac{85}{505} = 0.1684\,mA$$

$$\Delta\lambda_m' = \lambda\cdot\frac{\Delta n_{DBR} L_{peffF} + \Delta n_{DBR} L_{effB}}{\sum n_g L} =$$

$$= 1550nm\cdot\frac{\left(\frac{1}{1550}\cdot 3.8\right)\left[151+42.5\right]}{5940.6/2} = 0.2463\,nm$$

$\Delta\lambda_m'' = 0.7537nm$ $= 1.8638$ mode spacing

$$I_{ph} = 2.643mA\,(1.8638)^2 = 9.181\,mA$$

i) $\Gamma g_{th} = \langle\alpha_i\rangle + \alpha_m$

- effective mirror model

$$\langle\alpha_i\rangle = \frac{5cm^{-1}\overset{\mu m}{(75+151+42.5)}\cdot 3.8 + 15cm^{-1}\cdot 3.9\cdot 500\mu m}{(5940.6/2)\mu m}$$

$$= 11.56cm^{-1}$$

$$\alpha_m = \frac{1}{L}\ln\frac{1}{\sqrt{R_F R_B}} = \frac{1}{\frac{5940.6}{2\cdot 3.85}}\ln\frac{1}{\sqrt{0.8193\cdot 0.1780}} =$$

lossless

$$= 12.48\,cm^{-1}$$

$$\eta_d = 0.7\cdot\frac{12.48}{11.56+12.48} = 0.363$$ both ends, mostly front

from 1)

$$24.05cm^{-1} = \frac{500}{771.5}\cdot 28cm^{-1}\ln\left(\frac{I_A}{0.18\cdot 1.05cm}\right) \Rightarrow ?$$

$I_A = 33.84\,mA$

j) $\Delta n = 0.005$ $\Delta\alpha = 4.145\,\mathrm{cm}^{-1}$

$$\langle \alpha_i \rangle = \frac{9.145\,\mathrm{cm}^{-1}\,(75 + 151 + 42.5)\cdot 3.8 + 15\cdot 3.9\cdot 500\,\mu\mathrm{m}}{5940/2\,\mu\mathrm{m}} =$$

$$= 12.99\,\mathrm{cm}^{-1}$$

α_m – same $\Rightarrow$ $\eta_d = 0.343$ $I_{th} = 36.61\,\mathrm{mA}$