6.11:

Approximate index as a linear function of wavelength. Using the two data points given by the problem, we can write

$n_1(\lambda) = 3.81 - \frac{\lambda}{5}$

$n_2(\lambda) = 4.12 - \frac{2\lambda}{5}$

The propagation constants can then be written as a function of wavelength:

$\beta_{1,2}(\lambda) = \frac{2\pi}{\lambda} n_{1,2}(\lambda)$

$\Delta\beta(\lambda) = \frac{2\pi}{\lambda}(n_2(\lambda) - n_1(\lambda)) = \frac{2\pi}{\lambda}(0.31 - \frac{\lambda}{5})$

$s = \sqrt{\left(\frac{\Delta\beta}{2}\right)^2 + \kappa^2}$

Find the coupling length for a wavelength of 1.55μm.

$$L = \frac{\pi}{2\kappa_{12}} = \frac{\pi}{2(0.01\mu\text{m}^{-1})} = 157\mu\text{m} \tag{6.71}$$

Use Eq. 6.66 to find the coupling between guides for other wavelengths:

$$\frac{|a_2(L)|}{|a_1(0)|} = \frac{\kappa_{21}}{s}\sin(sL)$$

The bandwidth of the device can be found by finding the wavelengths at which the coupling between the guides is reduced by 3dB:

$$\frac{|a_2(L)|}{|a_1(0)|} = \frac{\kappa_{21}}{s}\sin(sL) = \frac{1}{\sqrt{2}}$$

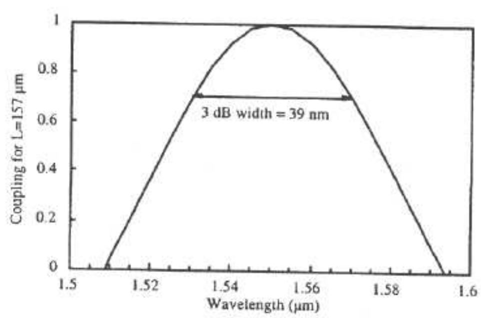

Figure 6.11. Coupling vs. wavelength.

$$\Delta\lambda = 39 \text{ nm}$$

~~17.22.6Hz~~

~~17.22 THz~~

4.92 THz

6.12:

Use Eq. 6.84 to calculate the proportion of power coupled from one waveguide to the next:

$$\frac{P_2(0^+)}{P_1(0^-)} = \left| t \int U_2^* U_1 \, dA \right|^2 \qquad (6.84)$$

From Table 1.1, the values for index in the materials of this waveguide are

InGaAsP(1.3μm): $n_g = 3.40$

InP: $n_c = 3.17$

$\lambda = 1.55\mu$m

$t = \sqrt{\frac{4\overline{n}_1\overline{n}_2}{(\overline{n}_1+\overline{n}_2)^2}}$ (6.82)

Calculate the effective indices for the two waveguides using equations from Appendix 3:

Smaller waveguide:

$$\begin{aligned} V &= \frac{2\pi}{\lambda} d\sqrt{n_g^2 - n_c^2} \\ &= \frac{2\pi}{1.55\mu\text{m}}(0.200\mu\text{m})\sqrt{3.40^2 - 3.17^2} \qquad (A3.12) \\ &= 0.996 \end{aligned}$$

$a = 0$

Find b from Fig. A3.2: $b = 0.19$

$$\overline{n}_1 = \sqrt{b(n_g^2 - n_c^2) + n_c^2} = \sqrt{0.19(3.40^2 - 3.17^2) + 3.17^2} = 3.215 \qquad (A3.12)$$

$$k_x = \sqrt{k_0^2(n_g^2 - \overline{n}^2)} = \sqrt{\left(\frac{2\pi}{1.55\mu m}\right)^2 (3.40^2 - 3.215^2)} = 4.48\mu\text{m}^{-1} \qquad (A3.7)$$

$$\gamma_x = \sqrt{k_0^2(\overline{n}^2 - n_c^2)} = \sqrt{\left(\frac{2\pi}{1.55\mu m}\right)^2 (3.215^2 - 3.17^2)} = 2.17\mu\text{m}^{-1} \qquad (A3.7)$$

From Eq. A3.8,

$$U_{g1}(x) = A_1 \cos((4.48\mu\text{m}^{-1})x) \qquad (|x| < 100 \text{ nm})$$

$$U_{c1}(x) = B_1 \, e^{\pm(2.17\mu\text{m}^{-1})x} \qquad (|x| \geq 100 \text{ nm})$$

Match boundary conditions at the edge of the waveguide:

$$\begin{aligned} B_1 &= A_1 \frac{\cos((4.48\mu\text{m}^{-1})(0.1\mu\text{m}))}{e^{\pm(2.17\mu\text{m}^{-1})(0.1\mu\text{m})}} \\ &= 1.12A_1 \end{aligned}$$

Larger waveguide:

$V = 1.993$

$a = 0$

$b = .43$

$\overline{n}_2 = 3.26$

$k_x = 3.91 \mu\text{m}^{-1}$

$\gamma_x = 3.08 \mu\text{m}^{-1}$

$$U_{g1}(x) = A_1 \cos((3.91 \mu\text{m}^{-1})x) \qquad (|x| < 100 \text{ nm})$$

$$U_{c1}(x) = B_1 \ e^{\pm(3.08 \mu\text{m}^{-1})x} \qquad (|x| \geq 100 \text{ nm})$$

$$B_1 = 1.31 A_1$$

$$t = 0.99998 \approx 1.$$

Do the overlap integral between the fields numerically:

$$\frac{P_2(0^+)}{P_1(0^-)} = \left| t \int U_2^* U_1 \, dA \right|^2 \approx 0.968 \tag{6.84}$$

Coupling (power) loss = 3.2%

Since U_1 and U_2 are real, the coupling loss is the same for both directions.

7.1:

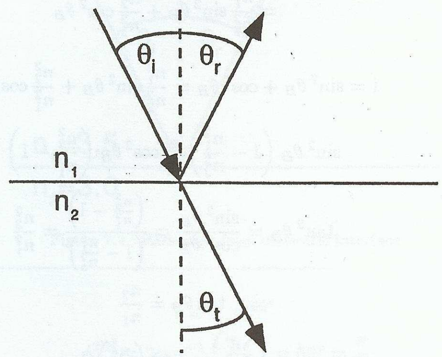

Figure 7.1. Reflection from dielectric interface

We have two equations and two unknowns, θ_B and θ_t:

1) $r^{TM} = \frac{k_{ix} - \frac{\epsilon_1}{\epsilon_2} k_{tx}}{k_{ix} + \frac{\epsilon_1}{\epsilon_2} k_{tx}}$ (Eq. 7.5)

2) $k_1 \sin\theta_B = k_2 \sin\theta_t$ (Snell's Law)

For zero reflection, the first equation leads to

$$0 = r^{TM} = \frac{k_{ix} - \frac{\epsilon_1}{\epsilon_2} k_{tx}}{k_{ix} + \frac{\epsilon_1}{\epsilon_2} k_{tx}} \tag{7.5}$$

$$k_{ix} = \frac{\epsilon_1}{\epsilon_2} k_{tx}$$

$$k_1 \cos\theta_B = \frac{n_1^2}{n_2^2} k_2 \cos\theta_t$$

$$\begin{aligned} \cos\theta_B &= \frac{n_1^2}{n_2^2} \frac{k_2}{k_1} \cos\theta_t \\ &= \frac{n_1^2}{n_2^2} \frac{\frac{2\pi n_2}{\lambda}}{\frac{2\pi n_1}{\lambda}} \cos\theta_t \\ &= \frac{n_1}{n_2} \cos\theta_t \end{aligned}$$

The second equation leads to

$$\begin{aligned} k_1 \sin\theta_B &= k_2 \sin\theta_t \\ \sin\theta_t &= \frac{n_1}{n_2} \sin\theta_B \end{aligned}$$

Combining these two equations, we have

$$1 = \sin^2\theta_t + \cos^2\theta_t$$

$$= \frac{n_1^2}{n_2^2}\sin^2\theta_B + \frac{n_2^2}{n_1^2}\cos^2\theta_B$$

$$1 = \sin^2\theta_B + \cos^2\theta_B = \frac{n_1^2}{n_2^2}\sin^2\theta_B + \frac{n_2^2}{n_1^2}\cos^2\theta_B$$

$$\sin^2\theta_B\left(1 - \frac{n_1^2}{n_2^2}\right) = \cos^2\theta_B\left(\frac{n_2^2}{n_1^2} - 1\right)$$

$$\tan^2\theta_B = \frac{\sin^2\theta_B}{\cos^2\theta_B} = \frac{\left(\frac{n_2^2}{n_1^2} - 1\right)}{\left(1 - \frac{n_1^2}{n_2^2}\right)} = \frac{n_2^2}{n_1^2}$$

$$\tan\theta_B = \frac{n_2}{n_1}$$

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

7.3:

a) $V = k_0 d\sqrt{n_{guide}^2 - n_{cladding}^2} = \frac{2\pi}{1.55\mu m}(0.4\mu m)\sqrt{3.4^2 - 3.2^2} = 1.86$ (A3.12)

$a = 0$

$b = 0.40$ from Fig. A3.2

Solve Eq. A3.12 for $\overline{n}$

$\overline{n} = \sqrt{n_c^2 + b(n_g^2 - n_c^2)} = \sqrt{3.2^2 + 0.40(3.4^2 - 3.2^2)} = 3.281$

$$\begin{aligned}\theta_i &= \tan^{-1}\left(\frac{\beta}{k_{ix}}\right)\\ &= \tan^{-1}\left(\frac{k_0\overline{n}}{k_0\sqrt{n_g^2 - \overline{n}^2}}\right)\\ &= \tan^{-1}\left(\frac{3.281}{\sqrt{3.4^2 - 3.281^2}}\right)\\ &= 74.7^o\end{aligned}$$

---

b) For the same θ_i, $\overline{n}$, a, and b, we have, from Fig A3.2 , for m=1,

$$V = 5.8$$

$$d = \frac{V}{k_0\sqrt{n_g^2 - n_c^2}} = \frac{5.8}{\frac{2\pi}{1.55\mu m}\sqrt{3.4^2 - 3.2^2}} = 1.24\mu m \quad (A3.12)$$

---

c)

$$a = \frac{n_{c2}^2 - n_{c1}^2}{n_g^2 - n_{c2}^2} = \frac{3.2^2 - 3^2}{3.4^2 - 3.2^2} = 0.939 \approx 1.0 \quad (A3.12)$$

$$b = 0.40$$

From Fig. A3.2,

$$V = 2.2$$

$$d = \frac{V}{k_0\sqrt{n_g^2 - n_c^2}} = \frac{2.2}{\frac{2\pi}{1.55\mu m}\sqrt{3.4^2 - 3.2^2}} = 0.47\mu m \quad (A3.12)$$