7.2:

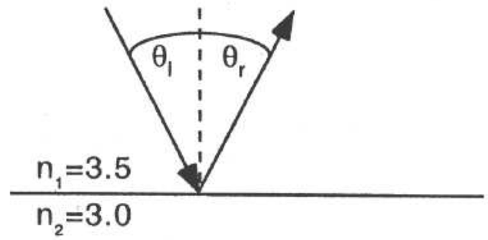

Figure 7.2. Reflection from dielectric interface

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a) TE polarization

$$r = 1 \quad e^{j60^o}$$

$$\phi^{TE} = 2\tan^{-1}\left(\frac{\gamma_{tx}}{k_{ix}}\right) = 60^o = \frac{\pi}{3} \tag{7.11}$$

$$\frac{\gamma_{tx}}{k_{ix}} = 0.577 = \frac{1}{\sqrt{3}}$$

$$\frac{\sqrt{\beta^2 - k_2^2}}{n_1 \cos\theta_i} = \frac{\sqrt{n_1^2 \sin^2\theta_i - n_2^2}}{n_1 \cos\theta_i} = \frac{1}{\sqrt{3}}$$

$$n_1^2 \sin^2\theta_i - n_2^2 = \frac{n_1^2}{3}\cos^2\theta_i$$

$$n_1^2 - n_1^2\cos^2\theta_i - n_2^2 = \frac{n_1^2}{3}\cos^2\theta_i$$

$$\frac{4}{3}n_1^2\cos^2\theta_i = n_1^2 - n_2^2$$

$$\cos\theta_i = \sqrt{\frac{3(n_1^2 - n_2^2)}{4n_1^2}}$$

$$\theta_i = \cos^{-1}\left(\sqrt{\frac{3(n_1^2 - n_2^2)}{4n_1^2}}\right)$$

$$\theta_i = \cos^{-1}(0.446)$$

$$\theta_i = 63.5^o$$

---

b) TM polarization

$$\phi^{TM} = 2\tan^{-1}\left(\frac{n_1^2\gamma_{tx}}{n_2^2 k_{ix}}\right) = 60^o = \frac{\pi}{3} \qquad \text{(p. 305)}$$

$$\frac{n_1^2}{n_2^2}\frac{\sqrt{n_1^2\sin^2\theta_i - n_2^2}}{n_1\cos\theta_i} = \frac{1}{\sqrt{3}}$$

$$\theta_i = \cos^{-1}\left(\sqrt{\frac{n_1^2 - n_2^2}{n_1^2 + \frac{n_2^4}{3n_1^2}}}\right)$$

$$\theta_i = 61.7^o$$

7.4:

As described by Fig. 7.13, modes occur at resonances of the layer structure. The resonances can be found by using the transmission matrices of chapter 3 (see Table 3.3).

Assuming lossless material, the propagation matrices are given by

$$T_p = \begin{bmatrix} e^{j\beta_t L_t} & 0 \\ 0 & e^{-j\beta_t L_t} \end{bmatrix}$$

where

$\beta_t = k_x = \sqrt{k_0^2 - k_z^2} = \frac{2\pi}{\lambda_0}\sqrt{n^2 - \bar{n}^2}$

L_t =distance of propagation (i.e. thickness of layer).

The reflection matrices are given by

$$T_{r_{12}} = \frac{1}{t_{12}}\begin{bmatrix} 1 & r_{12} \\ r_{12} & 1 \end{bmatrix}$$

$$r_{12} = \frac{n_1 - n_2}{n_1 + n_2}$$

$$t_{12} = \sqrt{1 - r_{12}^2}$$

The reflection values shown in Fig. 7.13,

$$r_A = r_B = \frac{T_{21}}{T_{11}}$$

$$T = T_{pa} T_{r_{a1}} T_{p1} T_{r_{12}} T_{p2} T_{r_{21}} T_{p1} T_{r_{12}}$$

Modes of the waveguide structure occur for

$$r_A^2 = 1$$

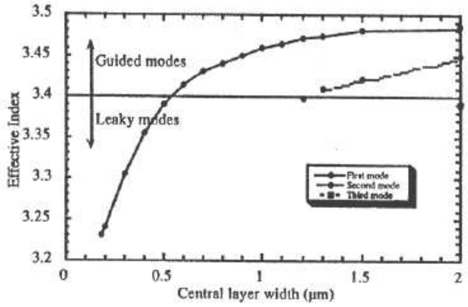

Figure 7.4. Effective index vs. central layer thickness

7.5:

For perfectly reflecting boundaries placed at $30\mu m$, $r=-1$. From Table 1.1, n(GaAs at $1.0\mu m$)= 3.52 and n($Al_{.2}Ga_{.8}As$ at $1.0\mu m$)= 3.39. This layer structure is shown in Fig. 7.11.

Solve for guided modes:

Guided modes can be solved for as done in Appendix 3. Using Eq. A3.12 and Fig. A3.2, we find that $V=\frac{2\pi}{1.0\mu m}(0.3\mu m)\sqrt{3.52^2-3.39^2}=1.79$

So there is one guided mode.

$b=0.36$

$\overline{n}=\sqrt{3.39^2+(0.36)(3.52^2-3.39^2)}=3.437$

$\beta(m=1)=\frac{2\pi}{1.0\mu m}3.437=21.598$

Solve for radiation modes:

$$k=\sqrt{k_x^2+\beta^2}$$

$$k_x=\sqrt{k^2-\beta^2}=\frac{2\pi}{\lambda}\sqrt{n^2-\overline{n}^2}$$

In order to satisfy the characteristic equation (round trip gain =1), we have

$$r_1\, e^{k_{1x}d_1}\; e^{k_{2x}d_2}\; e^{k_{3x}d_3} r_2\; e^{k_{1x}d_1}\; e^{k_{2x}d_2}\; e^{k_{3x}d_3}=1$$

$r_1=r_2=-1$

$d_1=d_3=30\mu m$

$d_2=0.3\mu m$

$k_1=k_3=\frac{2\pi}{1.0\mu m}\sqrt{3.39^2-\overline{n}^2}$

$k_2=\frac{2\pi}{1.0\mu m}\sqrt{3.52^2-\overline{n}^2}$

The phase of the characteristic equation can be simplified to

$$m\,\frac{\lambda}{2}=(30\mu m)\sqrt{3.39^2-\overline{n}^2}+(0.3\mu m)\sqrt{3.52^2-\overline{n}^2}+(30\mu m)\sqrt{3.39^2-\overline{n}^2} \qquad \text{for } m\geq 2$$

Solve for β using perturbation theory described in Chapter 6.

Simplify this expression by replacing the waveguide region with the same width of cladding material. This simplifies the expression for β significantly and allows us to write a closed form solution for β.

$$m\,\frac{\lambda}{2}\approx(60.3\mu m)\,\frac{\lambda}{2\pi}\sqrt{\left(\frac{2\pi n_1}{\lambda}\right)^2-\beta^2}$$

$$\beta(m)\approx\sqrt{\left(\frac{2\pi n_1}{\lambda}\right)^2-\left(\frac{m\pi}{60\mu m}\right)^2}$$

The perturbation due to the waveguide can then be added to the value of β for each mode:

$$\Delta\beta=\frac{\int\Delta\epsilon k_0^2|U|^2dA}{2\beta\int|U|^2dA} \qquad (6.9)$$

For odd numbered modes,

$$U(x)=A\cos(k_{x1m}x)$$

where A is a normalization coefficient and k_{x1m} is the value of the x component of the k vector for mode number m: $k_{x1m}=\sqrt{k_1^2-\beta_m^2}$

Similarly, for even numbered modes,

$$U(x)=A\sin(k_{x1m}x)$$

The index perturbation is $\Delta\epsilon=3.52^2-3.39^2=0.90$

$k_0=\frac{2\pi}{1.0\mu m}=6.28\mu m^{-1}$

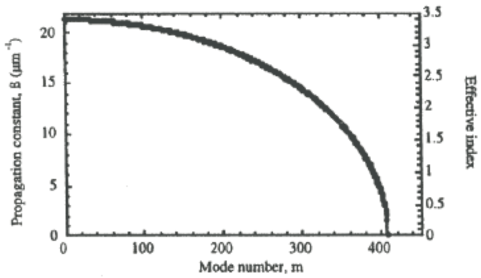

Figure 7.5. β and $\overline{n}$ vs. radiation mode number

Note: If better accuracy is desired, the values of β can be solved for numerically or graphically using the transcendental equations described by Eq. 7.26.