Lecture 13 - Optical Photodetectors and Receivers

Reading and Homework

- Read Chapter 4 of Agrawal
- Homework
  - Chapter 4 Problems 4.1, 4.2, 4.6, 4.7, 4.11, 4.12, 4.13, 4.15

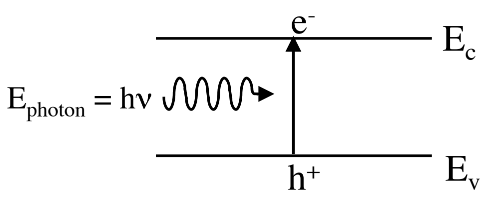
Detection of Optical Signals

- Thermal: Temperature change with photon absorption
  - Thermoelectric
  - Pyromagnetic
  - Pyroelectric
  - Liquid crystals
  - Bolometers
- Wave Interaction: Exchange energy between waves at different frequencies
  - Parametric down-conversion
  - Parametric up-conversion
  - Parametric amplification
- Photon Effects: Generation of photocarriers from photon absorption
  - Photoconductors
  - Photoemissive
  - Photovoltaics

Photoconductors

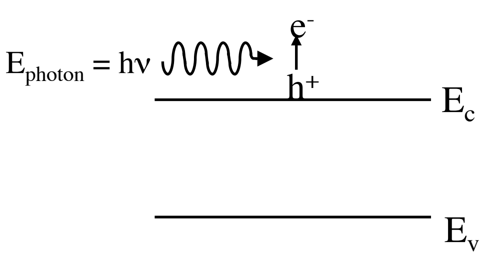
- Photon absorption in semiconductor materials.
- Three main absorption mechanisms: Intrinsic (band-to-band), Free-Carrier Absorption and Band-and-Impurity Absorption
- Intrinsic (band-to-band) is the dominant effect in most SC photoconductors

Intrinsic (band-to-band)

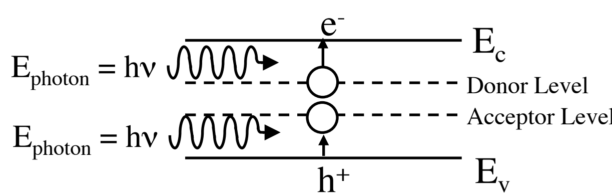


•Incident photon $E_{photon} = h\nu = E_c - E_v$

Free-Carrier Absorption



Band-and-Impurity Absorption



Photoconductors

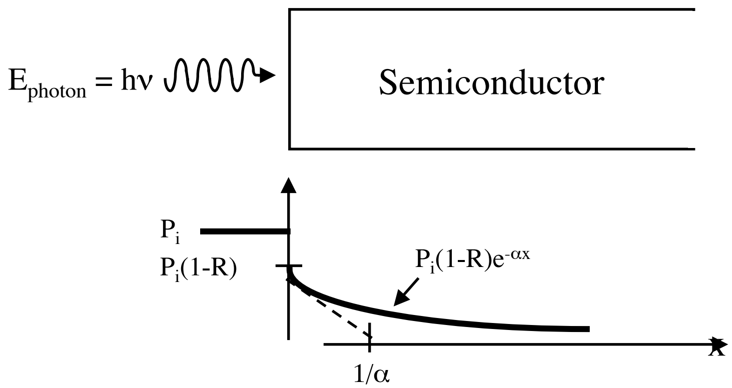
- For intrinsic absorption, photons can be absorbed if

$$\lambda(\mu m) > \frac{hc}{E_c - E_V} = \frac{1.24}{E_g(eV)}$$

$$\lambda(nm) > \frac{1240}{E_g(eV)}$$

Material	Bandgap (eV)	Maximum λ (nm)	Typical Operating Range (nm)
Si	1.12	1110	500-900
Ge	0.67	1850	900-1300
GaAs	1.43	870	750-850
$In_xGa_{1-x}As_yP_{1-y}$	0.38-2.25	550-3260	1000-1600

Photoconductors



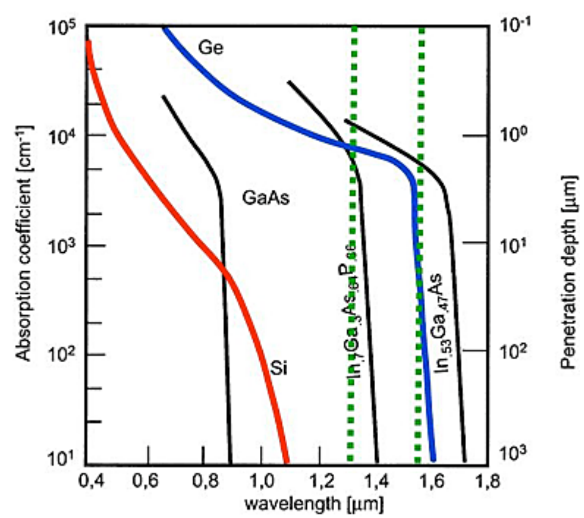
- Define:
  - P_i = incident optical power
  - $R(\lambda)$ power reflectivity from input medium to semiconductor
  - $\alpha(\lambda)$ = 1/e absorption length
  - $1/\alpha(\lambda)$ = penetration depth

- Power absorbed by the semiconductor is

$$P_{abs}(x) = P_i(1-R)(1-e^{-\alpha(\lambda)x})$$
$$= \eta(\lambda,x)P_i$$

- defining the efficiency

$$\eta(\lambda,x) = \frac{\text{number of photocarriers produced}}{\text{number of incident photons}}$$
$$= (1-R)(1-e^{-\alpha(\lambda)x})$$
$$0 \le \eta(\lambda,x) \le 1$$



Photoconductive Photodetectors

- Photogenerated current will have time and wavelength dependence

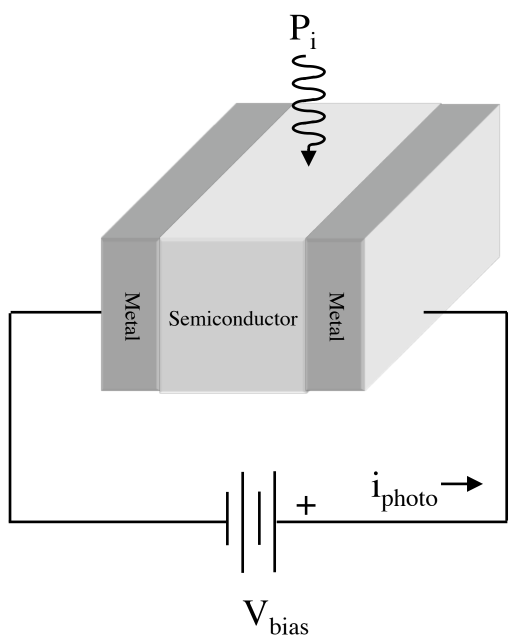
$$i_{photo}(t) = \frac{\eta q}{h\nu} G P_{rcvd}(t) + i_{dark}$$

$$\tau_{carrier} = \text{ mean free carrier lifetime}$$

$$\tau_{transit} = \text{ transit time between eletrical contacts}$$

$$G = \left(\frac{\tau_{carrier}}{\tau_{transit}}\right) = \text{ photoconductive gain}$$

$$i_{dark} = \text{ dark current}$$



- The transit time for electrons and holes can be different and in many SCs the eletron mobility is greater than that of the hole

$$\upsilon_e = \mu_e E > \mu_h E = \upsilon_h$$

- The SC must remain charge neutral, for every electron generated, multiple holes will get pulled in until the photogenerated electron reaches the other contact. The carrier and transit times are limited by the slower carrier and the photoconductive gain is given by the ratio of the transit times

$$\tau_{carrier} = \frac{L_a}{\upsilon_h}$$

$$\tau_{transit} = \frac{L_a}{\upsilon_e}$$

Photoconductive Photodetectors

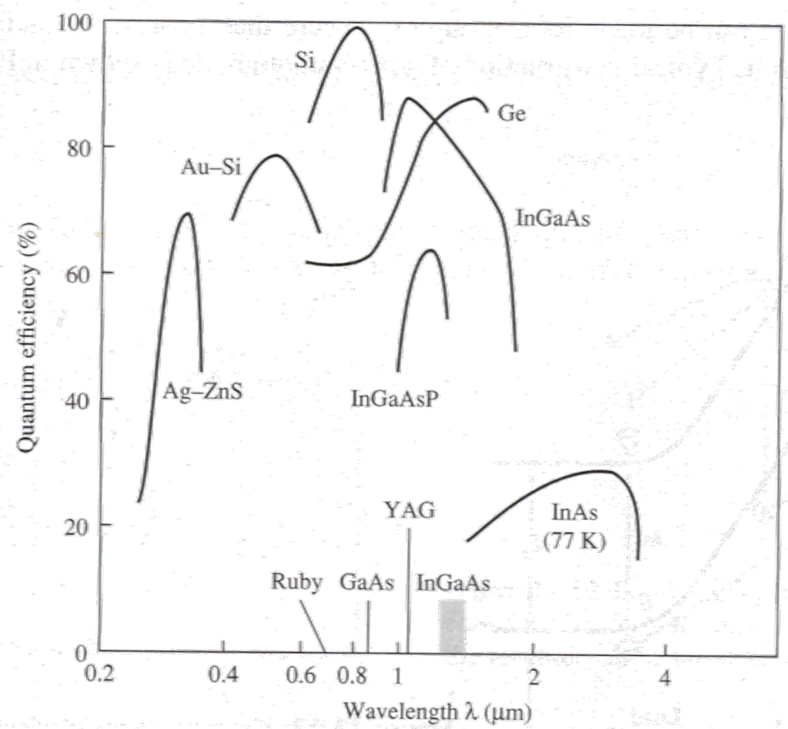
- The carrier velocity is a linear function of electric field strength up to a saturation velocity (which is the same for both electrons and holes)
  - Field strength of about 10^5 V/cm result in velocities in range of $6x10^6$ to 10^7 cm/s
  - Some materials have an electron drift velocity that peaks at $2x10^7$ cm/s at 10^4 V/cm
- When photoconductive gain is desirable, detector is operated at low voltages
- Carrier lifetime also impacts the frequency response of the photoconductive photodetector

$$i_{photo}(\omega) = \Re G \frac{P_{rcvd}(\omega)}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

$$\omega_c = \frac{1}{\tau_{carrier}} = \text{cutoff frequency}$$

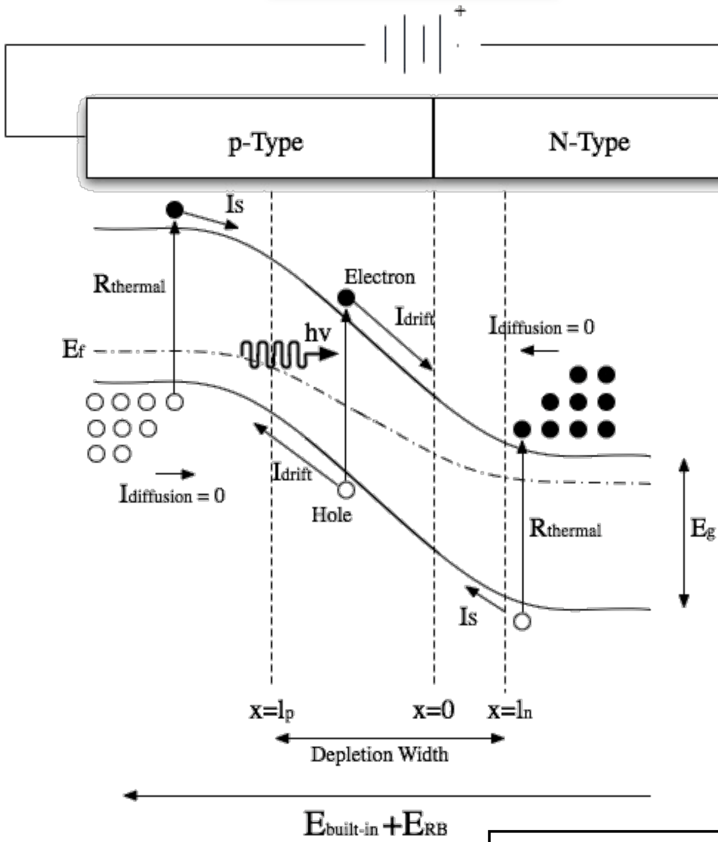
Pn Junction Photodetectors

- Photons absorbed within the junction diffusion length generate carriers that are swept down the barrier (drift current) adding to the thermal generation current (dark current) that is present in the absence of light.
- The semiconductor bandgap energy sets a lower limit on the frequency of light that can be absorbed, hence generating photocarriers.
  - Light with energy hn < Eg will not be absorbed and will not contribute to photocurrent
  - Light with energy hn >> Eg will be absorbed away from the
- The quantum efficiency and therefore the responsivity are wavelength dependent

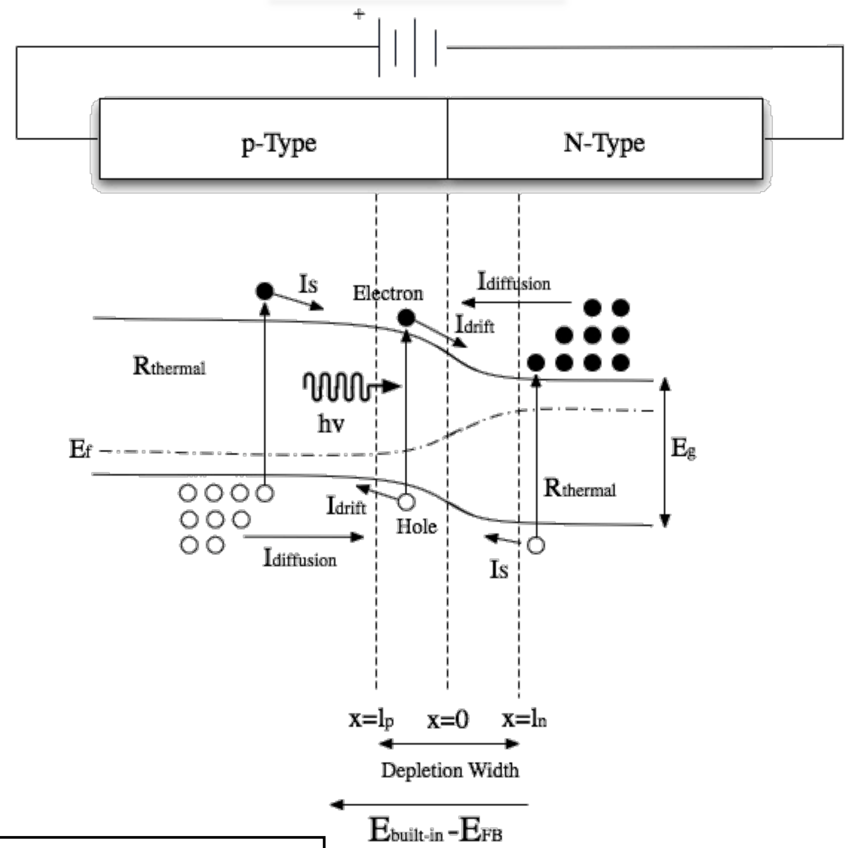


Biased p-n Junction Photodiodes

Reverse biased

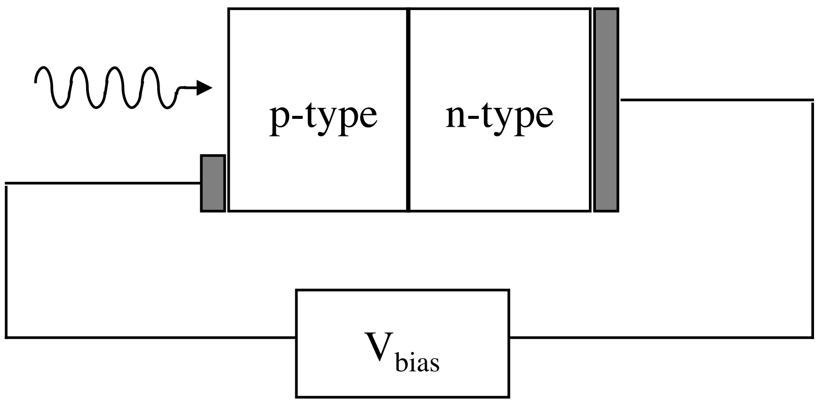


Forward biased



P-type : Semiconductor doped with acceptor atoms
N-type : Semiconductor doped with donor atoms

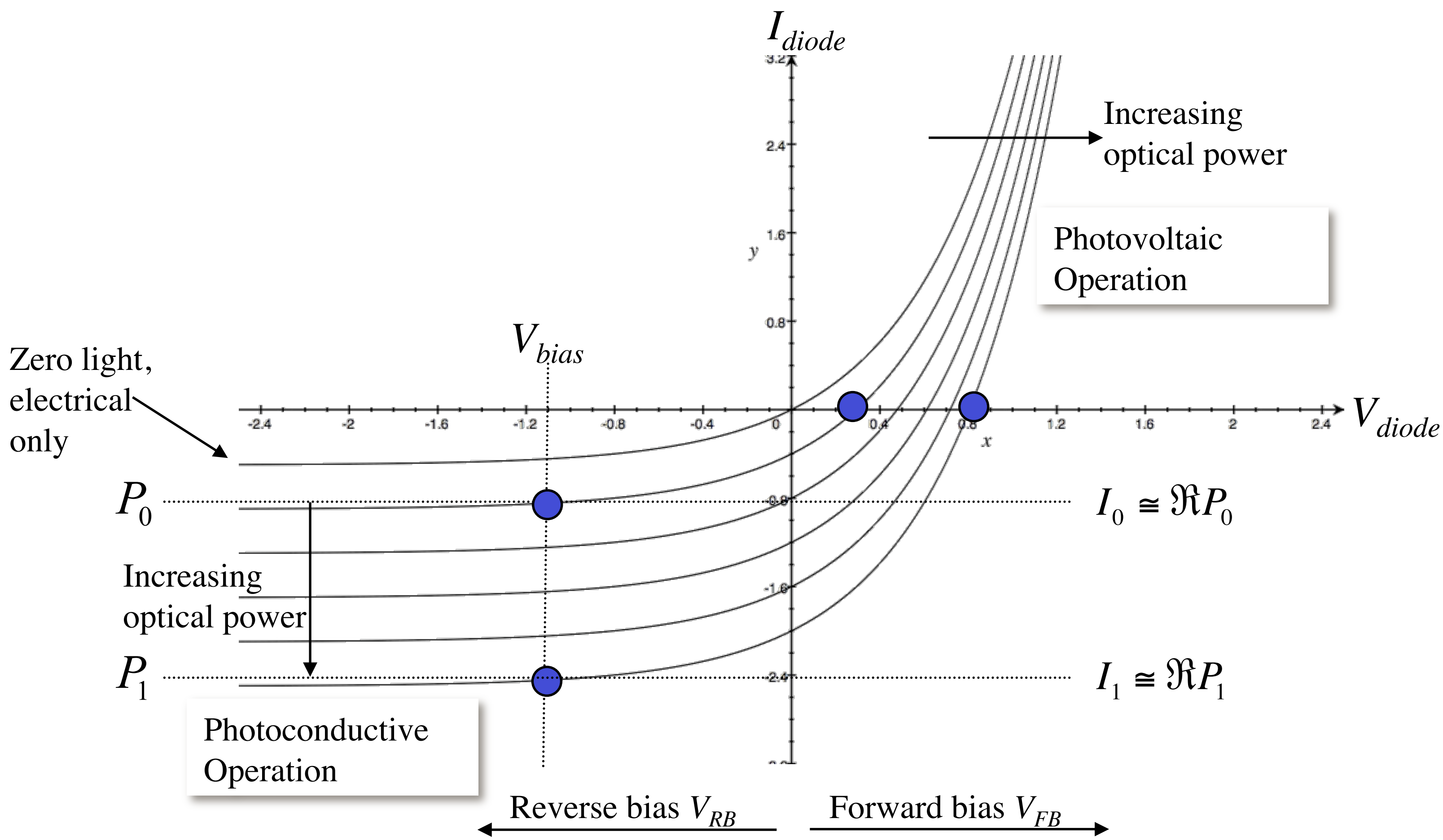
p-n Junction Photodiode Equation



$$I = (I_s)[\exp^{qV_{bias}/K_B T} - 1] - I_{photo}$$
$$= I_{dark} - I_{photo}$$

- I_{dark} = is the current that occurs with zero optical input
- $I_s = I_{th}$ is the thermal or saturation current that occurs in normal (non-illuminated) diode operating mode
- I_{photo} is photo-generated current = $\frac{\eta q}{h\nu} P_{rcvd}$
- q is the electron charge
- V_{bias} is applied bias voltage (positive = forward, negative=reverse)
- K_B is Boltzman's constant
- T is temperature (usually in Kelvin, depending on units of K_B)

p-n Junction Photodiode Regions of Operation



Discrete Train of Random Events

- For a time-dependent random variable made up of individual events occurring at discrete times $f(t\text{-}t_i)$, the events observed over a time interval T and its Fourier transform can be described by

$$i(t) = \sum_{i=1}^{N_T} f(t - t_i), \quad 0 \le t \le T$$

-

$$I_T(\omega) = \sum_{i=1}^{N_T} F_i(\omega)$$

$$F_i(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t - t_i)e^{-i\omega t}dt = \frac{e^{-i\omega t_i}}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt = e^{-i\omega t_i}F(\omega)$$

- If we take the magnitude square of the Fourier Transform

$$\left|I_T(\omega)\right|^2 = \left|F(\omega)\right|^2 \sum_{i=1}^{N_T}\sum_{j=1}^{N_T} e^{i\omega(t_i - t_j)}$$

$$= \left|F(\omega)\right|^2 \left(N_T + \sum_{i \ne j}^{N_T}\sum_{j}^{N_T} e^{i\omega(t_i - t_j)} \right)$$

Discrete Train of Random Events

- The last term averages out to zero for a large ensemble of random events

$$\left|I_T(\omega)\right|^2 = \overline{N_T}\left|F(\omega)\right|^2 = \overline{N}T\left|F(\omega)\right|^2$$

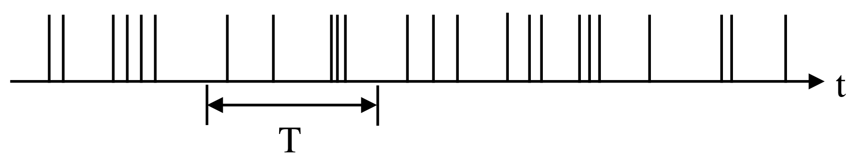
- The spectral density is defined by

$$S_i(\omega) = 4\pi\overline{N}\left|F(\omega)\right|^2$$
$$S_i(\nu) = 8\pi^2\overline{N}\left|F(2\pi\nu)\right|^2$$

- Where the result $S_i(\nu)$ is known as ***Carson's Theorem***

Photon Statistics

- Photon sources can in general be characterized as coherent or incoherent†
  - Coherent: Probability that a photon is generated at time t_0 is mutually independent of probability of photons generated at other times (Markov Process)
    - Poisson Process: Probability of finding *n* photons in time interval *T*
      - Bunching is a trait of the Poisson process
      - Interarrival time is decaying exponentially distributed



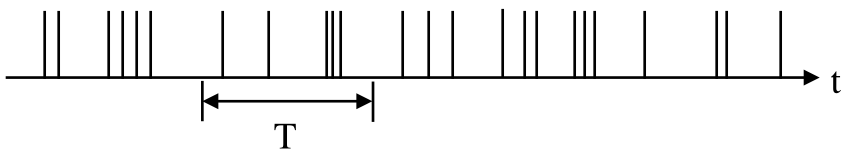
$$P(n \mid T) = \frac{(rT)^n e^{-rT}}{n!}$$

† Can also be a combination of these two types -> partially coherent

Where :
P(n|T) is probability of finding n photons in time interval T
R is mean photon arrival rate (photons/second)

Photon Statistics (II)

- Narrowband Thermal (Gaussian):
  - Bose-Einstein Process: Probability of finding *n* photons in time interval *T*



$$P(n) = \left(\frac{1}{1+n_b}\right)\left(\frac{n_b}{1+n_b}\right)^n$$

Where :

P(n) = probability of finding n photons given

n_b = mean number photons from incoherent source = $N_0/h\nu_0$

N_0 = spectral density of source = P_{opt}/B_0

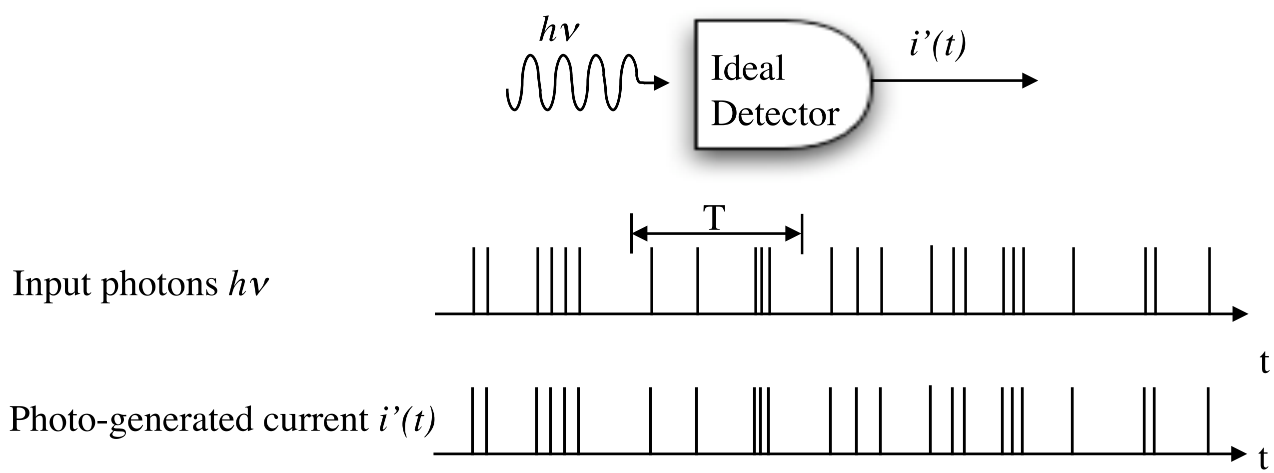
P_{opt} = total optical power from source

B_0 = source optical bandwidth

T = observation time ≤ $1/B_0$

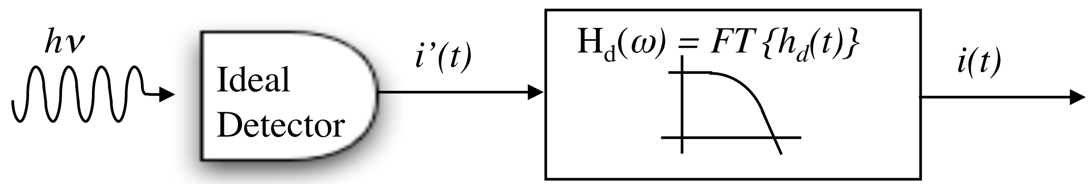
Detecting Photons (1)

- Any material that can respond to single photons can be used to count photons
- Ideal Detector
  - Generation of a electron-hole pair per absorbed photon results in an instantaneous current pulse



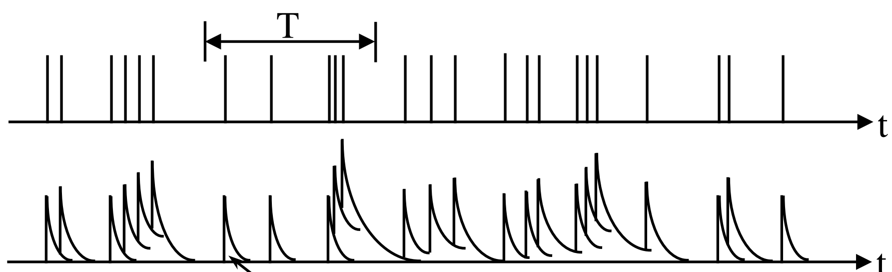
Detecting Photons (2)

- Real Detector
  - Has an inherent "impulse response," $h_d(t)$, due to built in resistance and/or capacitance.
  - Can be modeled as an RC filter with low pass response



Input photons $h\nu$

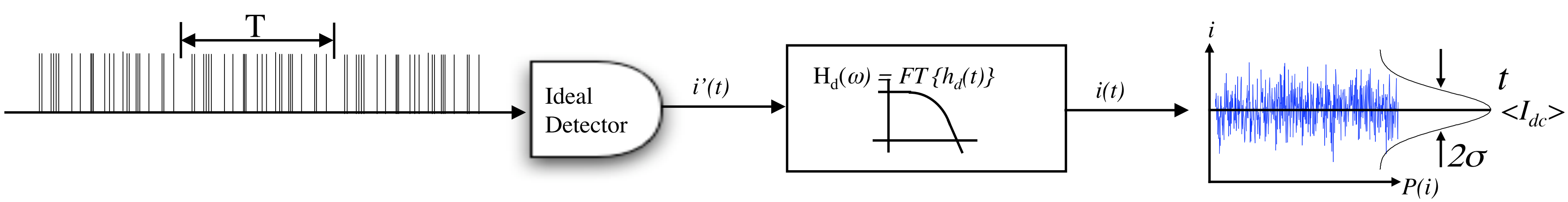
Observed photo-generated current $i(t)$



$$\text{Area} = q = \int_{-\infty}^{\infty} h_d(t)\,dt$$

Detecting Photons (3)

- As the average photon rate increases, the observed photo-current starts smoothing out, with a variance around the mean (average) count that is based on the statistics (which tends to Gaussian for large photon arrival rate)
- $P(i)$ is the probability function of measuring the current at a certain value at time t.



Detecting Photons (4)

- The detector output current $i(t)$ can be modeled as a discrete "filtered Poisson" process

$$i(t) = \sum_{j=1}^{N} h_d(t - \tau_j)$$

  - Where $h_d(t)$ is PD impulse response, N is total number e-h pairs generated, τ_j is the random time the j^{th} photocarrier is generated.

- Define: Quantum Efficiency (QE), unitless, as

$$\eta = \frac{\text{number of photocarriers produced}}{\text{number of incident photons}}, 0 \le \eta \le 1$$

- Define: Time varying photon rate parameter ($\lambda(t)$) in units of photocarriers/second as

$$\lambda(t) = \frac{\eta}{h\upsilon} P_{recvd}(t)$$

Detecting Photons (5)

- The power incident on a photodetector of area A, in units of Watts, is

$$P(t) = \int_A I(\vec{p},t)\,dA$$

  - where the instantaneous optical intensity at an observation point p is given by $\vec{p}$

$$I(\vec{p},t) = \frac{1}{Z_0}\left|E(\vec{p},t)\right|^2$$

- The time varying photon rate parameter $\lambda(t)$ can then be written in terms of $P(t)$

$$\lambda(t) = \frac{\eta}{h\nu}\frac{\left|E(t)\right|^2}{Z_0}$$

Detecting Photons (6)

- If we consider an observation interval, over which we are going to average our photon count over
  - This can be due either to the inherent bandwidth of the detector or (as we will see later) on purpose to match the receiver bandwidth to the data bit rate
- Then the number of photocarriers generated over the interval T counted at the j^{th} observation interval

$$N_j = \int_0^T \lambda_j(\tau)d\tau$$

- Assuming a coherent source, the ***conditional inhomogeneous Poisson process*** describes this photon count during the j^{th} observation interval

$$P(N_j = N) = \frac{\left(\int_0^T \lambda_j(\tau)d\tau\right)^N}{N!} e^{\left(-\int_0^T \lambda_j(\tau)d\tau\right)}$$

Detecting Photons (7)

- If we assume a constant rate parameter over the time interval T (independent of *j*), then the photo-generated current can be written as

$$i(t) = \lambda(t) q$$

$$\lambda(t) = \frac{N}{T}$$

- Then the photocurrent produced by the photodetector can be written in Amperes, assuming the observation time is normalized to one second

$$i(t) = \lambda(t) q = \frac{\eta q}{h\nu} P_{rcvd}(t)$$

$$= \Re P_{rcvd}(t)$$

  - Where we have defined the detector responsivity as

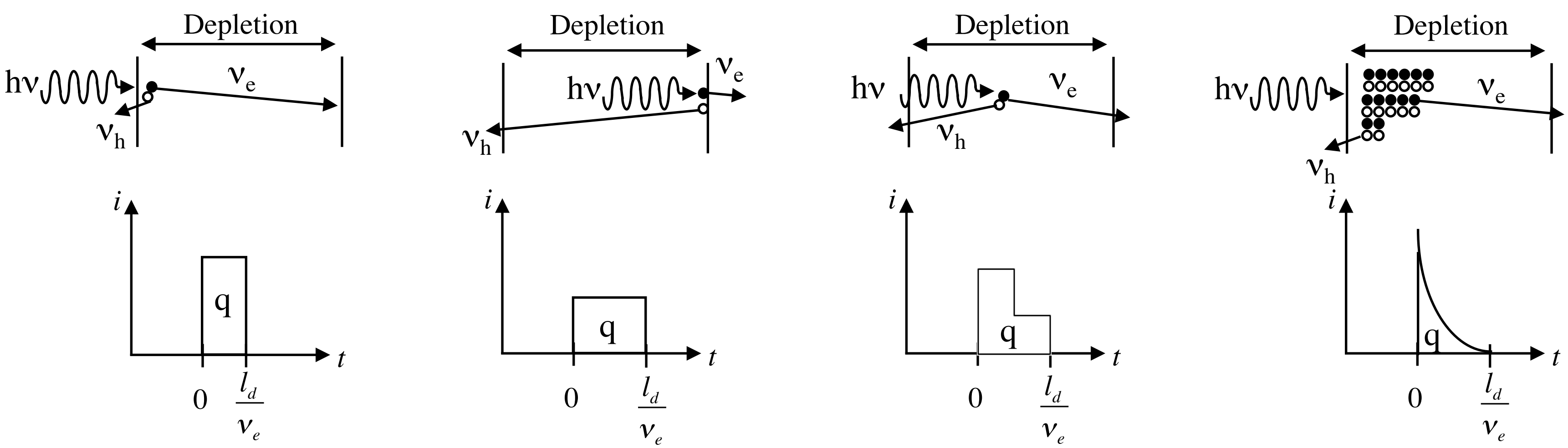
$$\Re = \frac{\eta q}{h\nu}$$

Photodiode Frequency Response

- The ability for the photocurrent to keep up with various modulation frequencies of the optical signal is an important metric in signal and data recovery
- There are three main effects that limit the detector frequency response
  - Finite diffusion time of photocarriers produced in the p and n doped regions of a pn-junction photodiode.
  - RC time constant associate with the pn-junction capacitance
  - Finite transit time of photocarriers drifting across the depletion layer

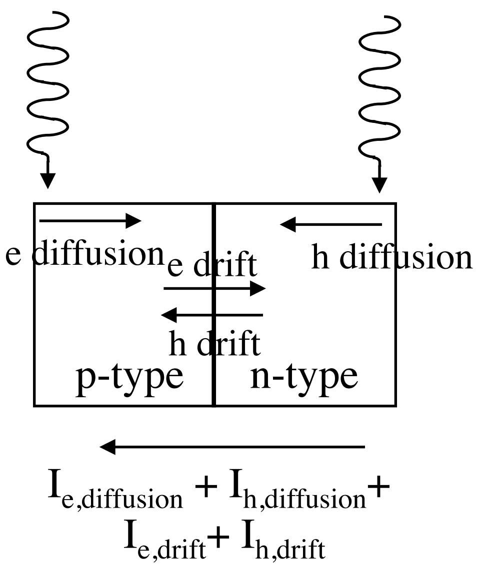
pn-Junction Carrier Dynamics

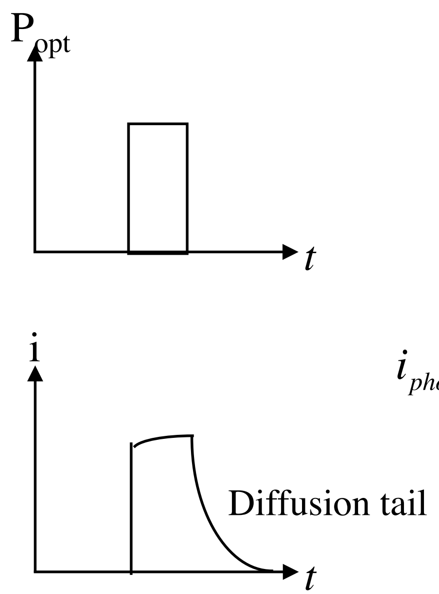
- Carrier diffusion time (~ns/μm) is typically much longer than carrier transit time (~10ps/μm)
- Electron and hole velocities saturated in depletion region due to high field strength
- Once away from depletion region carrier velocities fall below saturation
- Space charge barrier prevents carriers from entering the depletion region, therefore the multiple carrier effect seen in photoconductors does not occur when carrier velocities are mismatched

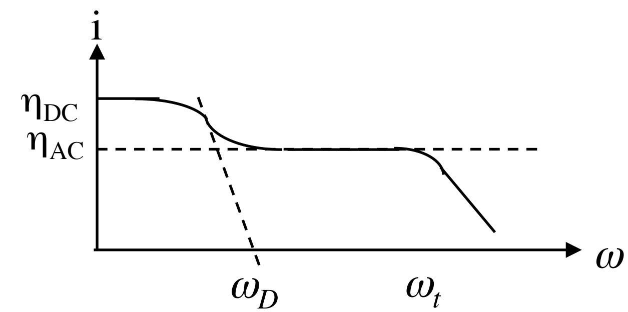


pn-Junction Carrier Dynamics

- Photons absorbed within one diffusion length outside the depletion region will be absorbed and the current contributing carriers will suffer both diffusion time and transit time delays
- Effect is geometry and material dependent



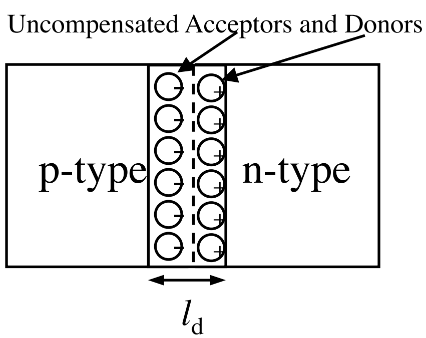




$$i_{photo}(\omega) = \frac{\eta_{DC}-\eta_{AC}}{h\nu}\frac{P_{rcvd}(\omega)}{\sqrt{1+\left(\frac{\omega}{\omega_D}\right)^2}} + \frac{q\eta_{AC}}{h\nu}P_{rcvd}(\omega)$$

pn-Junction Carrier Dynamics

- The separation of charge in the depletion region (due to uncompensated Donors and Acceptors) leads to a capacitive effect that also impacts the detector bandwidth



$$C_j = \frac{\varepsilon_0 \varepsilon_r A}{l_d}$$

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m = vacuum permitivitty
ε_r = semiconductor relative permitivitty
A = area of depletion region
l_d = depletion region length

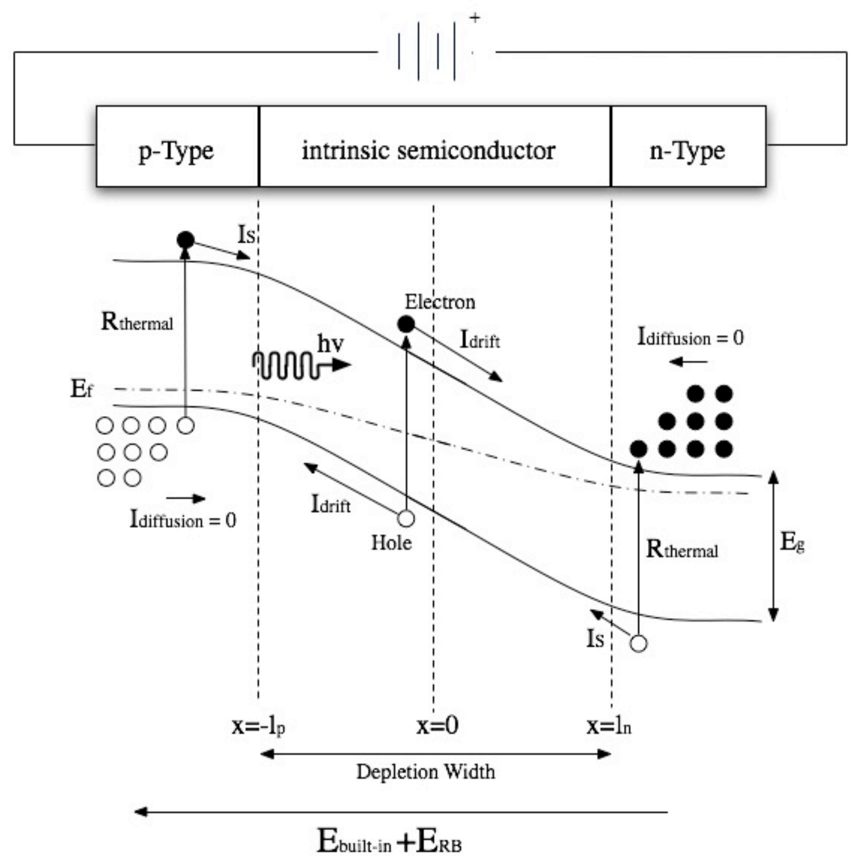
- The frequency at which the detector bandwidth rolls off by 3-dB due to the junction capacitance is

$$\omega_{RC} \frac{1}{R_S C_j}$$

Cj = area of depletion region
l_d = depletion region length

p-i-n Photodiodes

- To increase the photon absorption region, a layer of ***intrinsic*** semiconductor material can be added beteween the p and n material.
- The pin photodetector gain-bandwidth product improves of the pn-junction
  - The detector quantum efficiency can be increased over that of a simple pn junction since the depletion region is almost entirely contained in the intrinsic region and the intrinsic region can be made long.
  - Carrier diffusion effects minimized since all light absorbed in intrinsic region
  - The junction capacitance is reduced compared to a pn-junction because the distance between the effective plates is increased.
  - Carriers reach saturation velocity while traveling in intrinsic region, so even though pin depletion length $l_p + l_d$ is longer than pn-junction depletion length lower transit time than pn-junction where carrier velocity drops below saturation not far from metallurgical junction



p-i-n Photodiodes

- As with the pn-junction, the quantum efficiency is defined by the following equation, however the distance can now be integrated over the larger intrinsic region

$$P_{abs}(x) = P_i(1-R)(1-e^{-\alpha(\lambda)x}) \\ = \eta(\lambda,x)P_i$$

- As the depletion region length is increased, η increases, the junction capacitance C_j decreases, and the transit time τ_{trans} increases. The detector design must be optimized to maximize both efficiency and bandwidth. An estimate of the bandwidth is given by

$$B_{pin} = \frac{1}{\sqrt{\left(\frac{1}{f_{RC}}\right)^2 + \left(\frac{1}{f_{trans}}\right)^2}} = \frac{1}{\sqrt{\left(2\pi R_S \varepsilon_0 \varepsilon_r \frac{A}{l_d}\right)^2 + \left(\frac{1}{0.44 v_s}\right)^2}}$$

Bandwidth-Efficiency Tradeoffs in p-i-n Photodiodes

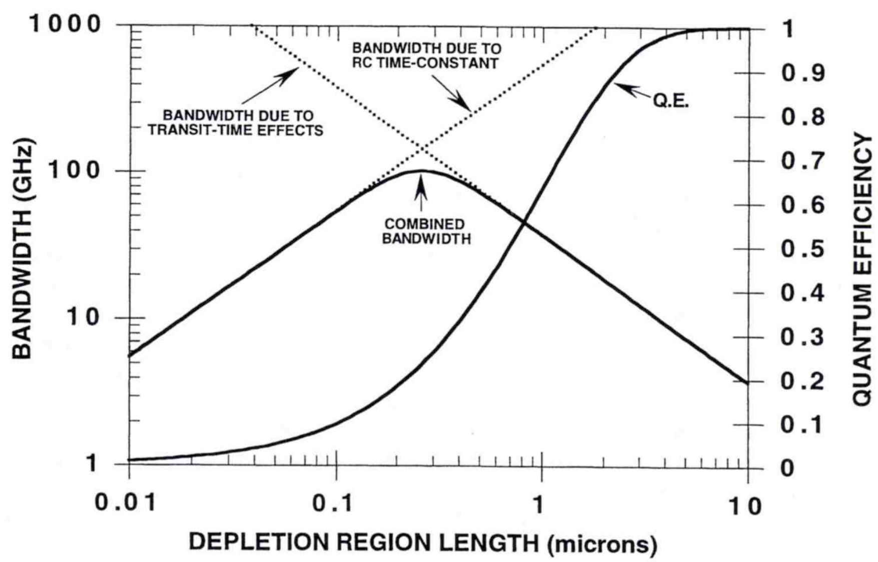
- The quantum efficiency, η, can be approximated assuming R=0 (high quality anti-reflection coating) and intrinsic region length l_d.
- For small l, bandwidth is transit time limited
- For large l, bandwidth it RC limited
- Optimal bandwidth length where two effects are equal
- QE keeps increasing with increased length

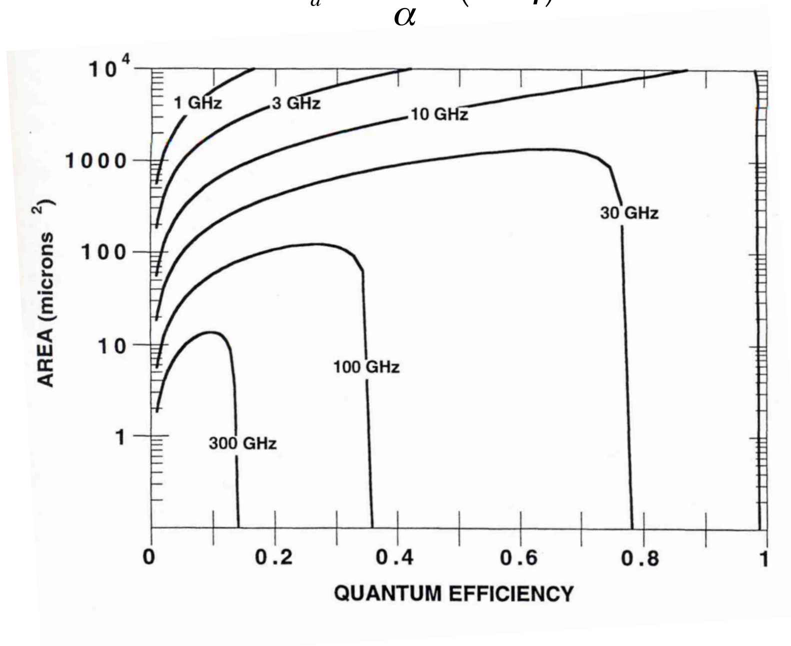
$$\eta = 1 - e^{-\alpha l_d}$$

- If the detector area A and length l_d are both optimized, then bandwidth and quantum efficiency can both be maximized

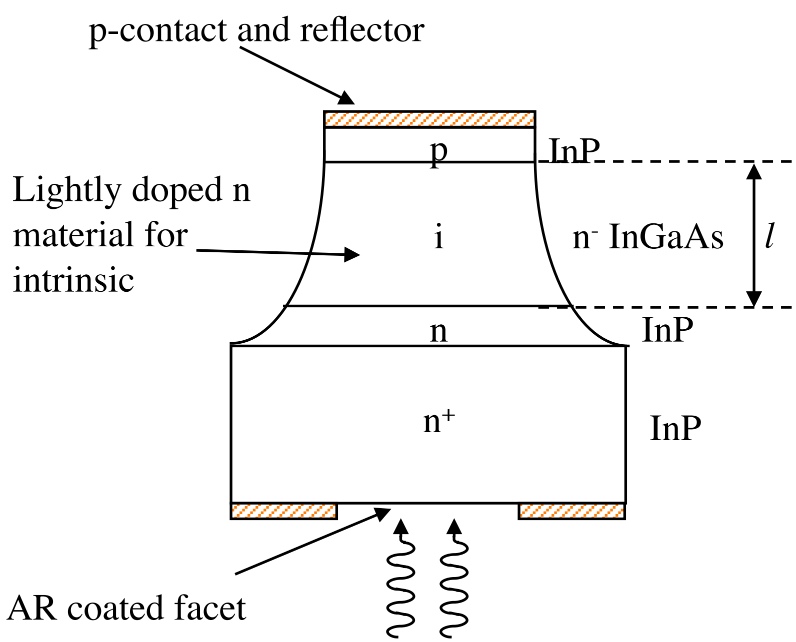
$$A = \frac{l_d}{2\pi R_l \varepsilon_0 \varepsilon_r}\sqrt{\frac{1}{B^2} - \left(\frac{l_d}{0.44 v_s}\right)^2}$$

$$l_d = -\frac{1}{\alpha}\ln(1-\eta)$$





Vertically Illuminated p-i-n Photodiodes



- For a double pass vertically illuminated pin detector (see left figure), the quantum efficiency is

$$\eta = \left(1 + re^{-\alpha l}\right)\left(1 - e^{-\alpha l}\right)$$

- When the carrier transit distance is approximately equal to l, and $\alpha l \ll 1$, the bandwidth-efficiency for a double-pass vertically illuminated pin photodiode is approximately

Optical Detection and Noise

- We turn our attention to converting optical signals to electrical signals
- We want to understand the physical mechansisms that govern "photodetection" and how well the signal is recovered
- One of the primary ways a signal gets degraded is when noise gets added to the information modulation on the optical signal, so we need to understand the different noise sources and how they interact with signal detection
- Noise impacts an optical system in the following ways
  - Measurement: Noise causes random fluctuations in the detected signal and limits the smallest power that can be detected accurately. This noise is generated in the detection devices and systems.
  - Generation: Noise is present in optical sources which will interfere with the accurate measurement of optical signals using a photodetector.
  - Amplification and Transmission: There is noise in the process of trying to increase the level of a signal using optical or electrical amplifiers. The medium used to transmit signals (e.g. optical fiber) can also manifest certain types of noise.

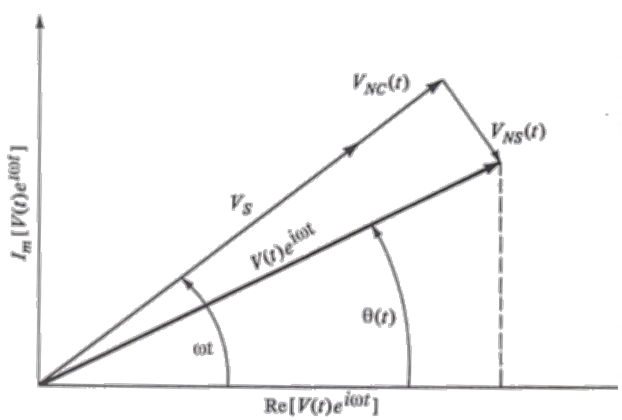
Limitations due to Noise Power

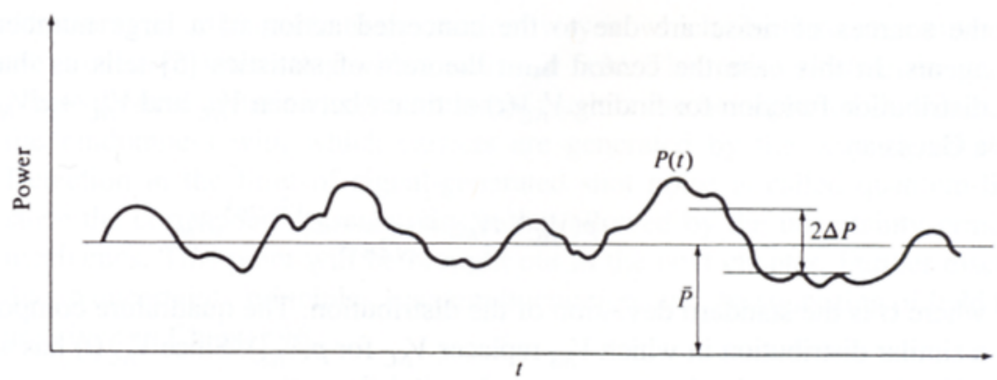
• The signal and additive noise can be represented in either the time domain or using phasor notation as shown below.

• Let V_s be the magnitude of a sinusoidally modulated signal $v_s(t)$ oscillating at ω: $v_s(t)= V_s\cos(\omega t)$.

• Let the noise be additive to the signal in both amplitude and phase and represented by an in-phase component $V_{NC}(t)$ and a quadrature component $V_{NS}(t)$. Also assume These noise components vary slowly w.r.t. cos(ωt).

• The total signal in phasor notation can be written as $v(t) = \mathrm{Re}\left\{\left[V_S + V_{NC}(t) - iV_{NS}(t)\right]e^{i\omega t}\right\}$





Effect of Noise Sources

• Since noise is due to many different random events that act independently, we can invoke the central limit theorem to describe the probability distribution for the amplitude and phase of both the real (in-phase) and imaginary (quadrature) noise components.

• This additive randomness in amplitude and phase creates a sort of "bullseye" pattern where the vector *v(t)* has a probability landing as its "average" is spinning around the imaginary plane at frequency ω.

• For a large number of random independent events, we can utilize the central limit theorem and describe the noise using Gaussian statistics with variance σ_{NS} and σ_{NC} and averages $\langle V_{NS} \rangle$ and $\langle V_{NC} \rangle$

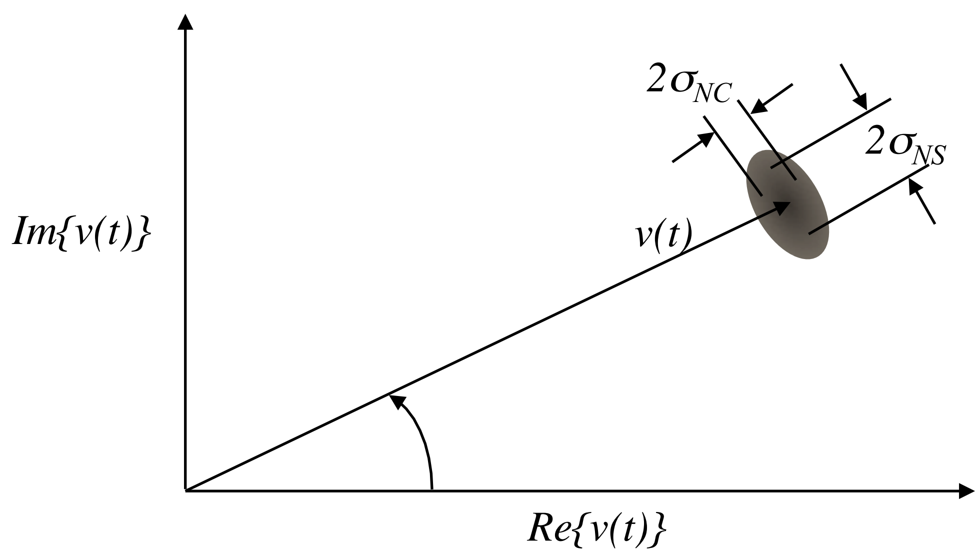
$$\rho(V_{NC}) = \frac{1}{\sqrt{2\pi}\sigma_{NC}} e^{-V_{NC}^2 / 2\sigma_{NC}^2}$$

$$\rho(V_{NS}) = \frac{1}{\sqrt{2\pi}\sigma_{NS}} e^{-V_{NS}^2 / 2\sigma_{NS}^2}$$

$$\sigma_{NC}^2 = \overline{V_{NC}^2} = \int_{-\infty}^{\infty} V_{NC}^2 \rho(V_{NC}) dV_{NC}$$

$$\sigma_{NS}^2 = \overline{V_{NS}^2} = \int_{-\infty}^{\infty} V_{NS}^2 \rho(V_{NS}) dV_{NS}$$

$$\langle V_{NC}(t) \rangle = \langle V_{NS}(t) \rangle = 0$$



Noise Power

• The average power associated with the signal and noise is given by the following, which can be reduced by assuming the variance of the in-phase and quadrature components are equal ($\sigma_{NC} = \sigma_{NS}$)

$$
\begin{aligned}
\bar{P} &= \langle P(t) \rangle \\
&= \left\langle \frac{1}{2}\left(V(t)e^{i\omega t}\right)\left(V^*(t)e^{-i\omega t}\right)\right\rangle \\
&= \left\langle \frac{1}{2}\left(V_S^2 + 2V_S V_{NC} + V_{NC}^2 + V_{NS}^2\right)\right\rangle \\
&= \frac{1}{2}\left(\overline{V_S^2} + \overline{V_{NC}^2} + \overline{V_{NS}^2}\right) \\
&= \frac{1}{2}\left(\overline{V_S^2} + \sigma_{NC}^2 + \sigma_{NS}^2\right) \\
&= \frac{1}{2}\left(\overline{V_S^2} + 2\sigma^2\right)
\end{aligned}
$$

Noise Power

- When measuring the power, there is an uncertainty due to the noise components, which can be described by a deviation in the average power $\Delta P(t)$

$$P(t) = \overline{P} + \Delta P(t)$$

- Using the rms values

$$\Delta P = \left[\overline{\left(P(t) - \overline{P} \right)^2} \right]^{1/2}$$

-

- And defining the signal power P_S as the power measured in the absence of noise and converting to Gaussian variance

$$\Delta P = \sigma \left(2P_S + \sigma^2 \right)^{1/2}$$