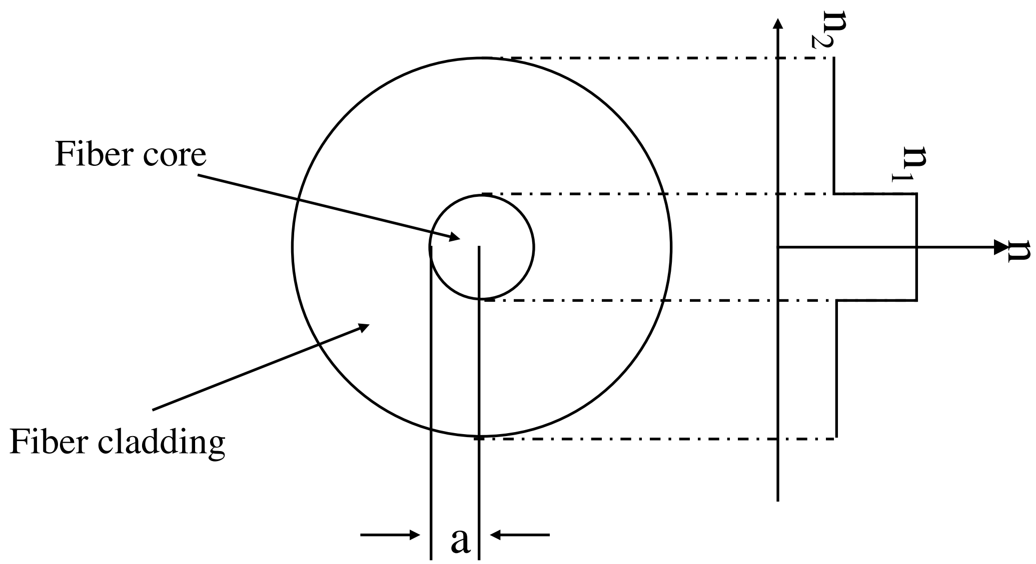
Lecture 4 - Propagation in Optical Fibers

Step Index Fibers



Definition: Fractional refractive index difference $\Delta = (n_1 - n_2)/n_1$

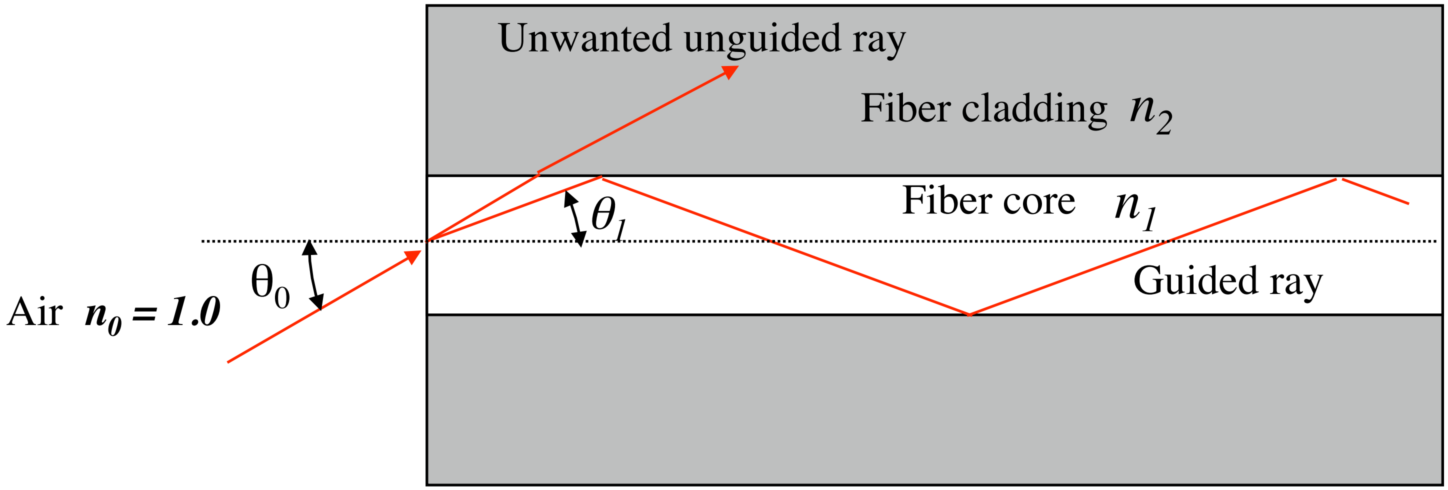
Typical value for silica (glass) fibers

$n_1 = 1.48, n_2 = 1.46$

$\Delta = .0135 \approx 1\%$

Geometrical Optics Model

- Use of total internal refraction for optical field guiding



Light rays that enter the fiber with an angle smaller than an "acceptance angle" θ_0 will be **guided** by **total internal reflection** within the fiber when:

$$\theta_0 \leq \theta_{0,\max} = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right)$$

Numerical Aperture

<u>Definition:</u> The light collecting capacity of the optical fiber is measured by the **Numerical Aperature (NA)**

$$NA = n_0 \sin\theta_{0,\max}$$
$$= \sqrt{n_1^{\ 2} - n_2^{\ 2}}$$
$$\approx n_1\sqrt{2\Delta}$$

(For small Δ)

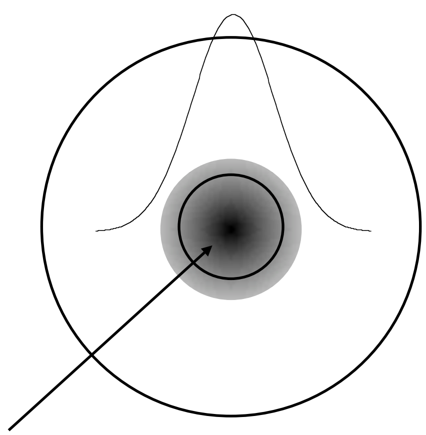
Example: if we couple light from air into a fiber with $\Delta = .01$ and $n_1 = 1.5$, then the NA $\approx$ 0.2121 and $\theta_{0,max} \approx 12^\circ$

The maximum acceptable "angular error" when launching an optical beam into a fiber is consequently of the order of $\theta_{0,max} \approx 12^\circ$

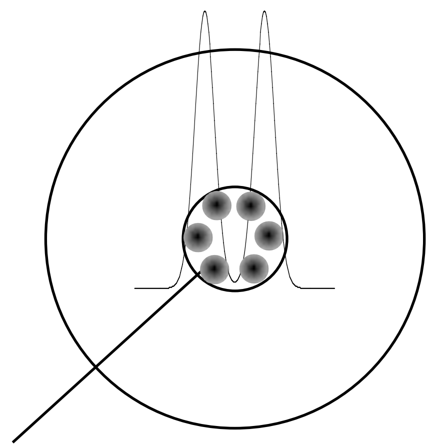
Modes in Step Index Fibers

Definition: Modes are light intensity profiles (patterns) that propagate down the fiber maintaining their transversal field shape

- Multimode fibers can support many thousands of modes.
- Single mode fibers support one mode.



Gaussian first order mode intensity profile



Gaussian secon order mode intensity profile

$$E(x,y,z,t) = J(x,y)Cos(\omega_0 t - \beta(\omega_0)z)$$

In order to accurately study optical modes, the complete Maxwell equations are to be solved.

Anyway, for multimode fibers, the following intuitive explanation can be given:

Each mode corresponds to a light beam traveling inside the fiber core with different angles

Normalized Frequency Parameter *V*

V is a design parameter that takes into account the fiber parameters (n_1, n_2 and a) and the free space wavelength λ_0.

$$V = \kappa_0 a \left(n_1^2 - n_2^2\right)^{1/2}$$

$$= \left(\frac{2\pi}{\lambda}\right) a n_1 \sqrt{2\Delta}$$

It can be shown that:

In order to have a Single Mode Fibers: $V \le 2.405$

In order to have a Multimode Fibers: $V > 2.405$

Important consequence:

Given the parameters n_1, n_2 and a fixed wavelength, a fiber is single mode if the core radius a is smaller than a given value (of the order of 10 µm at 1550 nm)

Multimode Fibers

For large V, the number of modes propagating in a multimode fiber is approximately

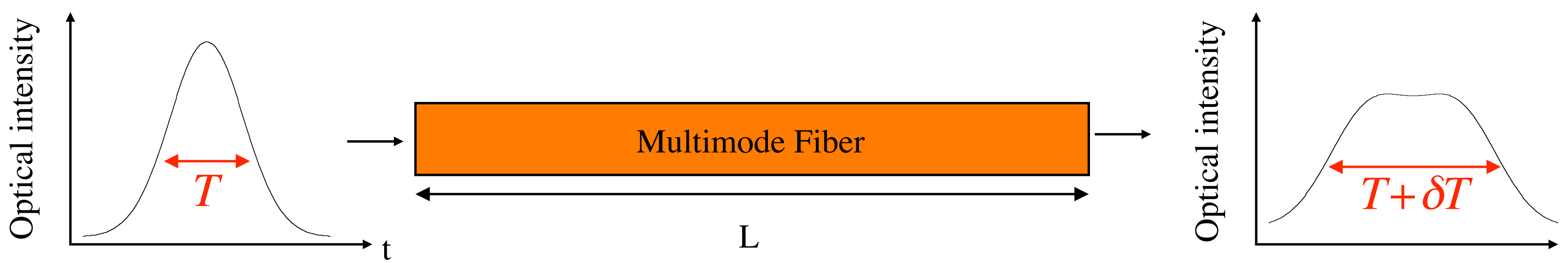
$$N = \frac{V^2}{2}$$

Example: A multimode fiber with core diameter $2a=50\mu m$, $\Delta=5x10^{-3}$ and $\lambda=1.3\mu m$ supports about *160* modes

- Each mode will propagate in the fiber at as if it had its own index of refraction n.
  - The index of refraction for each mode n lies between n_1 and n_2 (from the solution of the Maxwell equations)
  - Intuitive explanation: each mode has different portions of the field overlap with different amounts of the core and cladding
- Consequence: each mode will travel along the fiber at slightly different speeds, giving rise to multimode fiber dispersion

Multimode Fiber Dispersion

Since each mode travels at a different velocity on the fiber, an optical bit launched into the fiber will distort as it propagates.

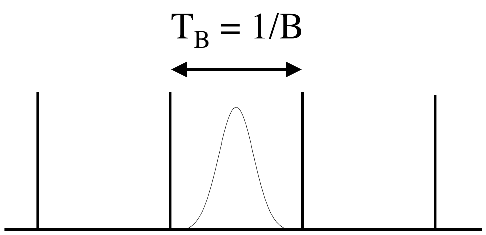


- The resulting distortion is actually a pulse broadening
- The amount of pulse broadening in a multimode fiber is given by:

$$\delta T = \frac{L n_1^2}{c n_2} \Delta$$

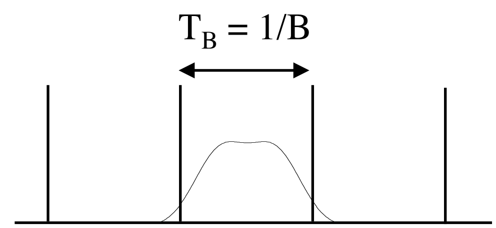
Bit Rate Limit for Multimode Fibers

- When dealing with digital transmission, each pulse represent a bit
- A pulse spreading leads to intersymbol interference (ISI)
- Let's assume a bit cannot spread by more than half the allocated bit period in order to have an acceptable ISI level



$$\Delta T < \frac{1}{2B}$$

$$\frac{L}{c}\frac{n_1^2}{n_2}\Delta < \frac{1}{2B}$$

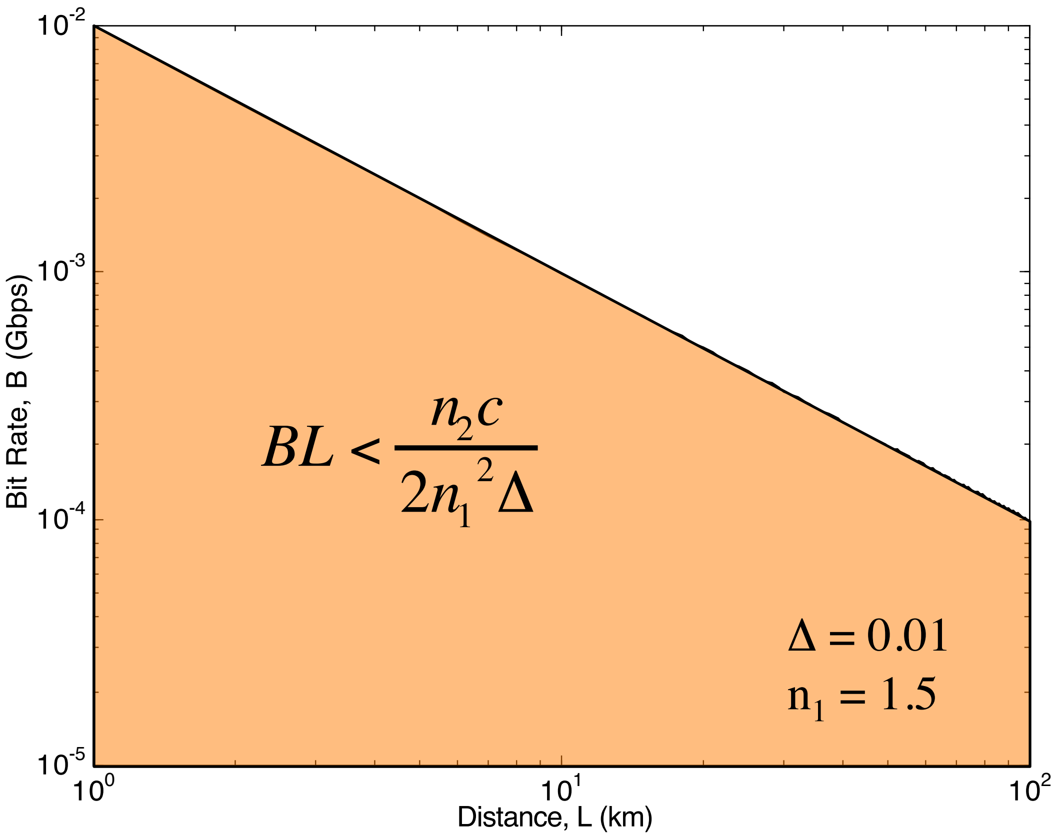


We can define the **Bandwidth-Distance** product (BL) for multimode fibers as:

$$BL < \frac{n_2 c}{2 n_1^2 \Delta}$$

Bit Rate Limit for Multimode Fibers

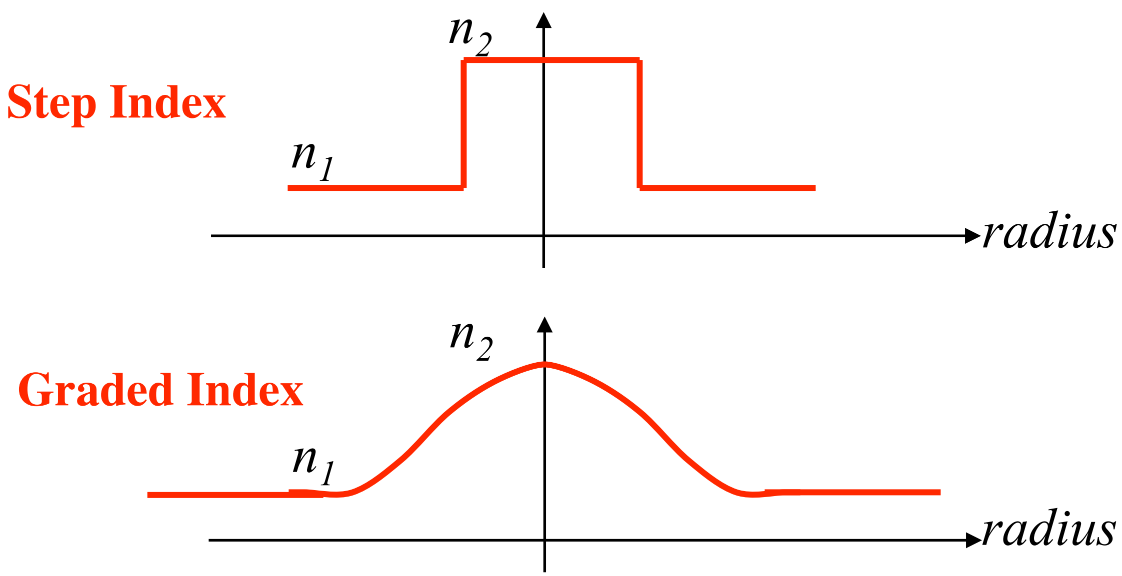
- The previous formula give rise to the ultimate bit-rate limitation of a standard multimode fiber



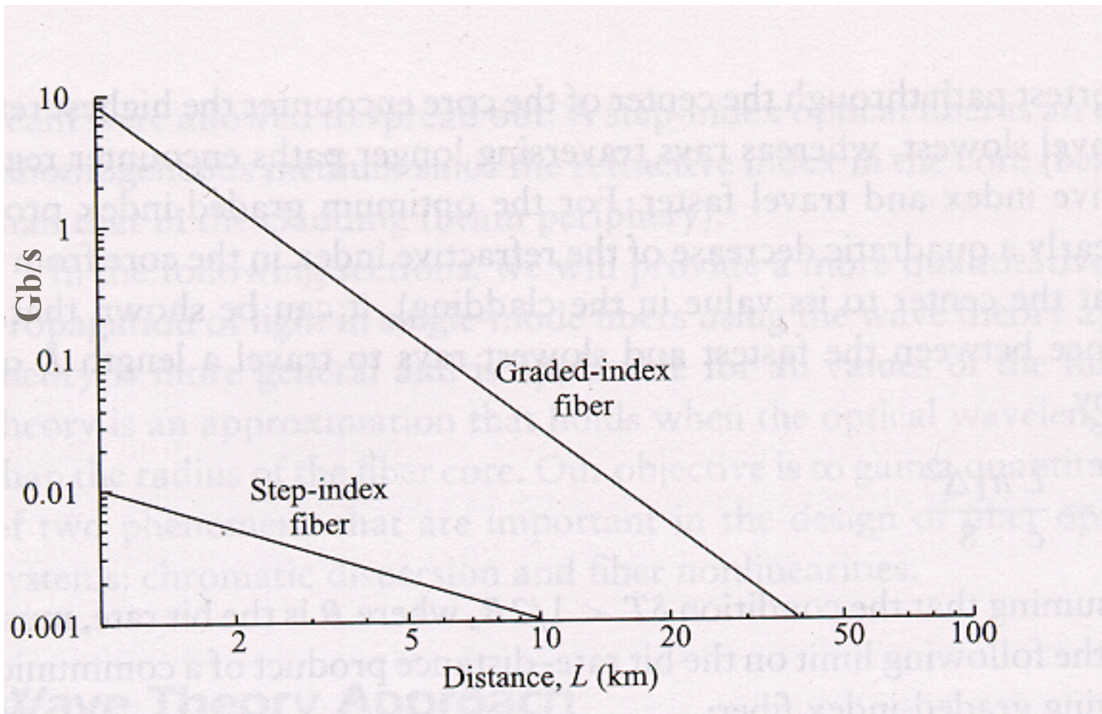
On a standard 1-km long step-index multimode fiber, the resulting maximum bit rate is 1 Mbit/s only !!

Graded index fibers

- The multimode dispersion limit can be drastically changed by using a proper index of refraction profile



Dispersion limit for graded index fibers



- The limit for graded index fibers is of the order of (for example) 1 Gbit/s at 2 Km
- With particular techniques (misplaced launch) this limit can be somehow increased
- Anyway, high bit rates and long haul link are NOT feasible on multimode fibers, even using graded index profiles

Multimode vs. single mode fiber

- **Multimode fiber**
  - They have a limit in terms of maximum bit rate of the order of 1Gbit/1Km, due to multimode dispersion
  - They have a relatively large core
    - Splicing is easier
    - Connectors are less expensive
  - Installation is simpler
  - They are intrinsically more resilient to mechanical and environmental stress
- <u>They are thus mostly used in LAN application</u>

- **Single mode fibers**
  - We will see that they are not affected by multimode dispersion, and their bandwidth limit is extremely higher
  - They have a small core
    - Splicing is more difficult
    - Connectors are more expensive
  - Installation is more difficult
- They are thus used in all applications where the distance to be covered is significantly higher than 1Km
- <u>In the rest of the course, we will mostly focus on single mode fibers</u>

Step Index Circular Waveguide (lossless, isotropic)

- Simplest type of fiber
- (Most fiber these days is far more complex)
- Cylindrical symmetry

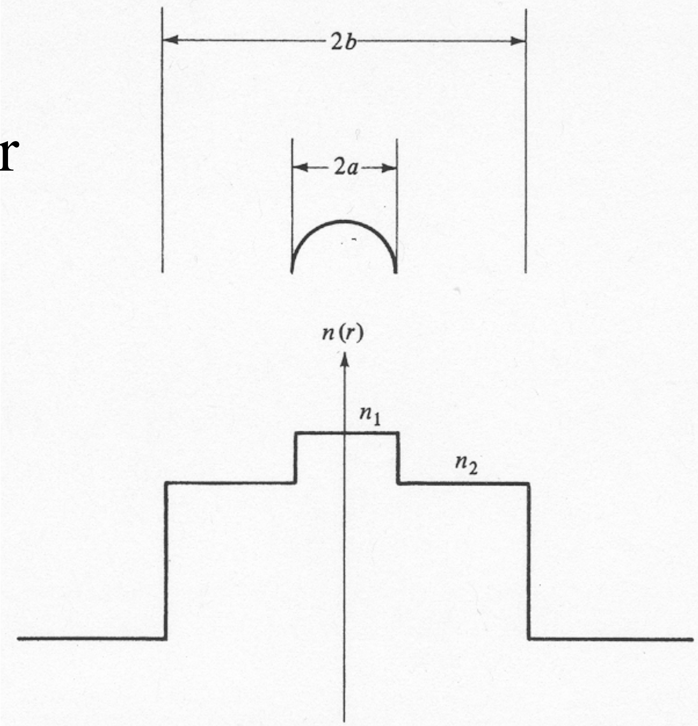


Figure 3-1 Structure and index profile of a step-index circular waveguide.

Step Index Circular Waveguide (lossless, isotropic)

- Simplest type of fiber
- (Most fiber these days is far more complex)
- Cylindrical symmetry
- Express Laplacian operator in cylindrical coordinates

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}$$

Separate variables

$$E_r = \psi(r)\Phi(\phi)e^{i(\omega t - \beta z)}$$

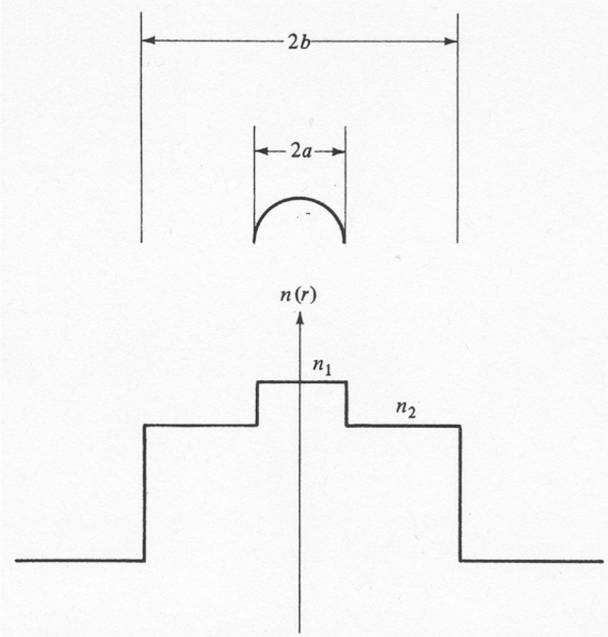


Figure 3-1 Structure and index profile of a step-index circular waveguide.

Separable Solutions

$$[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + (k^2 - \beta^2)]E_z = 0$$

$$E_r = \psi(r)\Phi(\phi)e^{i(\omega t - \beta z)}$$

$$\Phi(\phi) = e^{\pm i l \phi} \quad \textit{where} \quad l = 0,1,2\ldots$$

Separable Solutions

$$[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}+(k^2-\beta^2)]E_z=0$$

$$E_r=\psi(r)\Phi(\phi)e^{i(\omega t-\beta z)}$$

$$\Phi(\phi)=e^{\pm il\phi} \quad where \quad l=0,1,2...$$

$$[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+(k^2-\beta^2-\frac{l^2}{r^2})]\psi=0$$

Bessel differential equation

$$\psi=c_1J_l(hr)+c_2Y_l(hr) \quad k^2-\beta^2=h^2>0$$

$$\psi=c_1I_l(qr)+c_2K_l(qr) \quad k^2-\beta^2=-q^2>0$$

J Bessel function of the first kind
Y Bessel function of the second kind
I Modified Bessel function of the first kind
K Modified Bessel function of the second kind

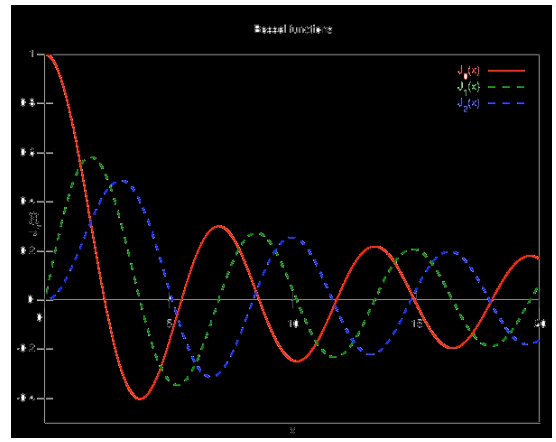
Bessel Functions

- For equations of the form

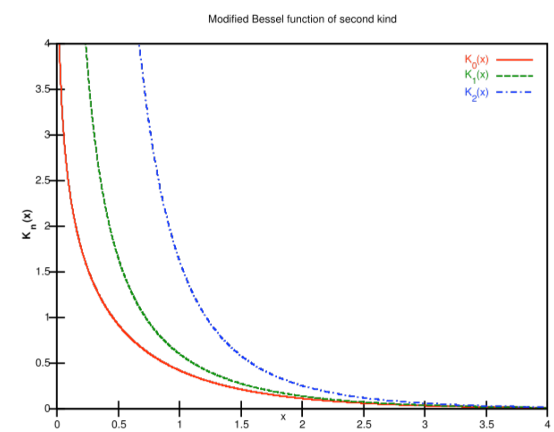
$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + \left(x^2 - \alpha^2\right) y = 0$$

- and non-negative integer α, the solution that represents a propagating mode confined within the core is the Bessel function of the first kind $J_a(x)$ and is finite at x = 0.
- and negative integer α, the modified Bessel function of the second kind is a decaying exponential that represents the evanescent field of the propagating mode in the cladding.

Bessel Function of the first kind



Modified Bessel Function of the second kind



Boundary Conditions

Decaying fields for r>a
q>0

$$q^2 = \beta^2 - k^2 = \beta^2 - n_2{}^2 k_0{}^2$$

$$k_0 = \omega / c$$

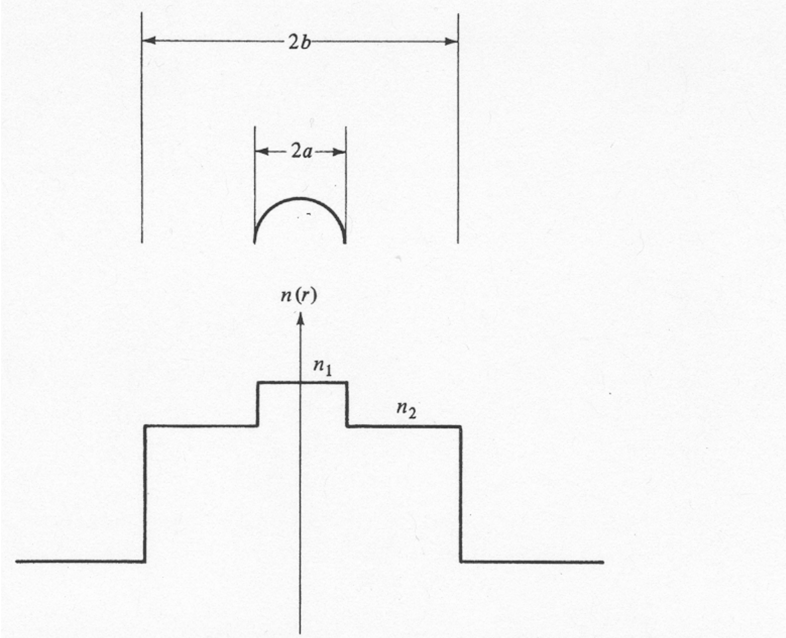


Figure 3-1 Structure and index profile of a step-index circular waveguide.

For fields in the core r<a, we need finite fields
(which eliminates Y and K which go to infinity as r approaches 0.

Boundary Conditions

- In order for the mode to be supported, it must be a standing wave pattern along r inside the core and a decaying exponential along r inside the cladding, with the boundary conditions supported at the step interface.
- β is therefore bounded by

$$n_1 k_0 \leq \beta \leq n_2 k_0$$