ECE 228A
HW 2

2.1

From Eq. (2.1.5), $\Delta T/L = (n_1^2/n_2 c)\Delta = 10$ ns/km, or

$$\Delta = \frac{n_2 c}{n_1^2} \times (10^{-11} \text{ s/m}).$$

Using $n_1 \approx n_2 = 1.45$ and $c = 3 \times 10^8$ m/s, we find $\Delta = 2.07 \times 10^{-3}$.

From Eq. (2.1.4), numerical aperture is found to be NA = $n_1(2\Delta)^{1/2} = 0.093$.

From Eq. (2.1.6), $BL < n_2 c/(n_1^2 \Delta)$. The maximum value of bit rate is limited to B = 10 Mb/s.

2.3

Writing Eq. (2.2.1) in the frequency domain and using $\mathbf{B} = \mu_0 \mathbf{H}$, we obtain

$$\nabla \times \mathbf{E} = i\omega\mu_0 \mathbf{H}.$$

The next step is to write this equation in the cylindrical coordinates using

$$\mathbf{H} = \hat{\rho} H_\rho + \hat{\phi} H_\phi + \hat{z} H_z$$

$$\nabla \times \mathbf{E} = \hat{\rho}\left(\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z}\right) - \hat{\phi}\left(\frac{\partial E_z}{\partial \rho} - \frac{\partial E_\rho}{\partial z}\right) + \hat{z}\left(\frac{E_\phi}{\rho} + \frac{\partial E_\phi}{\partial \rho} - \frac{1}{\rho}\frac{\partial E_\rho}{\partial \phi}\right).$$

Equating the three components on the two sides, we obtain three relations:

$$\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} = i\omega\mu_0 H_\rho,$$

$$-\frac{\partial E_z}{\partial \rho} + \frac{\partial E_\rho}{\partial z} = i\omega\mu_0 H_\phi,$$

$$\frac{E_\phi}{\rho} + \frac{\partial E_\phi}{\partial \rho} - \frac{1}{\rho}\frac{\partial E_\rho}{\partial \phi} = i\omega\mu_0 H_z.$$

Now follow the same procedure for Eq. (2.2.2) by writing it first in the frequency domain as $\nabla \times \mathbf{H} = -i\omega\varepsilon\mathbf{E}$. Using again the cylindrical coordinates, we obtain the following three additional relations:

$$\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} = -i\omega\varepsilon E_\rho,$$

$$-\frac{\partial H_z}{\partial \rho} + \frac{\partial H_\rho}{\partial z} = -i\omega\varepsilon E_\phi,$$

$$\frac{H_\phi}{\rho} + \frac{\partial H_\phi}{\partial \rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial \phi} = -i\omega\varepsilon E_z.$$

All derivatives with respect to z can be eliminated because all field components have the z dependence of the form $\exp(i\beta z)$. Thus, $\frac{\partial E_\rho}{\partial z} = i\beta E_\rho$, and so on.

We can express E_ρ, E_ϕ, H_ρ, and H_ϕ in terms of E_z and H_z using these six equations. For example, using H_ρ from the first equation in the fifth equation for E_ϕ, we obtain

$$E_\phi = \frac{i}{\omega\varepsilon}\left[-\frac{\partial H_z}{\partial\rho} + \frac{i\beta}{i\omega\mu_0}\left(\frac{1}{\rho}\frac{\partial E_z}{\partial\phi} - i\beta E_\phi\right)\right].$$

Combining E_ϕ terms, this equation becomes

$$\left(1 - \frac{\beta_2}{\omega^2\mu_0\varepsilon}\right)E_\phi = \frac{i}{\omega\varepsilon}\left(-\frac{\partial H_z}{\partial\rho} + \frac{\beta}{\omega\mu_0}\frac{1}{\rho}\frac{\partial E_z}{\partial\phi}\right).$$

Using $\varepsilon = \varepsilon_0 n^2$ with $\mu_0\varepsilon_0 = 1/c^2$ and $k_0 = \omega_0/c$,

$$1 - \frac{\beta^2}{\omega^2\mu_0\varepsilon} = 1 - \frac{\beta^2}{k_0^2 n^2} = \frac{p^2}{k_0^2 n^2} = \frac{p^2}{\omega^2\mu_0\varepsilon}.$$

Hence, we find that

$$E_\phi = \frac{i}{p^2}\left(\frac{\beta}{\rho}\frac{\partial E_z}{\partial\phi} - \mu_0\omega\frac{\partial H_z}{\partial\rho}\right).$$

The other three relations are obtained using a similar procedure.

2.5

The cut-off condition $V = 2k_0(n_1^2 - n_2^2)^{1/2} = 2.405$ with $k_0 = 2\pi/\lambda$ provides

$$a = \frac{V\lambda}{2\pi(n_1^2 - n_2^2)^{1/2}} = 3.18\ \mu\text{m},$$

where we used $(n_1^2 - n_2^2) \approx 2n_1(n_1 - n_2) = 0.0145$ with $\lambda = 1\ \mu$m.

At $\lambda = 1.3\ \mu$m, V becomes 1.85 because it is reduced by a factor of 1.3. From Eq. (2.2.45), we find $w = 4.368\ \mu$m for this value of V.

Fraction of the mode power inside the core is calculated using Eq. (2.2.46) and is found to be $\Gamma = 1 - \exp(-1.073) = 0.658$. Thus, only 65.8% of the total power resides in the core.

2.9

We first write Eq. (2.4.17) in a simpler form using $x = z/L_D$ and $y = (T_1/T_0)^2$ as

$$y = (1 + sCx)^2 + x^2,$$

where $s = \mathrm{sgn}(\beta_2 C)$. To find the minimum, we set the derivative to zero so that

$$dy/dx = 2(1 + sCx)sC + 2x.$$

The minimum occurs only if $s = -1$ or $\beta_2 C < 0$. The value of x at the minimum point is found to be

$$x = \frac{C}{1+C^2}, \quad \text{or} \quad z = \frac{CL_D}{1+C^2}.$$

We can verify that y is a minimum by noting that $d^2y/dx^2 > 0$. Since $y = 1/(1+C^2)$ at this value of x, the pulse is compressed by a factor of $\sqrt{1+C^2}$.

2.10

We start with Eq. (2.4.23) and set $V_\omega = 0$ and $C = 0$ so that

$$\frac{\sigma^2}{\sigma_0^2} = 1 + \left(\frac{\beta_2 L}{2\sigma_0^2}\right)^2 + \left(\frac{\beta_3 L}{4\sqrt{2}\sigma_0^3}\right)^2.$$

We are given that $L = 60$ km and $\sigma_0 = T_{\mathrm{FWHM}}/(2\sqrt{2\ln 2}) = 21.23$ ps.

At 1.3 μm, $\beta_2 = 0$ and $\beta_3 = 0.1$ ps^3/km. Using these values, $\sigma = 21.23$ ps (no pulse broadening), and the limiting bit rate $B = 1/4\sigma = 11.8$ Gb/s.

At 1.55 μm, $\beta_2 = -20$ ps^2/km and $\beta_3 = 0$. Using these values, $\sigma = 35.347$ ps, and the limiting bit rate $B = 1/4\sigma$ = 7.1 Gb/s.

2.12

We set $V_\omega \gg 1$, $\beta_2 = 0$, and $C = 0$ in Eq. (2.4.23) so that

$$\frac{\sigma^2}{\sigma_0^2} = 1 + \left(\frac{\beta_3 L V_\omega^2}{4\sqrt{2}\sigma_0^3}\right)^2.$$

We can relate β_3 to dispersion slope S using Eq. (2.3.13) as

$$\beta_3 = \left(\frac{\lambda^2}{2\pi c}\right)^2 S.$$

Noting that $V_\omega = 2\sigma_\omega\sigma_0$ and using $\sigma_\omega = (2\pi c/\lambda^2)\sigma_\lambda$, we can rewrite the σ equation as

$$\sigma = (\sigma_0^2 + \sigma_D^2)^{1/2}, \quad \text{where} \quad \sigma_D = |S|L\sigma_\lambda^2/\sqrt{2}.$$

Typically $\sigma_0 \ll \sigma_D$, and $\sigma \approx \sigma_D$. Using the criterion $B < 1/(4\sigma)$, we finally obtain the condition

$$BL|S|\sigma_\lambda^2 < 1/\sqrt{8}.$$

2.15

At $B = 5$ Gb/s, the maximum allowed value of RMS pulse width is $\sigma = 1/4B = 50$ ps. For 100-ps FWHM, the input value is $\sigma = 100/2\sqrt{2\ln 2} = 42.466$ ps. Using $\beta_3 = 0$ and $V_\omega = 0$ in Eq. (2.4.23), we find the relation

$$\sigma^2/\sigma_0^2 = (1 + |C|\xi)^2 + \xi^2 = (50/42.466)^2 = 1.386,$$

where $\xi = L/L_D = |\beta_2|L/2\sigma_0^2$. This is a quadratic equation in ξ and its solution is given by

$$\xi = \frac{-|C| \pm [C^2 + 0.386(1+C^2)]^{1/2}}{1+C^2}.$$

For $C = -6$, we obtain $\xi = 0.0295$ and $L = 5.3$ km (choosing + sign to ensure $L > 0$).
For $C = 0$, we obtain $\xi = 0.6215$ and $L = 112$ km. Thus, fiber length increases by a factor of about 20 for unchirped pulses.

2.16

Total fiber loss = (0.5 dB/km) $\times$ 50 km = 25 dB.
Total splice loss = (0.2 dB) $\times$ 9 = 1.8 dB.
Total connector loss = (1 dB) $\times$ 2 = 2 dB.
Total cable loss = (25+1.8+2) dB = 28.8 dB.
Required receiver power = 0.3 μW = -35.23 dBm.
Minimum launched power = $-35.23 + 28.8 = -6.43$ dBm = 0.227 mW.