ECE228A
HW 3

2.13

We set $V_\omega = 0,\ \beta_2 = 0$, and $C = 0$ in Eq. (2.4.23) so that

$$\frac{\sigma^2}{\sigma_0^2} = 1 + \left(\frac{\beta_3 L}{4\sqrt{2}\sigma_0^3}\right)^2.$$

We can write this equation as

$$\sigma = (\sigma_0^2 + \sigma_D^2)^{1/2}, \quad \text{where} \quad \sigma_D = \frac{|\beta_3|L}{4\sqrt{2}\sigma_0^2}.$$

Since σ_D depends on σ_0, we need to minimize σ by choosing an optimum value of σ_0. Using $y = \sigma^2$ and $x = \sigma_0^2$, we need to minimize

$$y = x + \frac{a^2}{2x^2}, \quad \text{where} \quad a = \frac{|\beta_3|L}{4}.$$

Taking the derivative, we find that the minimum occurs for $x = a^{2/3}$, or for $\sigma_0 = a^{1/3}$, and the minimum value is given by

$$\sigma = \sqrt{\frac{3}{2}}\left(\frac{|\beta_3|L}{4}\right)^{1/3}.$$

Using the criterion $B < 1/(4\sigma)$, we finally obtain the condition

$$B(|\beta_3|L)^{1/3} < 0.324.$$

2.14

Using $\beta_3 = 0$ and $V_\omega \ll 1$ in Eq. (2.4.23), we can write this equation as

$$\sigma^2 = \sigma_0^2 + C\beta_2 L + (1+C^2)\left(\frac{\beta_2 L}{2\sigma_0}\right)^2.$$

Since σ depends on σ_0, we need to minimize σ by choosing an optimum value of σ_0. Setting the derivative to zero we obtain

$$\frac{d\sigma^2}{d\sigma_0^2} = 1 - (1+C^2)\left(\frac{\beta_2 L}{2\sigma_0^2}\right)^2 = 0.$$

The minimum occurs when

$$\sigma_0^2 = \frac{|\beta_2| L}{2}\sqrt{1+C^2},$$

and the minimum value is given by

$$\sigma = |\beta_2| L (C + \sqrt{1+C^2})^{1/2}.$$

Using the criterion $B < 1/(4\sigma)$, we finally obtain the condition

$$B\sqrt{|\beta_2| L}(C + \sqrt{1+C^2})^{1/2} < 1/4.$$

2.17

From Eq. (2.6.14), nonlinear phase shift

$$\phi_{NL} = \gamma P_{in} L_{eff} = \gamma P_{in} L,$$

when fiber loss can be neglected. We can calculate γ from $\gamma = 2\pi \bar{n}_2/(A_{eff}\lambda)$ as all parameter values are given. It is found to be γ = 2.1 W^{-1}/km. Using P_{in} = 6 dBm = 4 mW, and $\phi_{NL} = 2\pi$, we obtain L = 745 km.

2.18

From Eq. (2.6.6), the threshold condition is $g_B P_{th} L_{eff}/A_{eff} \approx 21$.

For $L = 50$ km and α = 0.5 dB/km = 0.115 km^{-1},

$$L_{eff} = \frac{1 - e^{-\alpha L}}{\alpha} = 8.67 \text{ km}.$$

Now we can calculate P_{th} using the values given to find that $P_{th} \approx 1$ mW.

At 1.55 μm, $L_{eff} = 19.56$ km because of reduced losses, resulting in a threshold of 0.42 mW.

2.19

From Eq. (2.6.14), nonlinear phase shift

$$\phi_{NL} = \gamma P_{in} L_{eff}.$$

Using $\alpha = 0.2$ dB/km = 0.046/km, and $L = 40$ km, the effective length is found to be

$$L_{eff} = \frac{1 - e^{-\alpha L}}{\alpha} = 18.28 \text{ km}.$$

We can calculate γ from $\gamma = 2\pi\bar{n}_2/(A_{eff}\lambda)$ since all parameter values are given. It is found to be $\gamma =$ 3.24 W^{-1}/km. Using $\phi_{NL} = \pi$, we obtain P_{in} = 53 mW.

2.20

From Eq. (2.6.14), SPM-induced phase shift is given by

$$\phi_{NL} = \gamma P(t) L_{eff} = \gamma P(t) L,$$

when fiber loss can be neglected. For a Gaussian pulse, $P(t) = P_0 \exp(-t^2/T_0^2)$. The frequency chirp is calculated as

$$\Delta\nu(t) = -\frac{1}{2\pi}\frac{\phi_{NL}}{dt} = \frac{\gamma P_0 L}{\pi T_0^2} t \exp(-t^2/T_0^2).$$

We maximize the chirp by setting its time derivative to zero. The maximum is found to occur at $t = T_0/\sqrt{2}$. The maximum chirp for this value of t is given by

$$\Delta\nu_{max} = \frac{\gamma P_0 L}{\pi T_0 \sqrt{2e}}.$$

Using $\gamma = 3.24$ W^{-1}/km from preceding problem, $P_0 = 5$ mW, $L = 100$ km, and T_0 = 20/1.665 =12 ps, we obtain $\Delta\nu_{max} = 18.2$ GHz.