ECE 228A
HW 4

3.1

We start with Eq. (3.2.3) for the external quantum efficiency of an LED:

$$\eta_{\text{ext}} = \frac{1}{4\pi}\int_0^{\theta_c} T_f(\theta)(2\pi\sin\theta)d\theta,$$

where $\theta_c = \sin^{-1}(1/n)$ is the critical angle. Typically, $n = 3.5$ and $\theta_c = 16°$. Since θ is relatively small, we replace $T_f(\theta)$ with $T_f(0)$ and use a well-known formula for transmission when the light is incident normal to the interface (see any book on electromagnetic theory)

$$T_F(\theta) \simeq T_f(0) = \frac{4n}{(n+1)^2}.$$

Using this value and performing the integration, we obtain

$$\eta_{\text{ext}} = \frac{2n}{(n+1)^2}\int_0^{\theta_c}\sin(\theta)d\theta = \frac{2n}{(n+1)^2}[1-\cos(\theta_c)].$$

Noting that $\cos^2(\theta_c) = 1-\sin^2(\theta_c) = 1-1/n^2$,

$$\eta_{\text{ext}} = \frac{2n}{(n+1)^2}\left(1-\sqrt{1-1/n^2}\right) \approx \frac{1}{n(n+1)^2},$$

if we expand the square root in a Taylor series assuming $1/n^2 \ll 1$ and keep the two leading terms. For $n = 3.5$, $\eta_{\text{ext}} = 1.4\%$

3.3

For $In_{1-x}Ga_xAs_yP_{1-y}$ layers, the ratio $x/y = 0.45$ to ensure lattice matching with the InP substrate. We can find y using the bandgap relation $E_g(y) = 1.35-0.72y+0.12y^2$. For a 1.3-$\mu$m laser, $E_g = h\nu = hc/\lambda = 0.9545$ eV. Thus, y satisfies the quadratic equation

$$0.9545 \simeq 1.35-0.72y+0.12y^2.$$

Since $0 \le y \le 1$, the only acceptable solution is $y = 0.6117$. Now we use $x = 0.45y$ to obtain $x = 0.2753$.

For a $1.55-\mu$m laser, $E_g = h\nu = hc/\lambda = 0.8005$ eV. Solving the quadratic equation

$$0.8005 \simeq 1.35-0.72y+0.12y^2,$$

with $0 \le y \le 1$, we obtain $y = 0.8974$ and $x = 0.4038$.

3.7

We start with the laser rate equations:

$$\frac{dP}{dt} = G(N)P + R_{sp} - \frac{P}{\tau_p},$$

$$\frac{dN}{dt} = \frac{I}{q} - \frac{N}{\tau_c} - G(N)P,$$

where $G(N) = G_N(N - N_0)$. We set the time derivatives to zero in the steady state and also neglect the spontaneous emission term.

From the photon equation, we conclude that $(G - 1/\tau_p)P$ must remain zero in the steady state. This equation is automatically satisfied when laser is below its threshold since $P = 0$. Using $P = 0$ in the N equation, we find that N increases linearly with I as

$$P = 0, \quad N = \frac{\tau_c}{q} I \quad \text{(below threshold)}.$$

As current increases, N and G increase until $G = 1/\tau_p$. Now, the photon equation is satisfied, and P begins to increase. Thus, threshold is reached when $G(N_{th}) = G_N(N_{th} - N_0) = 1/\tau_p$. Solving it, we find N_{th}, and using it in the preceding equation, the threshold current is found to be

$$I_{th} = \frac{q}{\tau_c} N_{th} = \frac{q}{\tau_c} \left[N_0 + \frac{1}{G_N \tau_p} \right].$$

When the laser is above threshold, N remains clamped to its threshold value N_{th}, and P is found from the N equation to be

$$P = \frac{\tau_p}{q}(I - I_{th}), \quad N = N_{th} \quad \text{(above threshold)}.$$

3.9

From Eq. (3.5.4), $\tau_p = (v_g \alpha_{\text{cavity}})^{-1}$. The group velocity is found using $v_g = c/\bar{n}_g = 8.8235 \times 10^7$ m/s. Cavity losses are found using

$$\alpha_{\text{cav}} = \alpha_{\text{mir}} + \alpha_{\text{int}}.$$

where $\alpha_{\text{int}} = 40\ \text{cm}^{-1}$. We can find α_{mir} by noting that the mirrors of the laser cavity are simply the cleaved facets of the InGaAsP semiconductor. Since we are given the modal index for this material, we can calculate the mirror's reflectivity using (assuming that the laser is surrounded by air)

$$R_1 = R_2 = \left(\frac{n+1}{n-1}\right)^2 = 0.2861.$$

Mirror losses are calculated using

$$\alpha_{\text{mir}} = -\frac{1}{2L}\ln(R_1 R_2) = 50.056\ \text{cm}^{-1}.$$

Photon lifetime can now be calculated and is found to be $\tau_p = 1.26$ ps.

To find the threshold value of the electron population, we use the threshold condition

$$N_{th} = \left(N_0 + \frac{1}{G_N \tau_p}\right) = 2.324 \times 10^8.$$

3.10

From Eq. (3.5.6):

$$I_{th} = \frac{q}{\tau_c} N_{th}.$$

Using N_{th} and τ_c from the previous problem, $I_{th} = 18.62$ mA.

The power emitted from a facet is obtained using Eqs. (3.5.7) and (3.5.8):

$$P_e = \frac{1}{2}(v_g \alpha_{\text{mir}})\hbar\omega \left[\frac{\tau_p}{q}(I - I_{th})\right].$$

When the laser is operating twice above threshold, $I = 2I_{th}$. Using the values of v_g, τ_p,and α_{mir} from the previous problem, and assuming that this laser is operating at 1.55 μm so that $\hbar\omega$ = 0.8 eV, we obtain $P_e = 4.14$ mW.