Lecture 2- Photon Statistics and Basics of Propagation in Dielectric Media

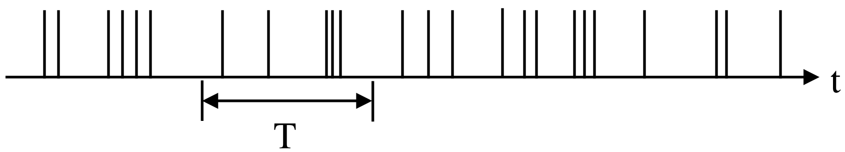
Photodetection

Detection of Optical Signals

- Thermal: Temperature change with photon absorption
  - Thermoelectric
  - Pyromagnetic
  - Pyroelectric
  - Liquid crystals
  - Bolometers
- Wave Interaction: Exchange energy between waves at different frequencies
  - Parametric down-conversion
  - Parametric up-conversion
  - Parametric amplification
- Photon Effects: Generation of photocarriers from photon absorption
  - Photoconductors
  - Photoemissive
  - Photovoltaics

Photon Statistics

- Photon sources can in general be characterized as coherent or incoherent†
  - Coherent: Probability that a photon is generated at time t_0 is mutually independent of probability of photons generated at other times (Markov Process)
    - Poisson Process: Probability of finding *n* photons in time interval *T*
      - Bunching is a trait of the Poisson process
      - Interarrival time is decaying exponentially distributed



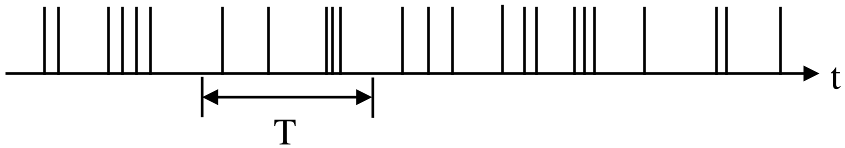
$$P(n \mid T) = \frac{(rT)^n e^{-rT}}{n!}$$

† Can also be a combination of these two types -> partially coherent

Where :
P(n|T) is probability of finding n photons in time interval T
R is mean photon arrival rate (photons/second)

Photon Statistics (II)

- Narrowband Thermal (Gaussian):
  - Bose-Einstein Process: Probability of finding *n* photons in time interval *T*

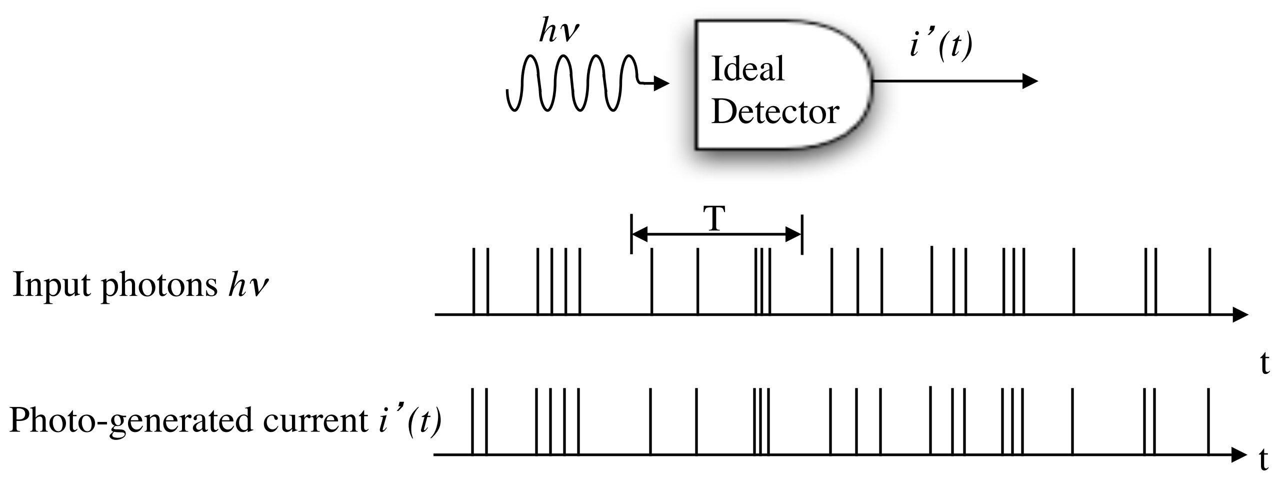


$$P(n) = \left(\frac{1}{1+n_b}\right)\left(\frac{n_b}{1+n_b}\right)^n$$

Where :
P(n) = probability of finding n photons given
n_b = mean number photons from incoherent source = $N_0/h\nu_0$
N_0 = spectral density of source = P_{opt}/B_0
P_{opt} = total optical power from source
B_0 = source optical bandwidth
T = observation time $\leq 1/B_0$

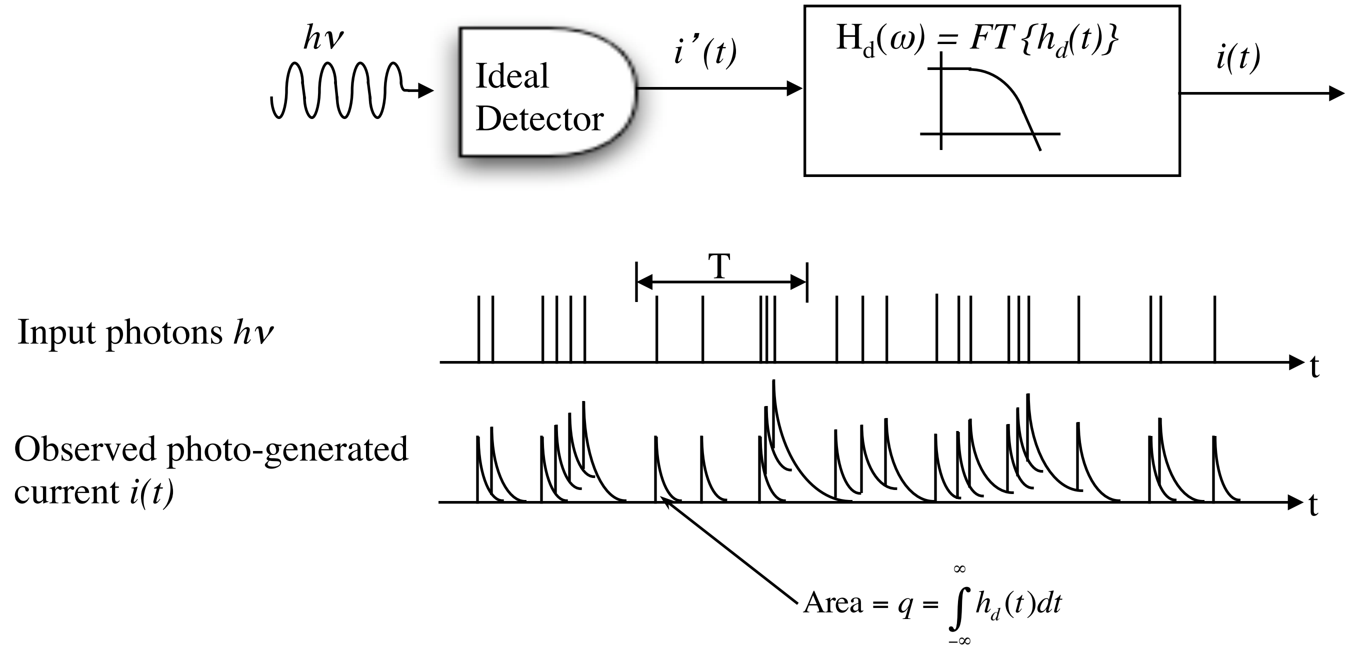
Detecting Photons (1)

- Any material that can respond to single photons can be used to count photons
- Ideal Detector
  - Generation of a electron-hole pair per absorbed photon results in an instantaneous current pulse



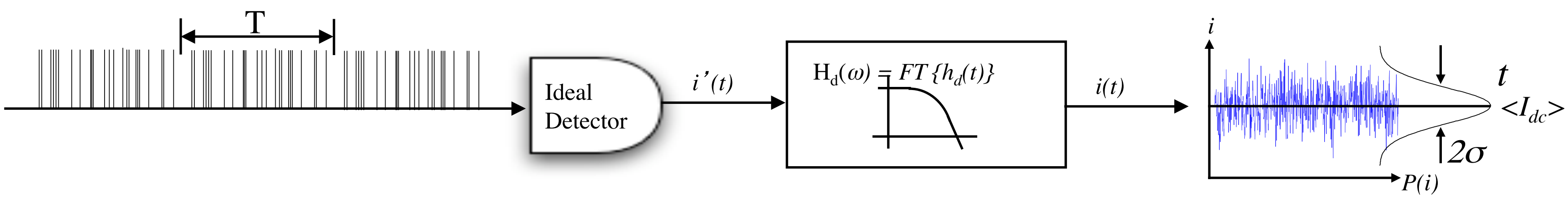
Detecting Photons (2)

- Real Detector
  - Has an inherent "impulse response," $h_d(t)$, due to built in resistance and/or capacitance.
  - Can be modeled as an RC filter with low pass response



Detecting Photons (3)

- As the average photon rate increases, the observed photo-current starts smoothing out, with a variance around the mean (average) count that is based on the statistics (which tends to Gaussian for large photon arrival rate)
- *P(i)* is the probability function of measuring the current at a certain value at time t.



Detecting Photons (4)

- The detector output current $i(t)$ can be modeled as a discrete "filtered Poisson" process

$$i(t) = \sum_{j=1}^{N} h_d(t - \tau_j)$$

  - Where $h_d(t)$ is PD impulse response, N is total number e-h pairs generated, τ_j is the random time the j^{th} photocarrier is generated.

- Define: Quantum Efficiency (QE), unitless, as

$$\eta = \frac{\text{number of photocarriers produced}}{\text{number of incident photons}}, 0 \le \eta \le 1$$

- Define: Time varying photon rate parameter ($\lambda(t)$) in units of photocarriers/second as

$$\lambda(t) = \frac{\eta}{h\upsilon} P_{recvd}(t)$$

Detecting Photons (5)

- The power incident on a photodetector of area A, in units of Watts, is

$$P(t) = \int_A I(\vec{p},t)\,dA$$

  - where the instantaneous optical intensity at an observation point $\vec{p}$ is given by

$$I(\vec{p},t) = \frac{1}{Z_0}\left|E(\vec{p},t)\right|^2$$

- The time varying photon rate parameter *λ(t)* can then be written in terms of *P(t)*

$$\lambda(t) = \frac{\eta}{h\nu}\frac{\left|E(t)\right|^2}{Z_0}$$

Detecting Photons (6)

- If we consider an observation interval, over which we are going to average our photon count over
  - This can be due either to the inherent bandwidth of the detector or (as we will see later) on purpose to match the receiver bandwidth to the data bit rate
- Then the number of photocarriers generated over the interval T counted at the j^{th} observation interval

$$N_j = \int_0^T \lambda_j(\tau)d\tau$$

- Assuming a coherent source, the ***conditional inhomogeneous Poisson process*** describes this photon count during the j^{th} observation interval

$$P(N_j = N) = \frac{\left(\int_0^T \lambda_j(\tau)d\tau\right)^N}{N!} e^{\left(-\int_0^T \lambda_j(\tau)d\tau\right)}$$

Detecting Photons (7)

- If we assume a constant rate parameter over the time interval T (independent of *j*), then the photo-generated current can be written as

$$i(t) = \lambda(t)q$$

$$\lambda(t) = \frac{N}{T}$$

- Then the photocurrent produced by the photodetector can be written in Amperes, assuming the observation time is normalized to one second

$$i(t) = \lambda(t)q = \frac{\eta q}{h\nu} P_{rcvd}(t)$$
$$= \Re P_{rcvd}(t)$$

  - Where we have defined the detector responsivity as

$$\Re = \frac{\eta q}{h\nu}$$

Review of The Wave Equation in Dielectric Media

Notation

- MKS units
- Lower case for time varying quantities
- Capitals for the amplitudes of time varying quantities
- Complex quantities used to represent amplitude and phase:

- (at least in Chapter 1. In Chapter 2, E(x,y,z,t)=Re [E(x,y,z) $e^{i\omega t}$]

$$a(t) = \mathrm{Re}[Ae^{i\omega t}]$$

Maxwell's Equations

$$\nabla \times \overline{h} = i + \frac{\partial d}{\partial t}$$

$$\nabla \times \overline{e} = -\frac{\partial b}{\partial t}$$

$$\nabla \bullet \overline{d} = 0$$

$$\nabla \bullet \overline{b} = 0$$

where e and h are the electric and magnetic field vectors
d and b are the electric and magnetic displacement vectors
No free charge.

Constitutive Relations

$$\bar{d} = \varepsilon_0 \bar{e} + \bar{p}$$

$$\bar{b} = \mu_0 (\bar{h} + \bar{m})$$

p and m are the electric and magnetic polarizations of the medium
ε_0 and μ_0 are the electric and magnetic permeabilities of vacuum
e and h are the electric and magnetic field vectors
d and b are the electric and magnetic displacement vectors

Electric Susceptibility χ (Isotropic)

Isotropic Media: χ is a complex number

$$\overline{P} = \varepsilon_0 \chi_{ij} \overline{E}$$

The real part determines the index (velocity) and the imaginary part determines the gain or absorption.

Isotropic media: Vacuum, gasses, glasses (optical fibers)
Anisotropic media: Semiconductors, crystalline materials.