Lecture 5 - Propagation in Optical Fibers and Dispersion

Boundary Conditions

Decaying fields for r>a
q>0

$$q^2 = \beta^2 - k^2 = \beta^2 - n_2^2 k_0^2$$

$$k_0 = \omega / c$$

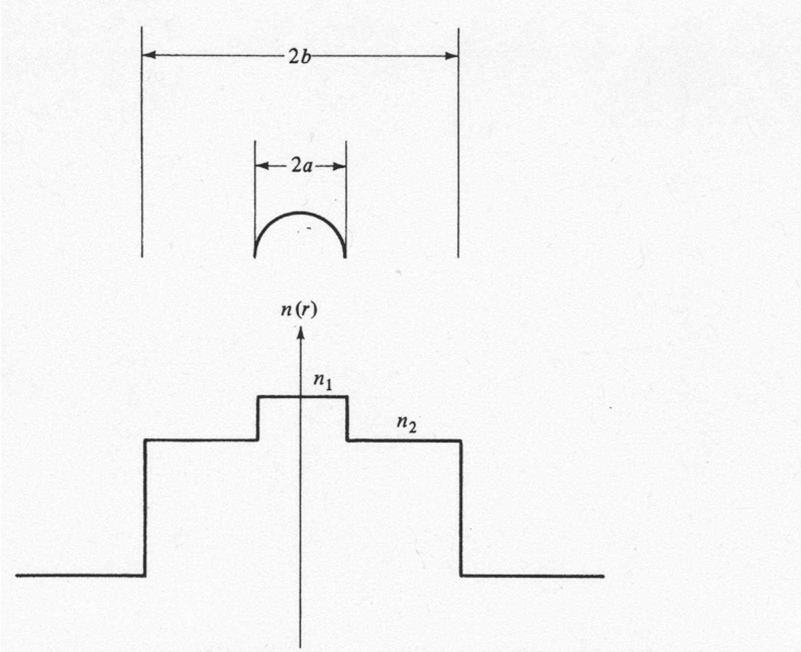


Figure 3-1 Structure and index profile of a step-index circular waveguide.

For fields in the core r<a, we need finite fields
(which eliminates Y and K which go to infinity as r approaches 0.

Boundary Conditions

- In order for the mode to be supported, it must be a standing wave pattern along r inside the core and a decaying exponential along r inside the cladding, with the boundary conditions supported at the step interface.
- β is therefore bounded by

$$n_1 k_0 \leq \beta \leq n_2 k_0$$

TE l=0 Modes

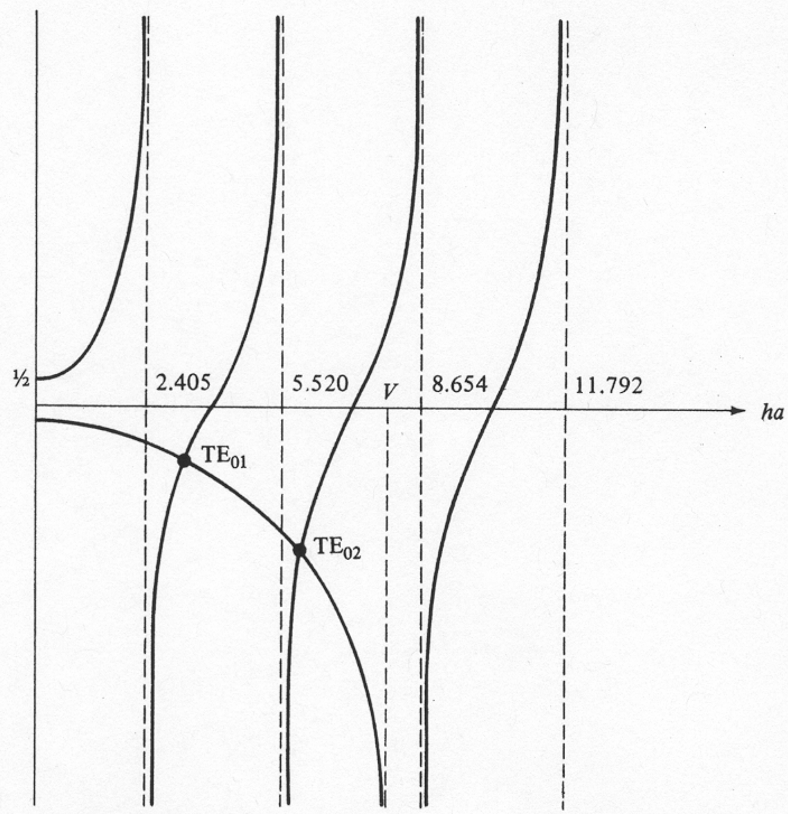


Figure 3-2 Graphical determination of the propagation constants of TE modes ($l = 0$) for a step-index waveguide.

l=1 (not TE or TM, but EH)

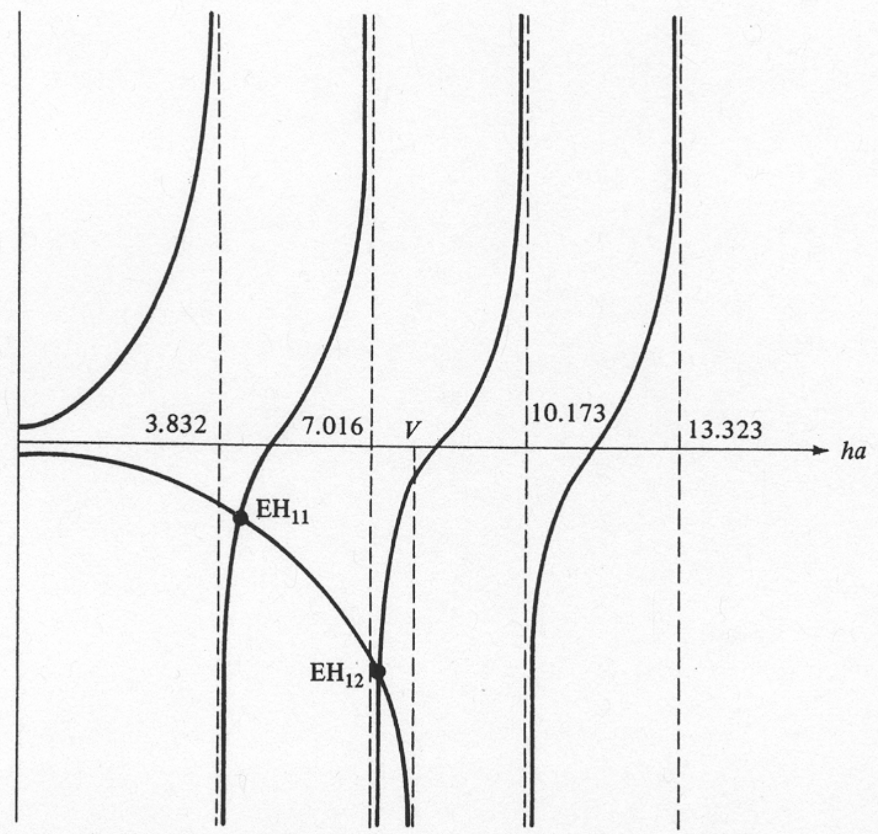


Figure 3-3 Graphical determination of the propagation constants of $l = 1$ EH modes for a step-index fiber.

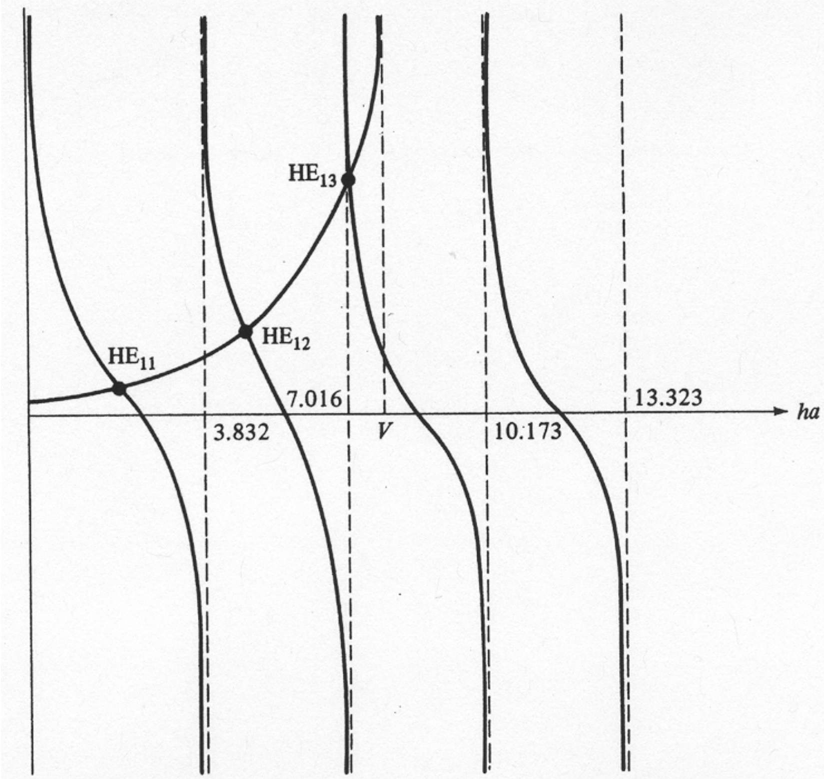


Figure 3-4 Graphical determination of the propagation constants of the $l = 1$ HE modes for a step-index dielectric waveguide.

V parameter

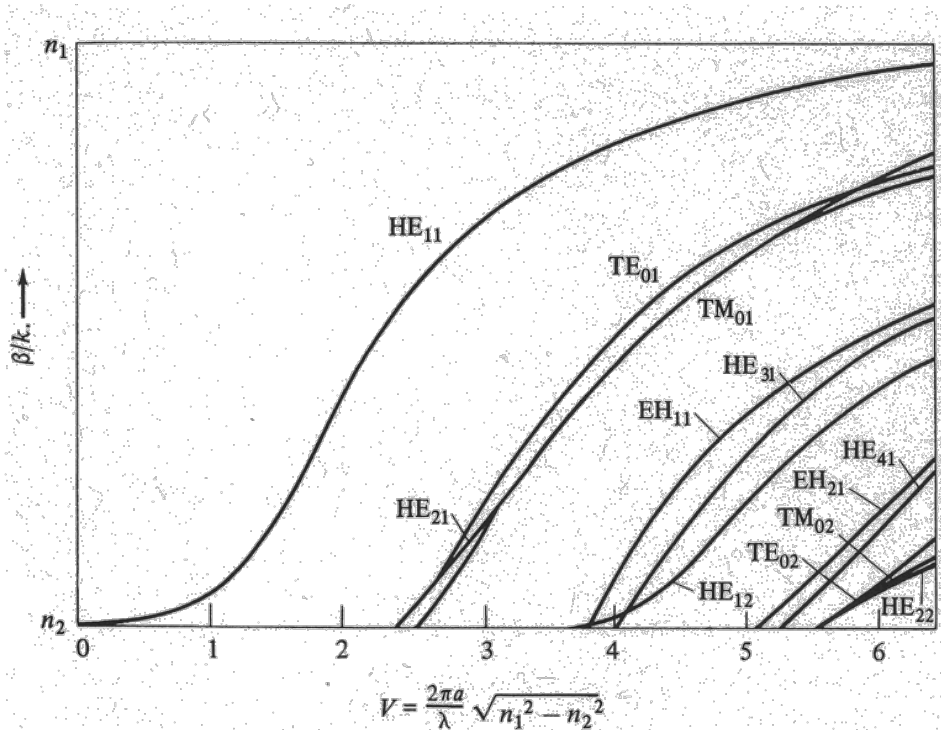


Figure 3-5 Normalized propagation constant as a function of V parameter for a few of the lowest-order modes of a step-index waveguide [4].

For n1-n2<<n1, LP approximation is valid.

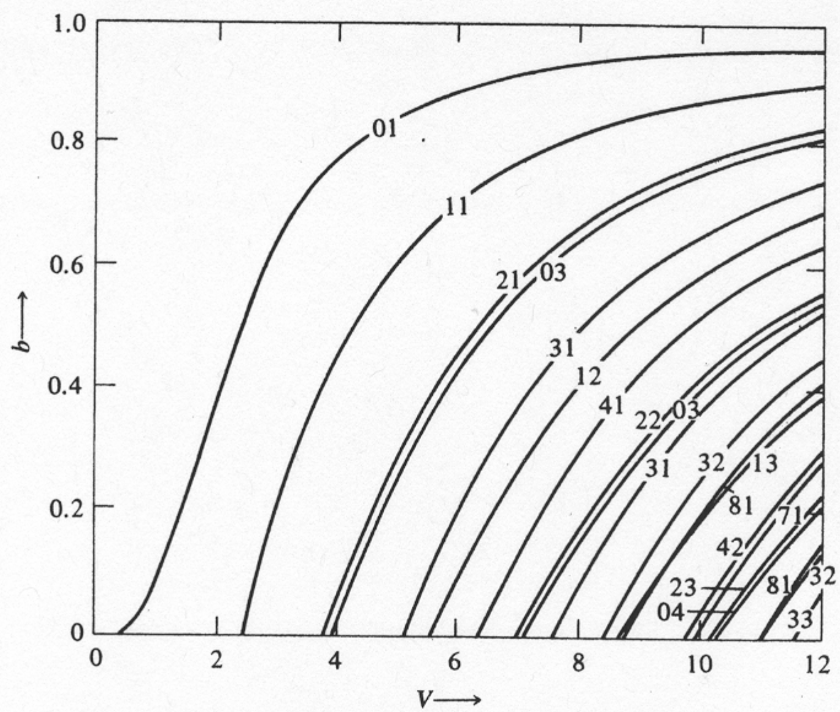


Figure 3-6 Normalized propagation constant b as function of normalized frequency V for the guided modes of the optical fiber, $b = (\beta/k_c - n_2)/(n_1 - n_2)$. (After Reference [5].)

Single mode cut off: V=2.405

Degenerate Modes LP_{11}

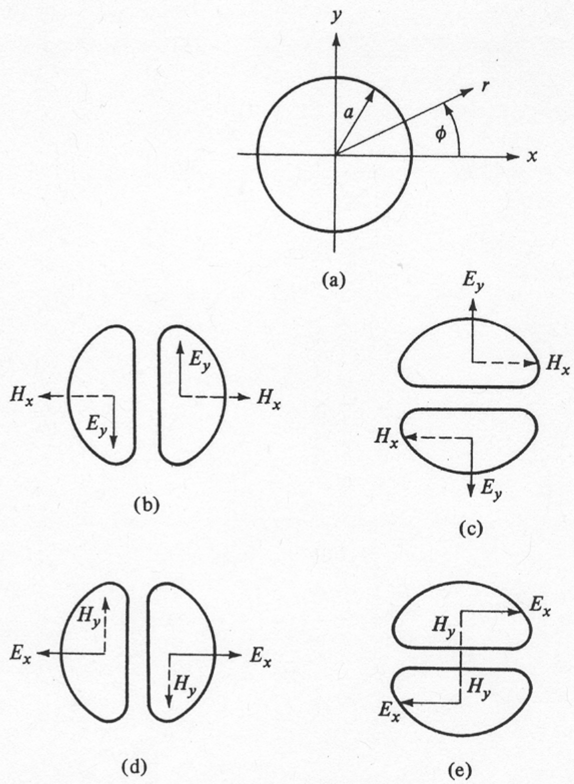


Figure 3-8 Sketch of the fiber cross section and the four possible distributions of LP_{11}.

Modes as a function of V parameter

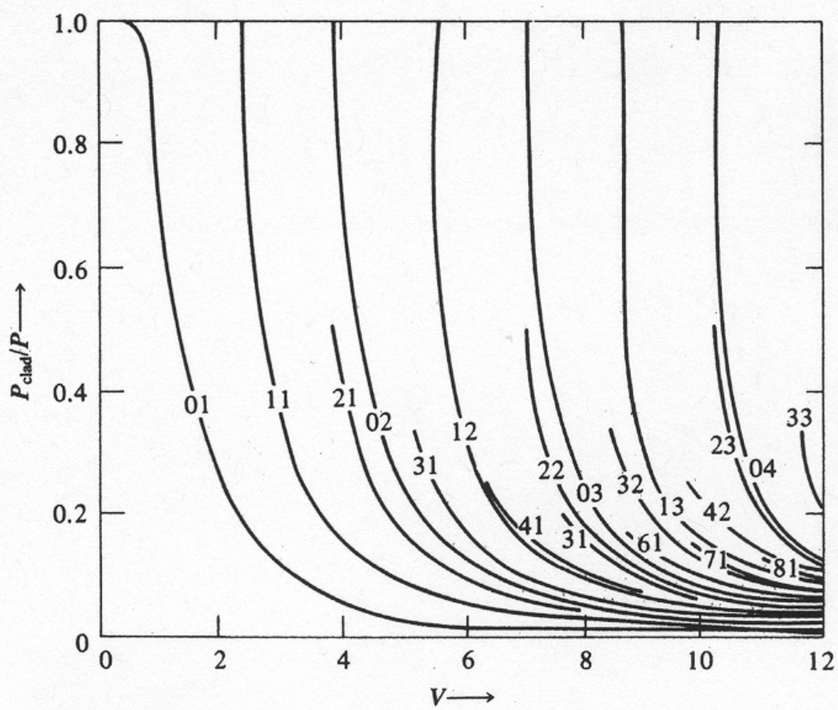


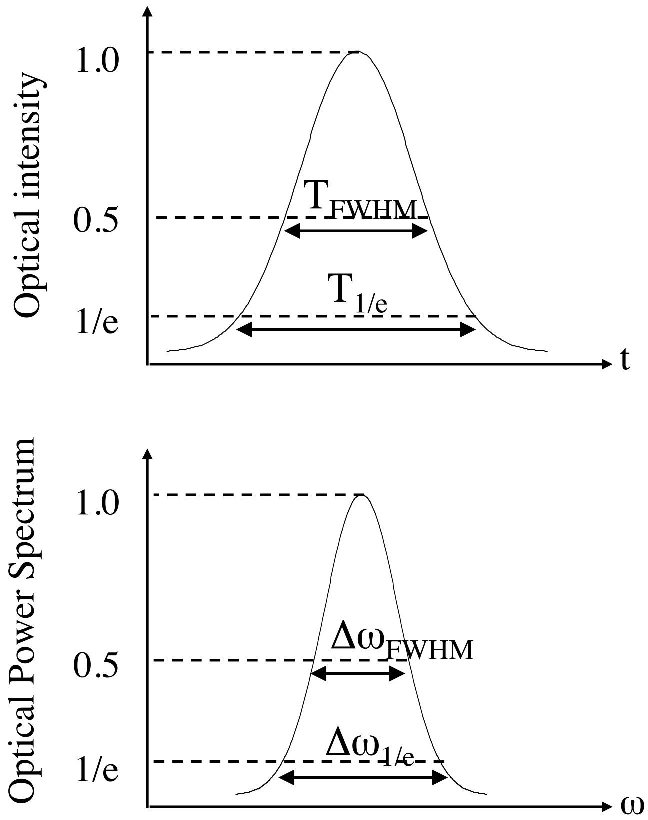
Figure 3-9 Fractional power contained in the cladding as a function of the frequency parameter *V*. (After Reference [5].)

At cutoff, all the power is in the cladding.

Dispersion in Single Mode Fibers

- Modal dispersion in multimode fibers is not present in single mode fiber (SMF)
- However, other types of dispersion are present in SMF
  - Material dispersion
  - Waveguide dispersion
  - Polarization mode dispersion
- The first two effects fall under the term "Chromatic Dispersion"
- The third effect is known as PMD
- Dispersion can set the ultimate bit-rate limit in single mode fiber when loss and fiber nonlinearities are not dominant

Gaussian Pulses



- Define rms pulsewidth and bandwidth values:
  - $\sigma_0 = T_{1/e}/\sqrt{2}$ (rms pulse width)
  - $\sigma_\omega = \Delta\omega_{1/e}/\sqrt{2}$ (rms spectral width)
- Define linear chirp factor
  - C
- Define relation between pulse width and bandwidth
  - $\Delta\omega_{1/e} = (1 + C^2)^{1/2}/T_0$

Non-Linear Schrodinger Equation

- Both linear (dispersive) and nonlinear effects must be taken into account for pulse propagation in the fiber
- The propagation of a signal in a single mode fiber is set (to a very high level of accuracy) by the following equation, called the nonlinear Schrodinger equation:

$$\frac{\partial A}{\partial z} = -\alpha A + j\frac{1}{2}\beta_2\frac{\partial^2 A}{\partial t^2} - \frac{1}{6}\beta_3\frac{\partial^3 A}{\partial t^3} - j\gamma|A|^2 A$$

- $-\alpha A$: Attenuation
- $j\frac{1}{2}\beta_2\frac{\partial^2 A}{\partial t^2} - \frac{1}{6}\beta_3\frac{\partial^3 A}{\partial t^3}$: Chromatic Dispersion
- $-j\gamma|A|^2 A$: Nonlinear Effects

- *A(z,t)* is the complex-envelope of the optical field
- The resulting optical power is $P(z,t)=| A(z,t) |^2$