Lecture 6 - Propagation in Optical Fibers and Dispersion

Non-Linear Schrodinger Equation

- Both linear (dispersive) and nonlinear effects must be taken into account for pulse propagation in the fiber
- The propagation of a signal in a single mode fiber is set (to a very high level of accuracy) by the following equation, called the nonlinear Schrodinger equation:

$$\frac{\partial A}{\partial z} = -\alpha A + j\frac{1}{2}\beta_2\frac{\partial^2 A}{\partial t^2} - \frac{1}{6}\beta_3\frac{\partial^3 A}{\partial t^3} - j\gamma\left|A\right|^2 A$$

Attenuation: $-\alpha A$

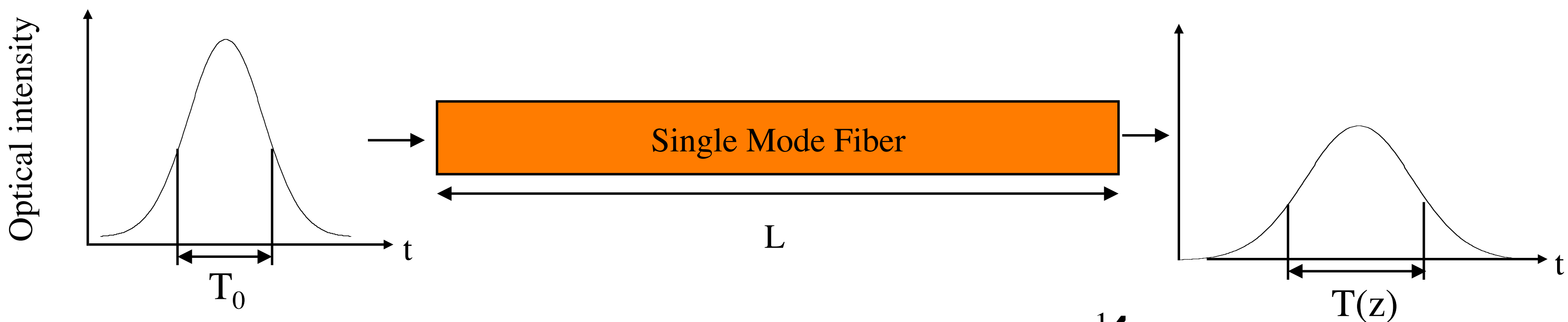
Chromatic Dispersion: $j\frac{1}{2}\beta_2\frac{\partial^2 A}{\partial t^2} - \frac{1}{6}\beta_3\frac{\partial^3 A}{\partial t^3}$

Nonlinear Effects: $-j\gamma\left|A\right|^2 A$

- $A(z,t)$ is the complex-envelope of the optical field
- The resulting optical power is $P(z,t)=| A(z,t) |^2$

Pulse Broadening

Assuming a Gaussian shaped input pulse and first order dispersion dominates ($\beta_2 \neq 0$)



$$\frac{T(z)}{T_0} = \left[\left(1 + \frac{C\beta_2 z}{T_0^{\,2}}\right)^2 + \left(\frac{\beta_2 z}{T_0^{\,2}}\right)^2 \right]^{1/2}$$

- Define **Dispersion Length**
  - An unchirped pulse (C=0) will broaden by a factor of $\sqrt{2}$ at $z = L_D$

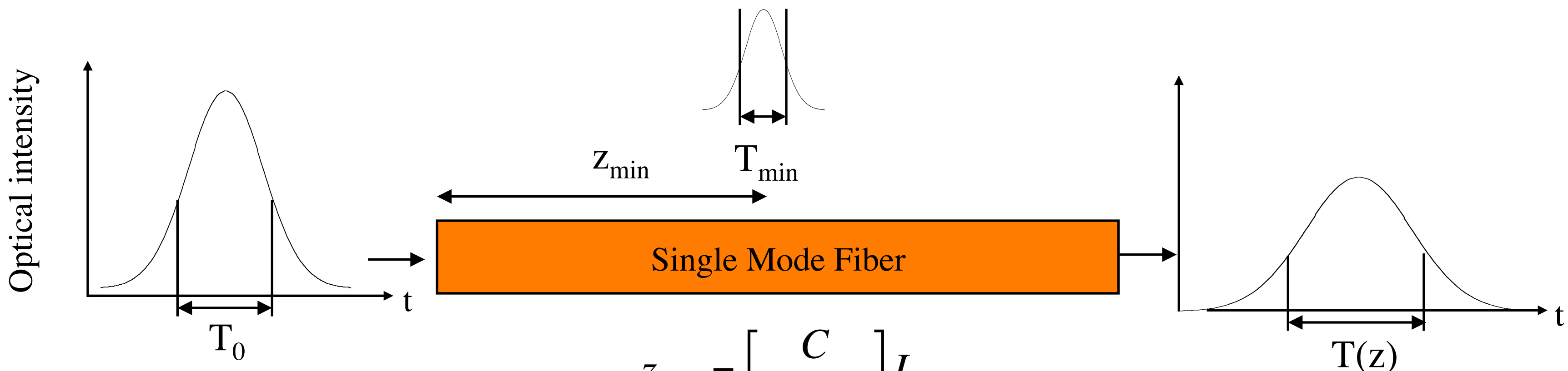
Pulse Compression

If $\beta_2 C < 0$, the pulse will initially decrease!

This will happen if the

(a) the initial pulse is positively chirped and propagates in the anomolous dispersion regime of the fiber OR

(b) if the pulse is initially negatively chirped and propagates in the normal dispersion regime of the fiber



$$z_{min} = \left[\frac{C}{1 + C^2}\right] L_D$$

$$T_{min} = \frac{T_0}{\left(1 + C^2\right)^{1/2}}$$

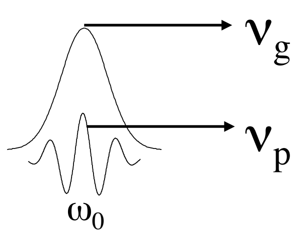
Chromatic Dispersion

- The two terms β_2 and β_3 of the previous equation are the derivative of the "mode propagation constant" $\beta(\omega)$
- The meaning of $\beta(\omega)$ is clear when considering a single pulse propagation

$$\beta(\omega) = \frac{\omega n(\omega)}{c} = \beta_0 + \beta_1 \Delta\omega + \frac{1}{2}\beta_2 \Delta\omega^2 + \frac{1}{6}\beta_3 \Delta\omega^3$$

$$v_p = \frac{\omega_0}{\beta_0} = \frac{c}{n(\omega_0)}$$

$$v_g = \frac{1}{\beta_1} = \left(\left. \frac{d\beta}{d\omega} \right|_{\omega=\omega_0} \right)^{-1}$$

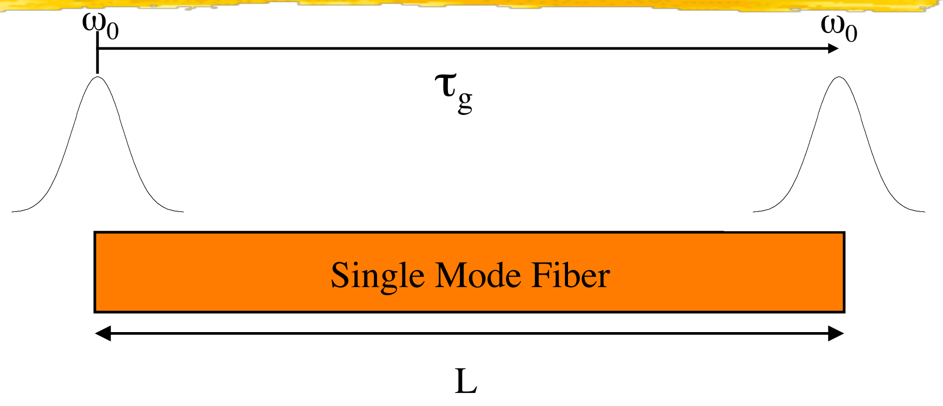


- It turns out that, considering the dispersion term only
  - The **phase velocity** (v_p) is the velocity of the center frequency ω_0,
  - The **group velocity** (v_g) is the velocity of the center of the pulse. It is the value that determine the practical "velocity" of the transmission of the information (energy) in the fiber

Group Delay

Group delay $$\tau_g = \frac{L}{v_g}$$

Group velocity $$v_g = \frac{c}{n_g}$$



- The group delay effective index n_g is approx. of the same order of the index of refraction of the fiber, i.e. , n_g =1.5
- As an example, the (group) delay of 100 Km of fiber is given by:

$$\tau_g = \frac{L}{v_g} = \frac{Ln_g}{c} = \frac{10^5 \cdot 1.5}{3 \cdot 10^8} \cong 500\ \mu s$$

$$v_g = \frac{c}{n_g} \cong 2 \cdot 10^8\ m/s$$

Group Velocity Dispersion (GVD)

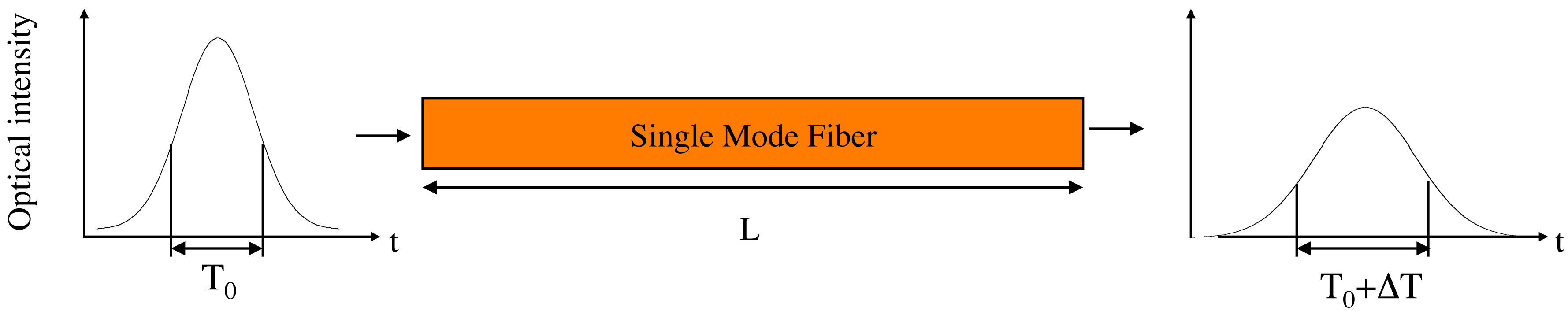
- Group velocity (GVD) is frequency-dependent
- Any communication signal (pulse) has a given bandwidth
  - Different frequencies in pulse => Different group delays => Leads to pulse distortion
- A more quantitative analysis can be carried out by considering that the fiber acts as a filter with the following transfer function:

$$A(z,\omega) = A(0,\omega)\cdot e^{-j\left(\frac{\beta_2}{2}\omega^2+\frac{\beta_3}{2}\omega^3\right)z}$$

- This equation is obtained after some mathematical manipulation that "extracts" the absolute group delay
- The coefficient β_2 and β_3 are evaluated on the pulse central frequency/wavelength ω_0

Group Velocity Dispersion (GVD)

- The previous equation can be exactly solved in some particular cases, among which the most important one is the propagation of a Gaussian pulse



- The Gaussian pulse is broadened after propagation of distance L by the amount:

$$\Delta T = L|\beta_2|\Delta\omega$$

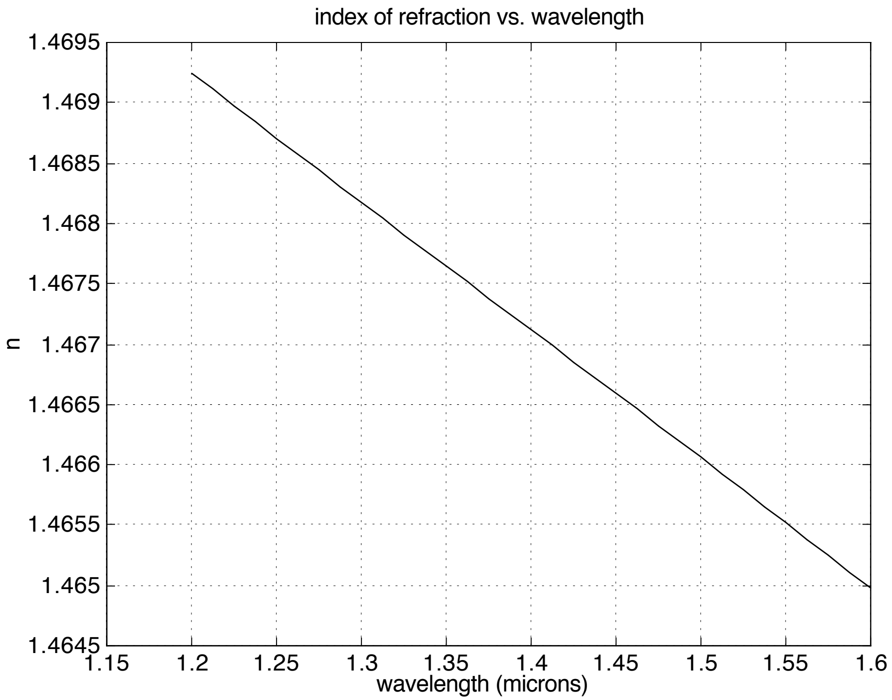
- where $\Delta\omega$ is the spectrum occupied by the pulse
- ανδ β_2 is the dispersion (material and waveguide) of the fiber

Refractive Index of Silica Fibers

- The index of refraction of bulk silica can be approximated using the Sellmeir equation with experimentally measured parameters.

$$n^2(\lambda) = 1 + \sum_{i=1}^{3} \frac{A_i \lambda_i^2}{\lambda^2 - \lambda_i^2}$$

$$\begin{aligned}
A_1 &= 0.401040; \\
\lambda_1 &= 0.064270; \\
A_2 &= 0.521885; \\
\lambda_2 &= 0.129408; \\
A_3 &= 0.904048; \\
\lambda_3 &= 9.425478;
\end{aligned}$$

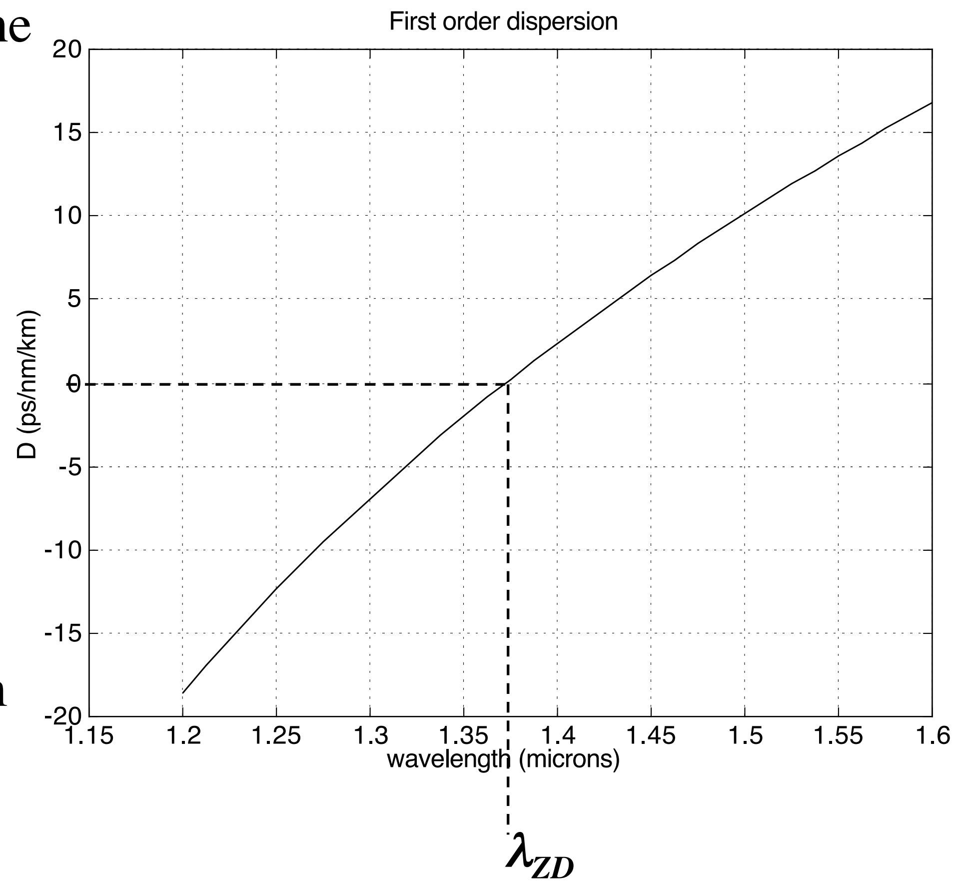


Material Dispersion Parameter

- The material refractive index wavlength dependance impacts the dispersion parameter :

$$D = \frac{d}{d\lambda}\left(\frac{1}{v_g}\right) \approx -\frac{\lambda}{c}\frac{\partial n^2(\lambda)}{\partial \lambda^2}$$

- The material "zero dispersion wavelength" is typically 1350 nm



Group-Velocity Dispersion

- The index of the mode is dependent on the wavelength (i.e. the fiber is dispersive).
- Two components: material dispersion and waveguide dispersion.
- These contribute to phase index.
- The group index is given by

$$n_g = n + \omega \frac{\partial n}{\partial \omega}$$

$$D = -\frac{2\pi c}{\lambda^2}\frac{d^2\beta}{d\omega^2} = -\frac{2\pi c}{\lambda^2}\beta_2$$

Units are ps/(km-nm)

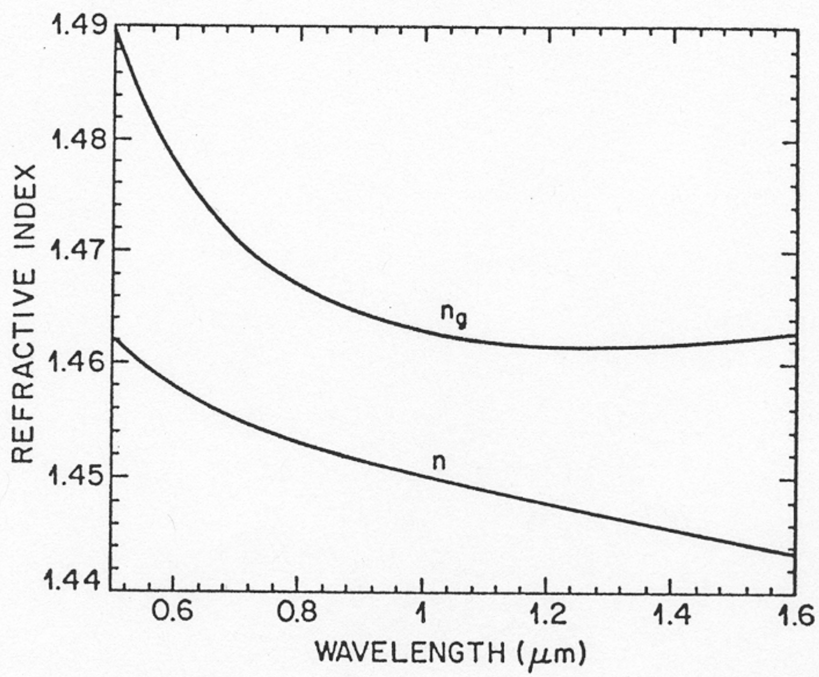


Figure 2.8: Variation of refractive index n and group index n_g with wavelength for fused silica.

Dispersion parameters: β_2 and D

- β_2 is called the "group velocity dispersion" GVD parameter
  - It is expressed in units of ps^2/km
- From a mathematical point of view, it is easier to handle equations dealing with β_2 and optical frequency
- It is also convenient to specify dispersion in terms of optical wavelength
- The "D" parameter is

$$D = \frac{d}{d\lambda}\left(\frac{1}{v_g}\right) \approx -\frac{\lambda}{c}\frac{\partial n^2(\lambda)}{\partial \lambda^2}$$

- D is called the "Dispersion parameter", and it is expressed in units of *ps/nm-km*

The Dispersion Parameter *D*

- The relation between the two parameters is given by:

$$D = -2\pi C/\lambda^2 \ \ \beta_2 \ [ps/nm\text{-}km]$$

- Physical meaning: given two wavelengths separated by $\Delta\lambda$, their different group velocities give rise to a (group) delay between the two components given by

$$\Delta_{delay} = D\,L\,\Delta\lambda$$

- The gaussian pulse spread, in terms of *D*, is given by:

$$\Delta T = L|D|\Delta\lambda$$

where $\Delta\lambda$ is the spectral width of the gaussian pulse

Frequency Dependance of Dispersion

- The frequency dependence of $\beta(\omega)$ is determined by the following physical effects:
  - Material dispersion
    - The index of refraction of the *bulk* material depends on frequency
  - Waveguide dispersion
    - Even for an ideal material with constant index of refraction, the solution of the Maxwell equation for a single mode propagating into a fiber gives a frequency-dependent $\beta(\omega)$
    - This waveguide effects depends on the profile of the index of refraction of the fiber
- The actual $\beta(\omega)$ is thus a combination of the two effects

General Dispersion Formula

- If we take into account more realistic source and fiber effects †
  - we include β_3
  - source with a generic spectral width σ_ω

$$\frac{\sigma(z)}{\sigma_0} = \left[\left(1 + \frac{C\beta_2 z}{2\sigma_0^{\,2}}\right)^2 + \left(1 + V^2\right)\left(\frac{\beta_2 z}{2\sigma_0^{\,2}}\right)^2 + (1 + C^2 + V^2)^2 \frac{1}{2}\left(\frac{\beta_3 z}{4\sigma_0^{\,3}}\right)^2\right]^{1/2}$$

Where $V = 2\sigma_0\sigma_\omega$

- This formula can be used to derive dispersion limits in several different transmission scenarios

†D. Marcuse, Applied Optics, Vol. 19, p. 1653, 1980 and Vol. 20, p. 3573, 1981.