University of California, Santa Barbara
Dept. of ECE

__Solutions___

ECE 228A
Midterm Exam
Winter 2011

This is a closed book exam. You are allowed to use both sides of a single 8.5"x10" sheet of paper for formula and constants. Calculators are only allowed for numerical computation only and not for storage of formulae. Clearly state all approximations and assumptions. You will only be graded for work shown. **Good Luck!**

Problem 1	Problem 2	Problem 3	Total
30 pts	30 pts	40 pts	100 pts

Problem 1 (30 points)
For parts (a) and (b) assume $\lambda_0 = 1.31$ μm, $\Delta = 2 \times 10^{-3}$, and $n_1=1.45$. Perform your calculations to three decimal places. With the aid of the attached Figure:

Part A, 10 pts Specify the maximum *V* and *b* that will allow 4 modes to propagate in a step index fiber. Calculate the effective mode index and the propagation constant β_{11} for the HE_{11} mode.

Using the figure provided, the maximum V and b allowed for 4 modes to propagate can be found on the HE_{11} curve:

$$\boxed{V \approx 3.8, \quad b \approx 0.8}$$

This allows HE_{11}, HE_{21}, TE_{01}, and TM_{01} modes to propagate
Calculating the effective mode index for HE_{11}:

$$\Delta = \frac{n_1 - n_2}{n_1} \Rightarrow n_2 = n_1(1-\Delta) = 1.45(1 - 2\times 10^{-3}) = 1.4471$$

$$b = \frac{n_{eff} - n_2}{n_1 - n_2} \Rightarrow n_{eff} = n_2 + b(n_1 - n_2) = 1.4471 + 0.8(1.45 - 1.4471)$$

$$\boxed{n_{eff} = 1.4494}$$

The propagation constant for HE_{11}:

$$\boxed{\beta_{11} = n_{eff}\frac{2\pi}{\lambda_0} = 6.95\times 10^6} \qquad \text{r/m}$$

Part B, 10 pts Specify the maximum value of *V* and *b* for single mode propagation. Calculate the fiber core radius, the effective mode index and propagation constant for the single mode case.

For single mode operation:

$$\boxed{V_{max} = 2.405, \quad b_{max} \approx 0.55}$$

Calculating the radius, effective mode index, and propagation constant::

$$\boxed{a = V\frac{\lambda_0}{2\pi}\frac{1}{n_1\sqrt{2\Delta}} = (3.8)\left(\frac{1.31\mu m}{2\pi}\right)\left(\frac{1}{1.45\sqrt{2\times 10^{-3}}}\right) = 5.47\mu m}$$

$$n_{eff}\Big|_{HE_{11}} = 1.4471 + 0.55(1.45 - 1.4471) = 1.4487$$

$$\boxed{\beta_{11} = n_{eff}\frac{2\pi}{\lambda_0} = 6.95\times10^{6}(r/m)}$$

Part C, 10 pts For the fiber in part (a) calculate the maximum bandwidth distance product assuming the bit spread at the fiber output must be less than 1/4 of the bit period and excitation of all fiber modes at the input.

To compute the maximum bandwidth distance product:

The difference in transmission times between the two modes:

$$n_{eff}\Big|_{HE11} = n_2 + b(n_1 - n_2) = 1.4471 + 0.8(1.45 - 1.4471) = 1.4494$$

$$n_{eff}\Big|_{HE21} = n_2 + b(n_1 - n_2) = 1.4471 + 0.4(1.45 - 1.4471) = 1.4282$$

$$\Delta T = \frac{L}{c}\left[n_{eff}\Big|_{HE11} - n_{eff}\Big|_{HE21}\right] = \frac{L}{c}\Delta n_{eff}$$

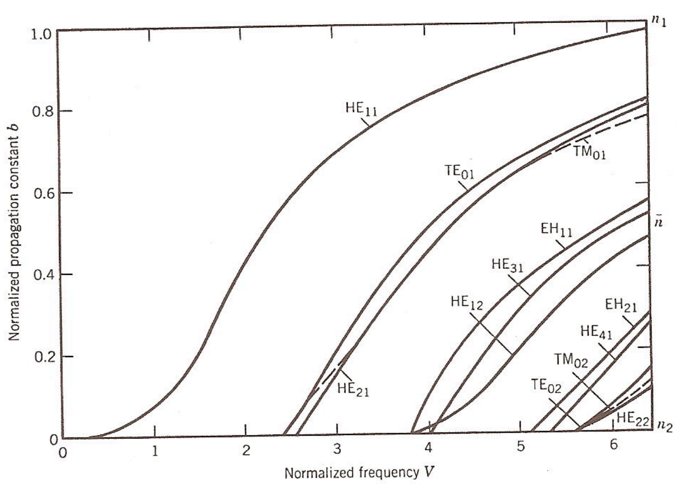
The bit (mode) spreading is required to meet the condition:

$$\Delta T \le \frac{1}{4B}$$

From this, the bandwidth distance product can be calculated:

$$BL \le \frac{c}{4}\left[\frac{1}{\Delta n_{eff}}\right] = \frac{3}{4}\times10^{8}\ \frac{1}{0.0212}(\text{bits/s})\cdot\text{m} = 3.54\times10^{9}(\text{bits/s})\cdot\text{m} = 3.54\text{Mb/s}\cdot k\text{m}$$

$$\boxed{BL \le 3.54\text{Mb/s}\cdot k\text{m}}$$



Problem 2 (30 pts)
A step index fiber has the following wavelength dependent refractive index (where wavelength is given in Angstroms).

$$n(\lambda) = 1.50 + \frac{1.4x10^6}{\lambda^2}$$

Assume two transform limited pulses launched into a fiber of length L = 5000 km. Pulse 1 has a center wavelength λ_1 = 1.550 μm and an initial FWHM pulse width T_1 = 1ps and Pulse 2 has a center wavelength λ_2 = 1.565 μm and an initial FWHM pulse width T_2 = 10ps.

Part A, 15 pts Calculate the pulse width of each pulse at the fiber output assuming material dispersion dominates.

We are given a formula for the index of refraction as a function of wavelength *in Angstroms*. Therefore, the center wavelengths of the pulses must be converted from microns to angstroms:

$$\lambda_1 = 1.550\mu m = 15{,}500\ \text{Å}$$
$$\lambda_2 = 1.565\mu m = 15{,}650\ \text{Å}$$

The *group index* is:

$$\overline{n_g(\lambda)} = \overline{n(\lambda)} - \lambda\frac{d\left[\overline{n(\lambda)}\right]}{d\lambda} = 1.50 + \frac{1.4\times10^6}{\lambda^2} + 2\lambda\left(\frac{1.4\times10^6}{\lambda^3}\right) = 1.50 + \frac{4.2\times10^6}{\lambda^2}$$

The *material dispersion parameter* is calculated as:

$$D = \frac{d}{d\lambda}\left(\frac{1}{\upsilon_g}\right) = \frac{d}{d\lambda}\left[\frac{n_g(\lambda)}{c}\right] = \frac{1}{c}\frac{d}{d\lambda}\left[n_g(\lambda)\right] = \frac{1}{c}\left(-\frac{8.4\times10^6}{\lambda^3}\right)$$

This is then used to calculate the *GVD parameter*:

$$\beta_2(\lambda) = -\frac{\lambda^2}{2\pi c}D = \left(-\frac{\lambda^2}{2\pi c}\right)\left(-\frac{8.4\times10^6}{c\lambda^3}\right)$$
$$= \frac{8.4\times10^6}{2\pi c^2\lambda} = \frac{8.4\times10^{-6}}{2\pi\left(3\times10^5\right)\left(3\times10^{18}\right)\lambda}\times10^{24}\left({ps^2}/{km}\right)$$
$$\therefore \beta_2(\lambda_1) = 95.835\left({\text{ps}^2}/{km}\right)$$
$$\therefore \beta_2(\lambda_2) = 94.917\left({\text{ps}^2}/{km}\right)$$

The change in pulse width is:

$$T(L) = \left[T_0^{\;2} + \left(\frac{\beta_2 L}{T_0}\right)^2\right]^{1/2}$$

$$\therefore T(\lambda_1, 5000km) = \left[1^2 + \left(\frac{95.835 \times 5000}{1}\right)^2\right]^{1/2} = \boxed{479.2 \text{ ns}}$$

$$\therefore T(\lambda_2, 5000km) = \left[10^2 + \left(\frac{94.917 \times 5000}{10}\right)^2\right]^{1/2} = \boxed{47.5 \text{ ns}}$$

Note, that since the broadening dominates the final pulse width and is linear with bandwidth (inversely linear to pulse width), the pulse that is ten times longer at the input broadens by ten times less at the output!

Part B, 15 pts What is the difference in arrival time of the pulses at the fiber output between the centers of the two pulses?

The group indices and group delays can be used to determine difference in arrival time:

$$\therefore \overline{N_g(\lambda_1)} = 1.50 + \frac{4.2 \times 10^6}{15500^2} = 1.51748$$

$$\therefore \overline{N_g(\lambda_2)} = 1.50 + \frac{4.2 \times 10^6}{15650^2} = 1.51715$$

$$\Delta T = \frac{L}{v_{g1}} - \frac{L}{v_{g2}} = \frac{L}{c}\left[\overline{N_g(\lambda_1)} - \overline{N_g(\lambda_2)}\right]$$

$$\Delta T = \frac{5000km}{3 \times 10^5 \; km/\text{sec}}\left[1.51748 - 1.51715\right]$$

$$\boxed{\Delta T = 5.5 \mu s}$$

Problem 3 (40 pts)
For a 10 Gbps, 1.55 μm transmission link with zero chirp input Gaussian pulses (C = 0) we specify the bits at the output cannot spread by more than 1/10 the bit time slot. Assume the input pulse full-width half max (FWHM) occupies half a bit slot, the fiber dispersion is +10ps^2/km, and the laser linewidth is 100kHz.

Part A, 20pts What is the dispersion-limited maximum fiber length
We want

$$\Delta T(L) \le T_B/10$$

Assume T(0) = T_B/2. At 10 Gbps T_B = 100ps. Since the laser linewidth $\Delta \nu_L = 100kHz << B$, we have a transform-limited pulse.

$$T_{FWHM} = \frac{1}{2}\frac{1}{10Gbps} = 5 \times 10^{-11}$$

$$\sigma_0 = \frac{5 \times 10^{-11}}{2} = 2.5 \times 10^{-11}$$

The maximum distance is calculated as

$$\frac{\sigma(L)}{\sigma_0} = 1.1 = \left[1 + \left(\frac{\beta_2 L}{2\sigma_0^2}\right)^2\right]^{1/2}$$

$$L = \frac{(.458)2\sigma_0^2}{\beta_2}$$

$$= \frac{(.458)2(25)^2}{10}$$

$$= 57.25km$$

Part B, 20pts What is the maximum bit rate if you are allowed to choose the fiber length? What is this fiber length? Use the assumptions from part (a).

If C were non-zero then we could design the link to have a minimum pulse width at some distance z_{min}. However, here C is zero, so there is no optimal length and in principle the bandwidth distance product says that the bandwidth can be increased arbitrarily high as the length is decreased to zero. But even assuming our modulator and detector could keep up with the increasing bit rate to very high speeds, we will need ever increasing bandwidth in the optical signal itself. At some point the optical bandwidth will be so large that the dispersion is not just the single value quoted above, but will be a wide range of dispersion values given by the complete dispersion curve, with the different dispersion values acting on different portions of the signal spectrum. So the dispersion itself will become increasing complex and distort the signal over even short distances. Of course as the length shrinks to zero there is no material so the exercise becomes irrelevant. There would be other practical issues to deal with like non-uniformity of loss over very wide bandwidths and the modulator and detector responses. But in principle over very short distances, even femtosecond (10^{-15}s) pulses are possible to transmit.