

rectangular array which is placed inside a polyethylene tube. The mechanical strength is provided by using steel rods in the two outermost polyethylene jackets. The outer diameter of such fiber cables is about 1–1.5 cm.

Connectors are needed to use optical fibers in an actual communication system. They can be divided into two categories. A permanent joint between two fibers is known as a fiber splice, and a detachable connection between them is realized by using a fiber connector. Connectors are used to link fiber cable with the transmitter (or the receiver), while splices are used to join fiber segments (usually 5–10 km long). The main issue in the use of splices and connectors is related to the loss. Some power is always lost, as the two fiber ends are never perfectly aligned in practice. Splice losses below 0.1 dB are routinely realized by using the technique of fusion splicing [93]. Connector losses are generally larger. State-of-the-art connectors provide an average loss of about 0.3 dB [94]. The technology behind the design of splices and connectors is quite sophisticated. For details, the reader is referred to Ref. [95], a book devoted entirely to this issue.

## Problems

- 2.1 A multimode fiber with a 50- $\mu\text{m}$  core diameter is designed to limit the intermodal dispersion to 10 ns/km. What is the numerical aperture of this fiber? What is the limiting bit rate for transmission over 10 km at 0.88  $\mu\text{m}$ ? Use 1.45 for the refractive index of the cladding.
- 2.2 Use the ray equation in the paraxial approximation [Eq. (2.1.8)] to prove that intermodal dispersion is zero for a graded-index fiber with a quadratic index profile.
- 2.3 Use Maxwell's equations to express the field components  $E_\rho$ ,  $E_\phi$ ,  $H_\rho$ , and  $H_\phi$  in terms of  $E_z$  and  $H_z$  and obtain Eqs. (2.2.29)–(2.2.32).
- 2.4 Derive the eigenvalue equation (2.2.33) by matching the boundary conditions at the core–cladding interface of a step-index fiber.
- 2.5 A single-mode fiber has an index step  $n_1 - n_2 = 0.005$ . Calculate the core radius if the fiber has a cutoff wavelength of 1  $\mu\text{m}$ . Estimate the spot size (FWHM) of the fiber mode and the fraction of the mode power inside the core when this fiber is used at 1.3  $\mu\text{m}$ . Use  $n_1 = 1.45$   $\mu\text{m}$ .
- 2.6 A 1.55- $\mu\text{m}$  unchirped Gaussian pulse of 100-ps width (FWHM) is launched into a single-mode fiber. Calculate its FWHM after 50 km if the fiber has a dispersion of 16 ps/(km-nm). Neglect the source spectral width.
- 2.7 Derive an expression for the confinement factor  $\Gamma$  of single-mode fibers defined as the fraction of the total mode power contained inside the core. Use the Gaussian approximation for the fundamental fiber mode. Estimate  $\Gamma$  for  $V = 2$ .
- 2.8 A single-mode fiber is measured to have  $\lambda^2(d^2n/d\lambda^2) = 0.02$  at 0.8  $\mu\text{m}$ . Calculate the dispersion parameters  $\beta_2$  and  $D$ .

- 2.9 Show that a chirped Gaussian pulse is compressed initially inside a single-mode fiber when  $\beta_2 C < 0$ . Derive expressions for the minimum width and the fiber length at which the minimum occurs.
- 2.10 Estimate the limiting bit rate for a 60-km single-mode fiber link at 1.3- and 1.55- $\mu\text{m}$  wavelengths assuming transform-limited, 50-ps (FWHM) input pulses. Assume that  $\beta_2 = 0$  and  $-20 \text{ ps}^2/\text{km}$  and  $\beta_3 = 0.1 \text{ ps}^3/\text{km}$  and 0 at 1.3- and 1.55- $\mu\text{m}$  wavelengths, respectively. Also assume that  $V_\omega \ll 1$ .
- 2.11 A 0.88- $\mu\text{m}$  communication system transmits data over a 10-km single-mode fiber by using 10-ns (FWHM) pulses. Determine the maximum bit rate if the LED has a spectral FWHM of 30 nm. Use  $D = -80 \text{ ps}/(\text{km}\cdot\text{nm})$ .
- 2.12 Use Eq. (2.4.23) to prove that the bit rate of an optical communication system operating at the zero-dispersion wavelength is limited by  $BL|S|\sigma_\lambda^2 < 1/\sqrt{8}$ , where  $S = dD/d\lambda$  and  $\sigma_\lambda$  is the RMS spectral width of the Gaussian source spectrum. Assume that  $C = 0$  and  $V_\omega \gg 1$  in the general expression of the output pulse width.
- 2.13 Repeat Problem 2.12 for the case of a single-mode semiconductor laser for which  $V_\omega \ll 1$  and show that the bit rate is limited by  $B(|\beta_3|L)^{1/3} < 0.324$ . What is the limiting bit rate for  $L = 100 \text{ km}$  if  $\beta_3 = 0.1 \text{ ps}^3/\text{km}$ ?
- 2.14 An optical communication system is operating with chirped Gaussian input pulses. Assume that  $\beta_3 = 0$  and  $V_\omega \ll 1$  in Eq. (2.4.23) and obtain a condition on the bit rate in terms of the parameters  $C$ ,  $\beta_2$ , and  $L$ .
- 2.15 A 1.55- $\mu\text{m}$  optical communication system operating at 5 Gb/s is using Gaussian pulses of width 100 ps (FWHM) chirped such that  $C = -6$ . What is the dispersion-limited maximum fiber length? How much will it change if the pulses were unchirped? Neglect laser linewidth and assume that  $\beta_2 = -20 \text{ ps}^2/\text{km}$ .
- 2.16 A 1.3- $\mu\text{m}$  lightwave system uses a 50-km fiber link and requires at least  $0.3 \mu\text{W}$  at the receiver. The fiber loss is 0.5 dB/km. Fiber is spliced every 5 km and has two connectors of 1-dB loss at both ends. Splice loss is only 0.2 dB. Determine the minimum power that must be launched into the fiber.
- 2.17 A 1.55- $\mu\text{m}$  continuous-wave signal with 6-dBm power is launched into a fiber with  $50\text{-}\mu\text{m}^2$  effective mode area. After what fiber length would the nonlinear phase shift induced by SPM become  $2\pi$ ? Assume  $\bar{n}_2 = 2.6 \times 10^{-20} \text{ m}^2/\text{W}$  and neglect fiber losses.
- 2.18 Calculate the threshold power for stimulated Brillouin scattering for a 50-km fiber link operating at 1.3  $\mu\text{m}$  and having a loss of 0.5 dB/km. How much does the threshold power change if the operating wavelength is changed to 1.55  $\mu\text{m}$ , where the fiber loss is only 0.2 dB/km? Assume that  $A_{\text{eff}} = 50 \mu\text{m}^2$  and  $g_B = 5 \times 10^{-11} \text{ m/W}$  at both wavelengths.
- 2.19 Calculate the power launched into a 40-km-long single-mode fiber for which the SPM-induced nonlinear phase shift becomes  $180^\circ$ . Assume  $\lambda = 1.55 \mu\text{m}$ ,  $A_{\text{eff}} = 40 \mu\text{m}^2$ ,  $\alpha = 0.2 \text{ dB/km}$ , and  $\bar{n}_2 = 2.6 \times 10^{-20} \text{ m}^2/\text{W}$ .

- 2.20** Find the maximum frequency shift occurring because of the SPM-induced chirp imposed on a Gaussian pulse of 20-ps width (FWHM) and 5-mW peak power after it has propagated 100 km. Use the fiber parameters of the preceding problem but assume  $\alpha = 0$ .

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