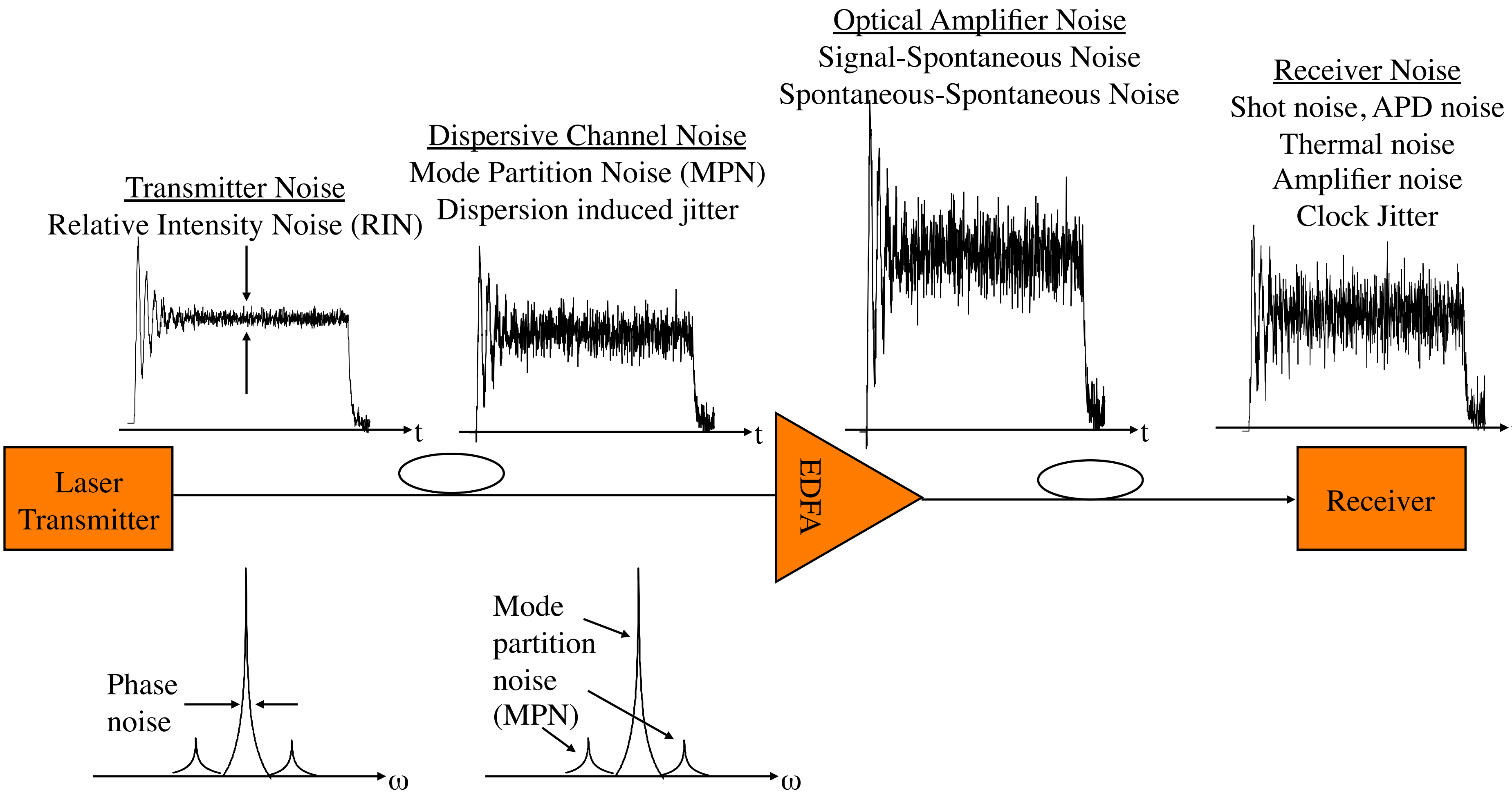
Lecture 15: Receiver Design

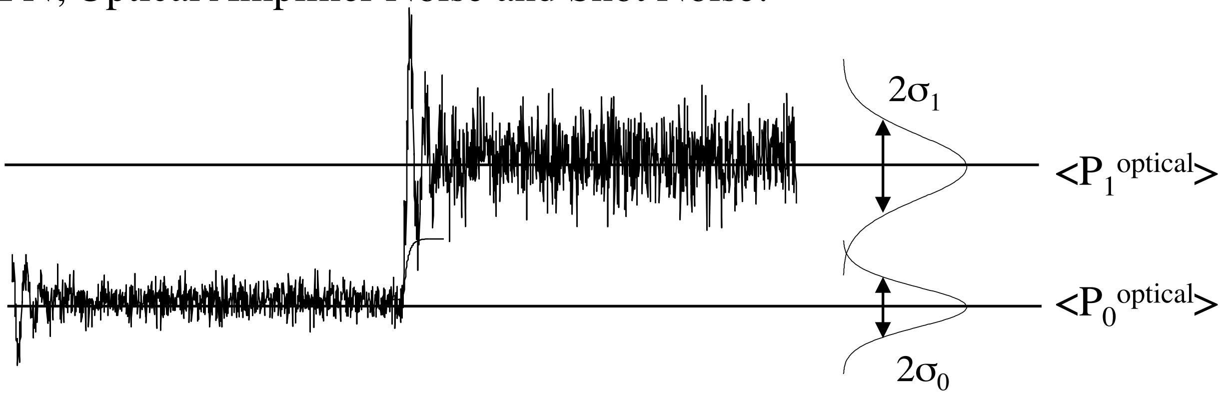
Receiver Design

Signal-to-Noise Ratio (SNR)



Optical Signal-to-Noise Ratio (OSNR)

- Noise is accumulated in the optical channel due to
  - RIN, MPN, Optical Amplifier Noise and Shot Noise.



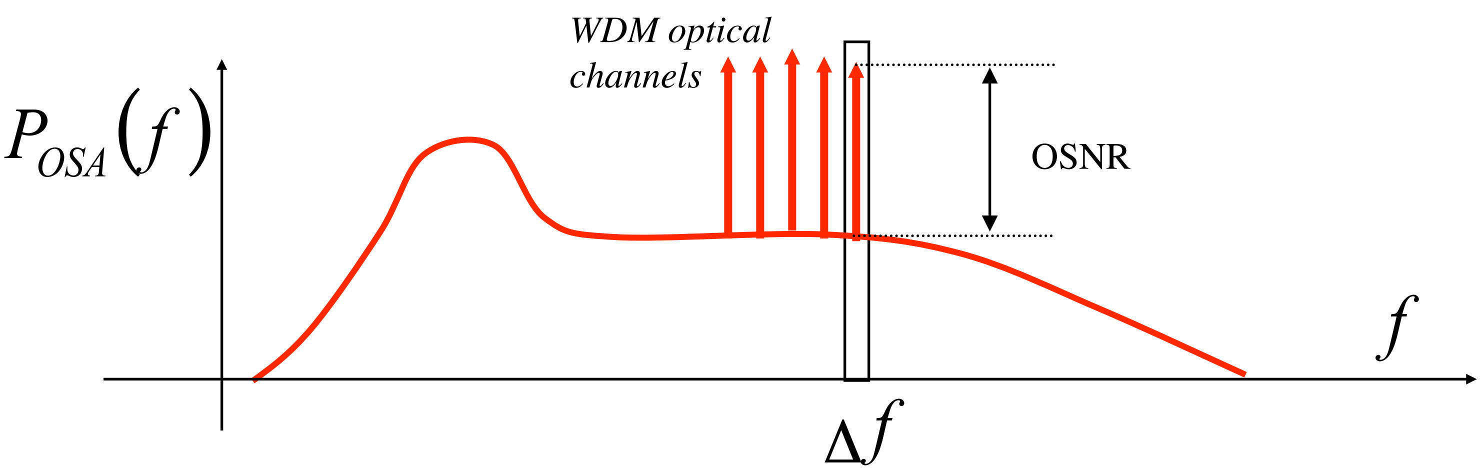
- OSNR for each level and for complete signal can be defined

$$OSNR_1 = \frac{\left\langle P_1^{Optical} \right\rangle^2}{\sigma_1^{\,2}}$$

$$OSNR_0 = \frac{\left\langle P_0^{Optical} \right\rangle^2}{\sigma_0^{\,2}}$$

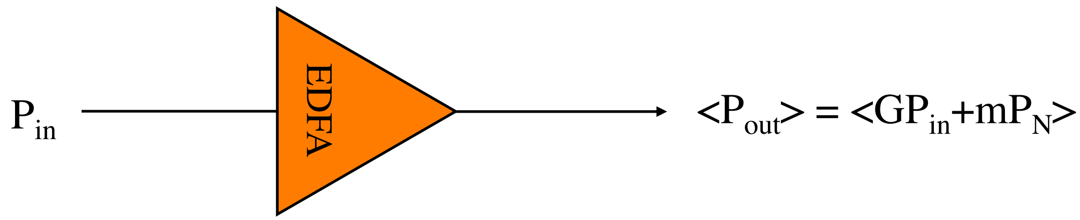
Optical Signal-to-Noise Ratio (OSNR)

- OSNR is an extremely important parameter in optically amplified systems
- A poor OSNR cannot in principle be improved at the receiver
- It is mainly determined by:
  - Useful signal level
  - ASE noise level
- OSNR is typically measured using an Optical Spectrum Analyzer (OSA)
  - The resulting quantities are thus time averaged
- The OSNR is defined on a given resolution bandwidth Δf (an example standard requires 0.1 nm =12.5 GHz)



Optical Amplifier OSNR

- The signal at the output of an optical amplifier in response to a noise free signal at the input is



- The following formulation accounts for all noise terms that can be treated as Gaussian noise due to the optical amplifier

$$P_N = m\, n_{sp} h\nu (G-1) B_{opt}$$

- G = amplifier gain
- n_{sp} = spontaneous emission factor
- m = number of polarization modes (1 or 2)
- P_N = mean noise in bandwidth B_{opt}

OSNR at the output of EDFA

- The optical OSNR on a 0.1 nm band around 1550 nm, at the output of an EDFA, is approx. given by:

$$OSNR \cong P_{signal}^{in} - F_{EDFA} + 58dB$$

- It is thus determined ONLY by:
  - The optical input power for the useful signal
  - The EDFA noise figure
- Typical values
  - P_{in}=-35dBm
  - F=5 dB
  - $OSNR$=-35-5+58=18 dB
  - This is the typical OSNR required at the receiver for a 10 Gbit/s system

Optical Amplifier Noise Figure

At the amplifier output

$$SNR_{out} = \frac{P_{in}}{P_{ASE}^{Total}}$$

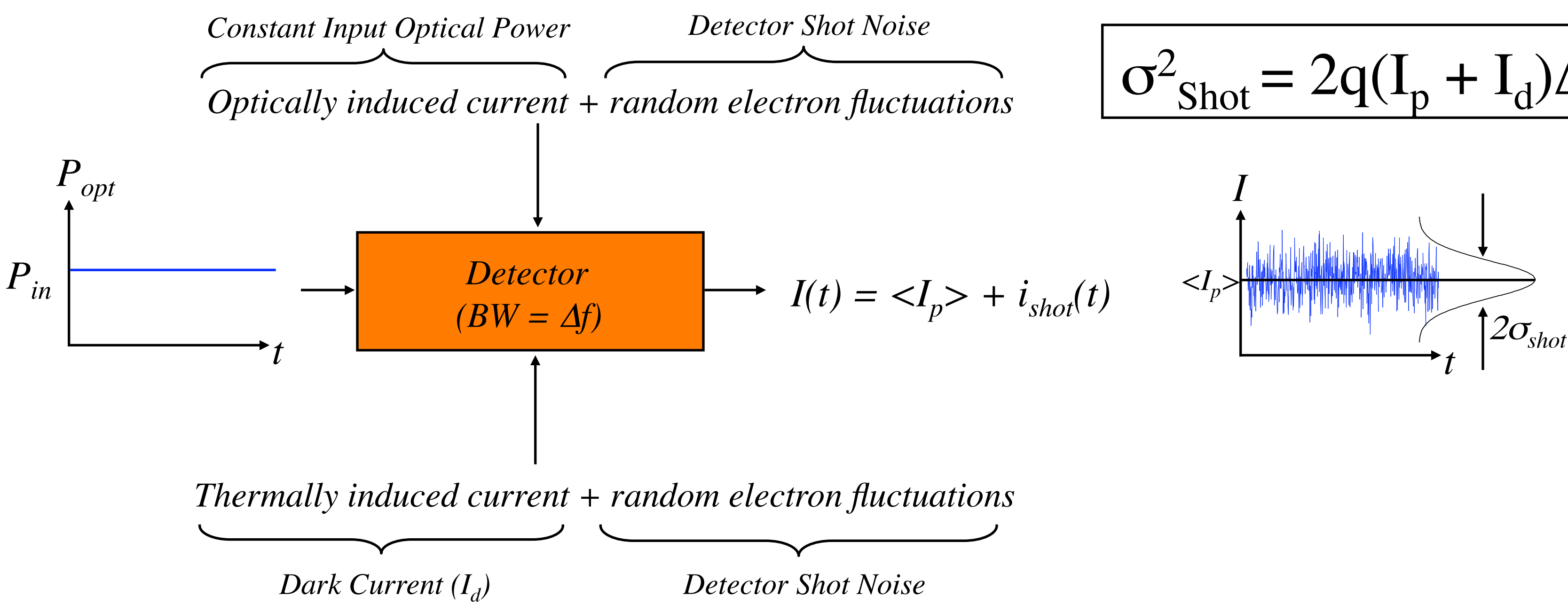
Amplifier Noise Figure (F_N)

$$F_N = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{in}^2}{\sigma_{in}^2} \frac{\sigma_{out}^2}{P_{out}^2}$$

$$\approx 2n_{sp} \text{ for } G \gg 1$$

Electrical Shot Noise

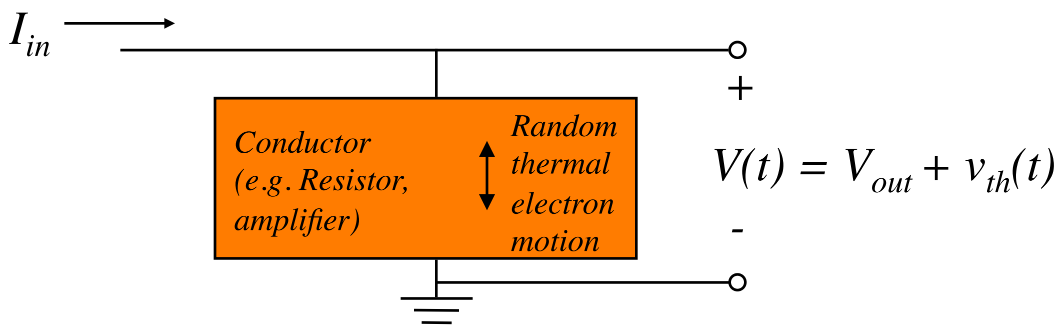
- The shot noise generated in the photodetection process is physically due to the "quantum granularity" of the received (and photo converted) optical signal
- It sets the ultimate limit of an optical receiver (only in theory, as shown later)
- It is a Poisson noise, but it is usually approximated as a Gaussian noise



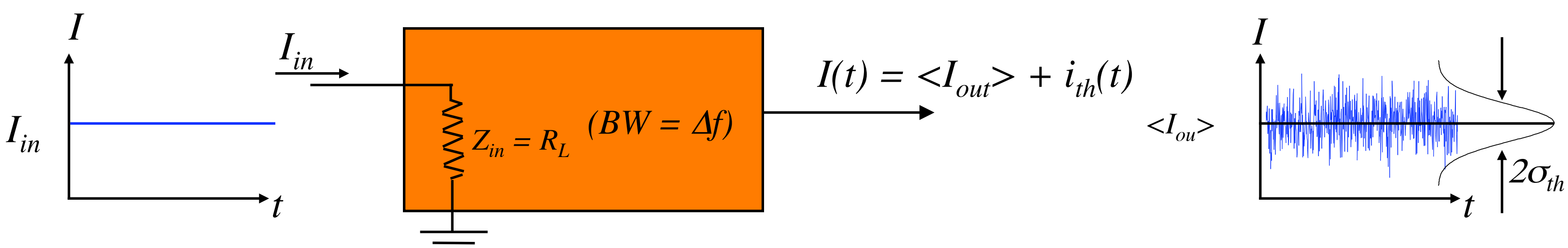
$$\sigma^2_{Shot} = 2q(I_p + I_d)\Delta f$$

Thermally Generated Noise

- Noise generated by any electrical component due to the thermal motion of electrical carriers inside conductive media
- It is a Gaussian noise source



$$\sigma^2_{th} = 4\, k_B T R_L \Delta f$$



Amplifier Noise

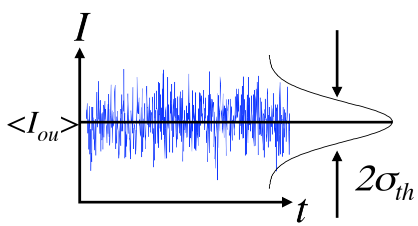
- The amplifier enhances the thermal noise at the input by a factor called the amplifier “Noise Figure

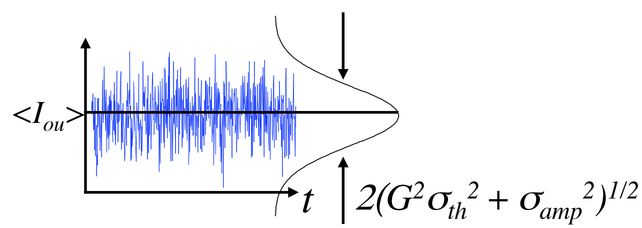
$SNR_{in} = \langle I_{in}\rangle^2/\sigma^2_{th}$

$SNR_{out} = \langle I_{out}\rangle^2/\sigma^2_{out}$

$I_{in}(t) = \langle I_{in}\rangle + i_{th}(t)$ → **Amplifier** *(BW = Δf)* *(Gain = G)* →

$$
\begin{aligned}
I_{out}(t) &= G\,[\langle I_{in}\rangle + i_{in}(t)] + i_{amp}(t) \\
&= G\,\langle I_{in}\rangle + [G\, i_{in}(t) + i_{amp}(t)] \\
&= \langle I_{out}\rangle + i_{out}(t)
\end{aligned}
$$





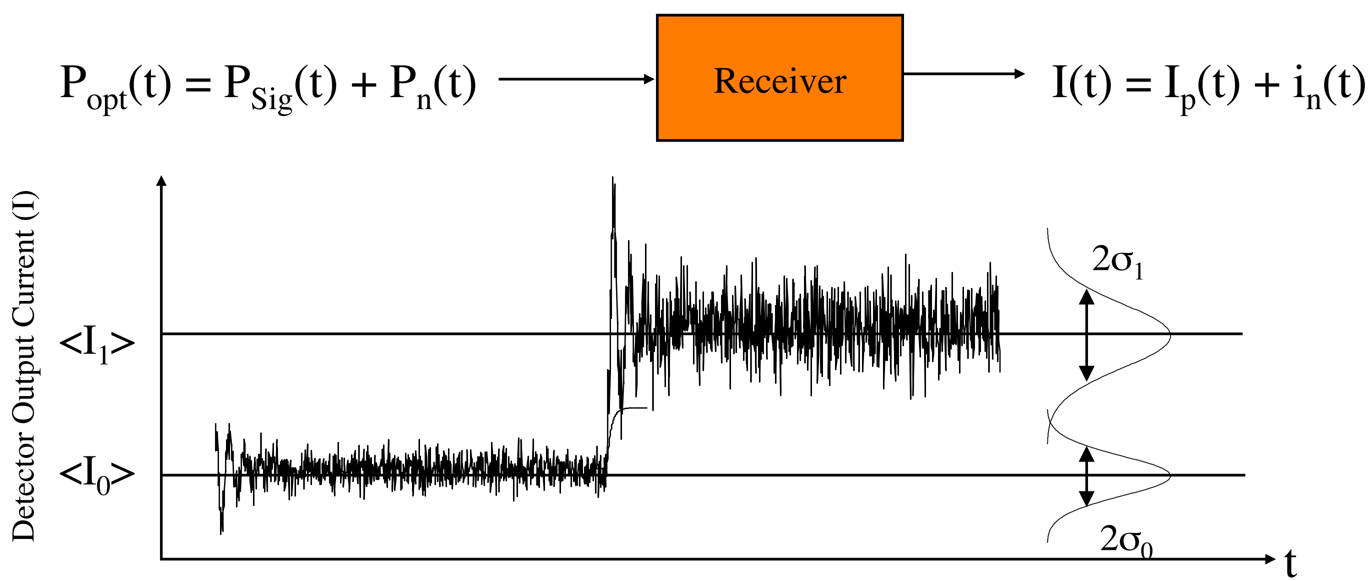
$$F_n = (SNR_{in})/(SNR_{out}) = \left(\langle I_{in}\rangle^2/\sigma^2_{th}\right)/\left(G^2 \langle I_{in}\rangle^2/(G^2\sigma_{th}^2 + G^2\sigma_{eff}^2)\right) = \sigma^2_{out}/\sigma^2_{th}$$

$$\sigma^2_{out} = \sigma^2_{th} F_n$$

$$\sigma^2_{out} = 4\, k_B T R_L F_n \Delta f$$

Electrical Signal-to-Noise Ratio (SNR)

- At the receiver, there is noise on the signal arriving at the input and and after detection added to that is noise that is injected at various stages of the receiver
  - The current output of the receiver $i_n(t)$ has current contributions from
    - Electrical shot noise
    - Thermal noise
    - APD detectors have additional multiplication noise
    - Amplifier noise



Electrical Signal-to-Noise Ratio (SNR)

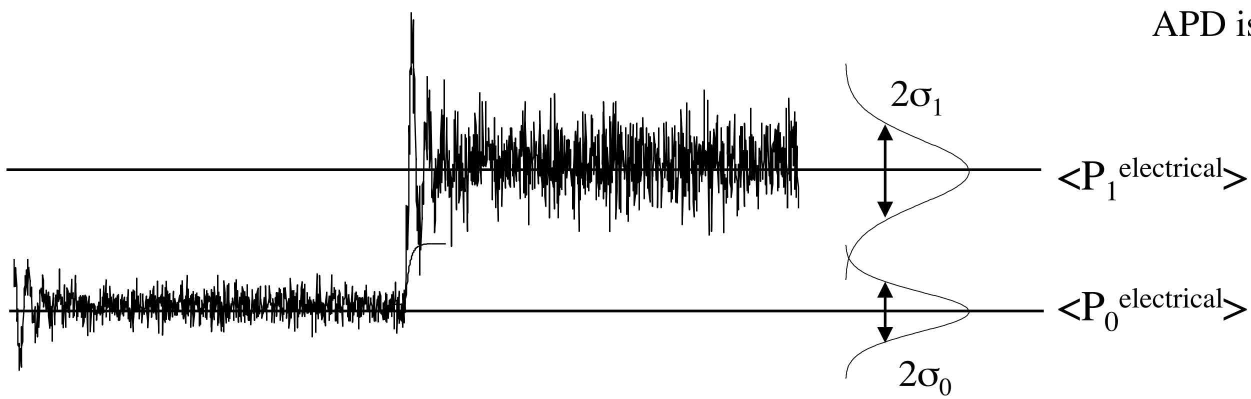
- At the receiver, there is noise on the signal arriving at the input and there is noise that is injected at various stages of the receiver

$P_{opt}(t) = P_{Sig}(t) + P_n(t)$ → **Receiver** → $I(t) = I_p(t) + i_n(t)$

$\sigma^2_{RIN} + \sigma^2_{MPN} + \sigma^2_{BEAT} + \sigma^2_{SIG\text{-}SPONT}$

$\sigma^2_{Thermal} + \sigma^2_{Shot} + \sigma^2_{BEAT} + \sigma^2_{SIG\text{-}SPONT}$

For a pin detector. Replace with σ_{APD} if an APD is used



SNR and system performance

- The resulting global electrical SNR at the receiver determines the performance of a system
- We show in the following slides
  - The SNR for different systems, assuming constant (non-modulated) input power
  - Starting from the SNR formulas, we derive the expression for the BER of a digital system

SNR in pin Receivers

- SNR in pin receivers, without optical amplification

$$\sigma^2_{Thermal} + \sigma^2_{Shot}$$

$$SNR = \frac{\text{Average signal power}}{\text{Noise power}} = \frac{I_p^2}{\sigma^2}$$

$$= \frac{\Re^2 P_{in}^2}{2q(\Re P_{in} + I_D)\Delta f + 4(\kappa_B T / R_L) F_n \Delta f}$$

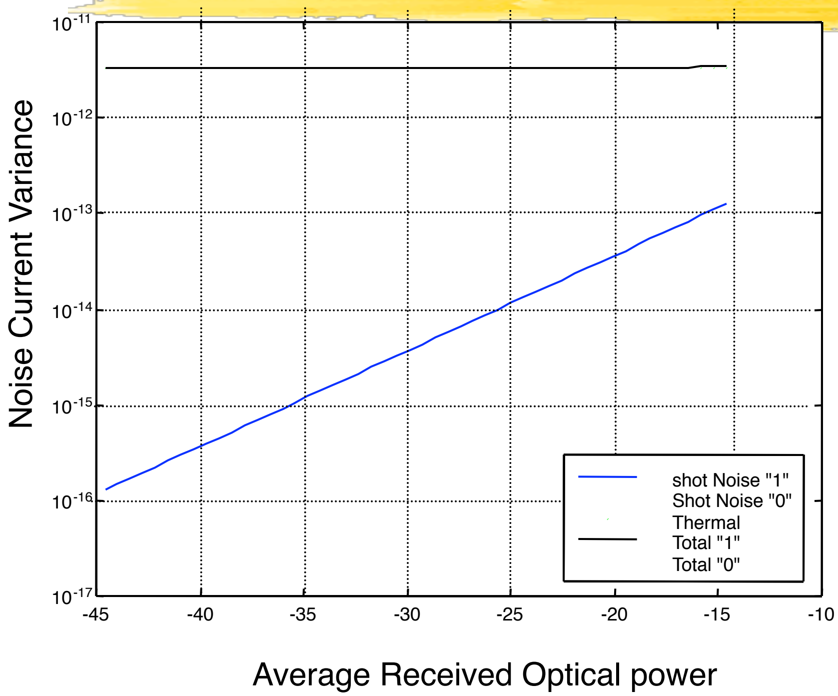
Detector Dark Current

Thermal noise

Electrical Amplifier Noise Figure

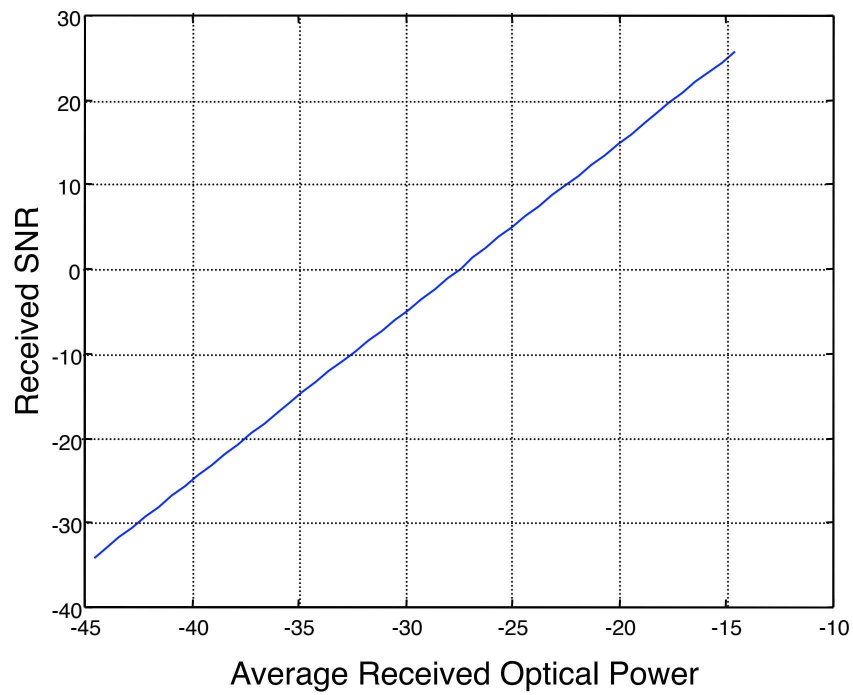
Receiver electrical bandwidth

SNR in pin Receivers



Bit Rate=10Gb/s

I_d =1nA
F_{ne} = 3dB



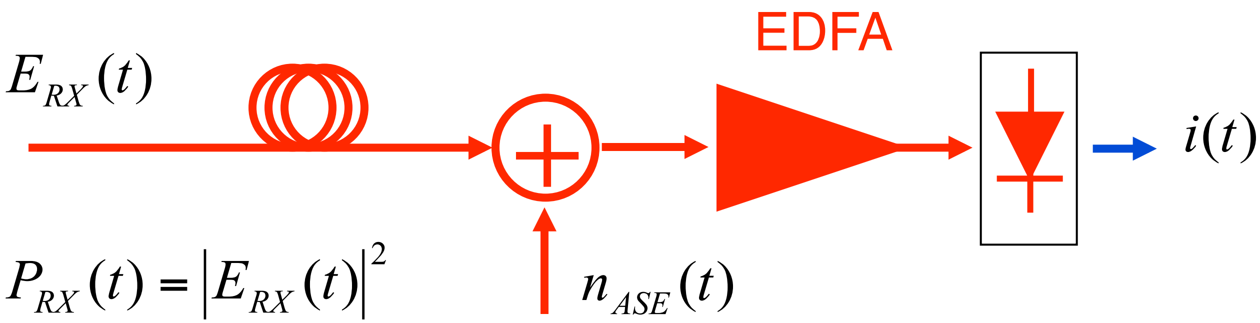
- Note that without optical amplification, the shot noise variance is well below the thermal noise variance
- This regime of operation is called **"thermal noise limited detection"**

SNR in Optically Preamplified Receivers (OPRs)

- The ASE noise levels on the electrical photodetection signal combines with all the electrical noise levels
  - The resulting equations for the resulting global electrical SNR are quite complex
  - Still, in most practical situations, only one noise source determines the system performance
  - We decided to skip the equations, but to show in the next slides the numerical results in practical situations

SNR due to Optical Amplifier ASE noise

- Effects of ASE noise (neglecting other noise sources)



$$E(t) = E_{RX}^F(t) + n_{ASE}^F(t) \quad \text{where: } n_{ASE}^F(t) = p_{ASE}^F(t) + jq_{ASE}^F(t)$$

$$i(t) = R|E(t)|^2 = R\left|E_{RX}^F(t) + p_{ASE}^F(t) + jq_{ASE}^F(t)\right|^2 =$$

$$= R\left(\underbrace{\left|E_{RX}^F(t)\right|^2}_{\text{Useful signal}} + \underbrace{2\cdot E_{RX}^F(t)\cdot p_{ASE}^F(t)}_{\text{S x N beating}} + \underbrace{\left(p_{ASE}^F(t)\right)^2}_{\text{N x N beating}} + \underbrace{\left(q_{ASE}^F(t)\right)^2}_{\text{N x N beating}}\right)$$

Useful signal | S x N beating | N x N beating | N x N beating

SNR in OPRs

$$\sigma^2_{Thermal} + \sigma^2_{Shot} + \sigma^2_{Spont\text{-}Spont} + \sigma^2_{Sig\text{-}Spont}$$

$$\sigma^2_{SHOT} = 2q\left[\Re(GP_S + P_{SP}) + I_d\right]\Delta f$$

$$\sigma^2_{SP-SP} = 4\Re^2 S^2_{SP}\Delta\nu_{opt}\Delta f$$

$$\sigma^2_{Sig-SP} = 4\Re^2 G P_S S_{SP}\Delta f$$

$$\sigma^2_{SHOT-SP} = 4q\Re S_{SP}\Delta\nu_{opt}\Delta f$$

$$SNR = \frac{\text{Average signal power}}{\text{Noise power}} = \frac{I_p^2}{\sigma^2}$$

$$= \frac{\Re^2 P_{in}^2}{2q(\Re P_{in} + I_D)\Delta f + 4(\kappa_B T / R_L) F_n \Delta f}$$

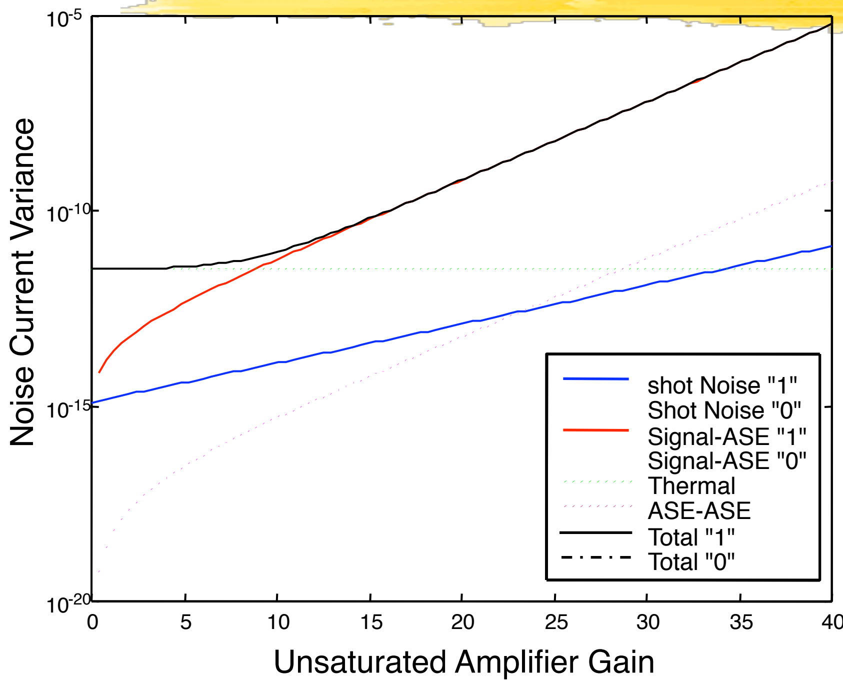
Detector Dark Current

Thermal noise

Electrical Amplifier Noise Figure

Receiver electrical bandwidth

SNR in OPRs



B=10Gbps

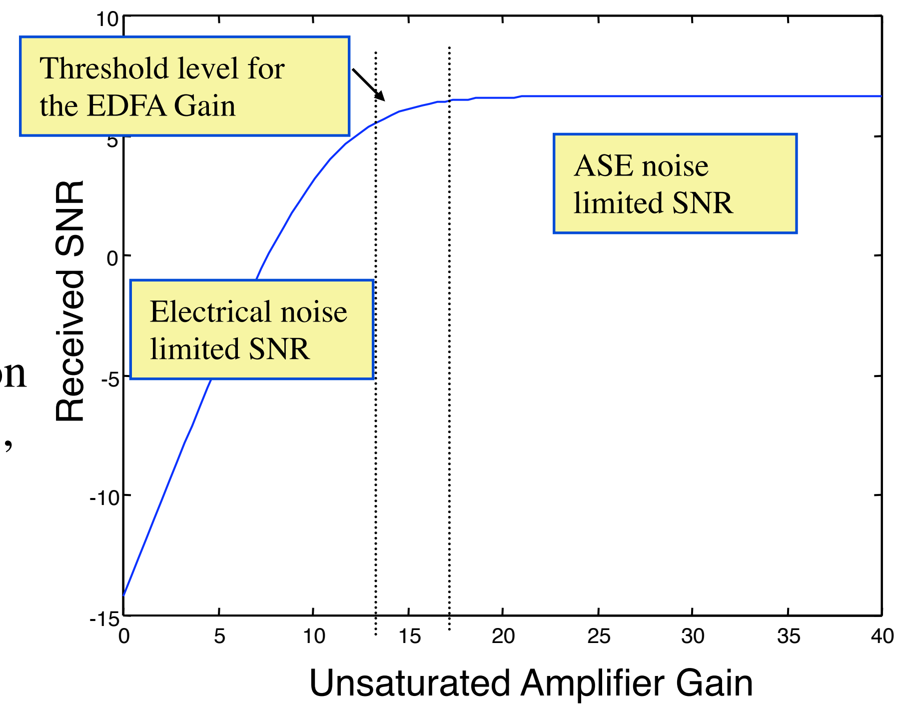
P_1 =-20dBm

P_0 = -30 dBm

I_d =1nA

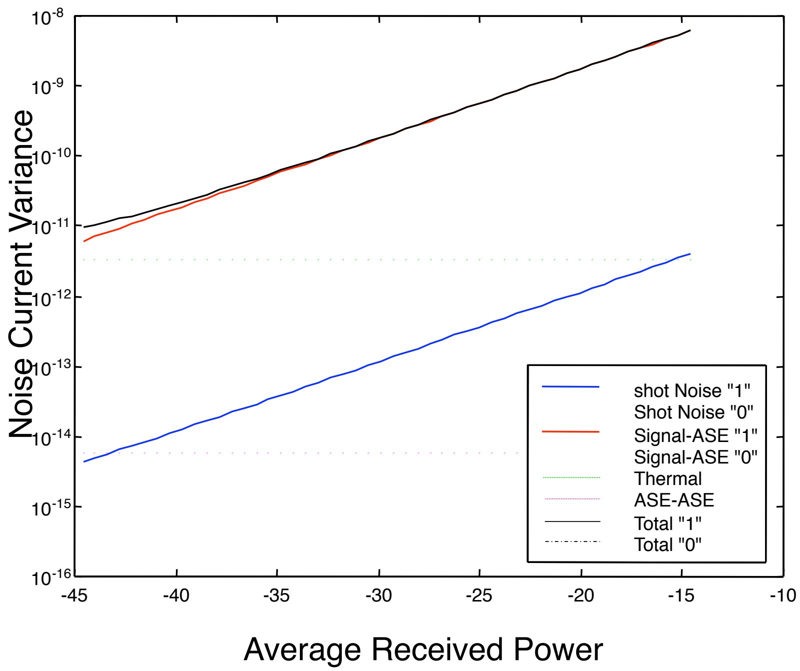
F_{ne} = 3dB

F_{nedfa} = 5dB



- Note that in any realistic situation, as soon as the EDFA gain is above a certain level, all electrical noises are negligible
- Thus, in an optically amplified systems, electrical noises are negligible in most cases

SNR in OPRs



L=60km

α=0.2dB/km

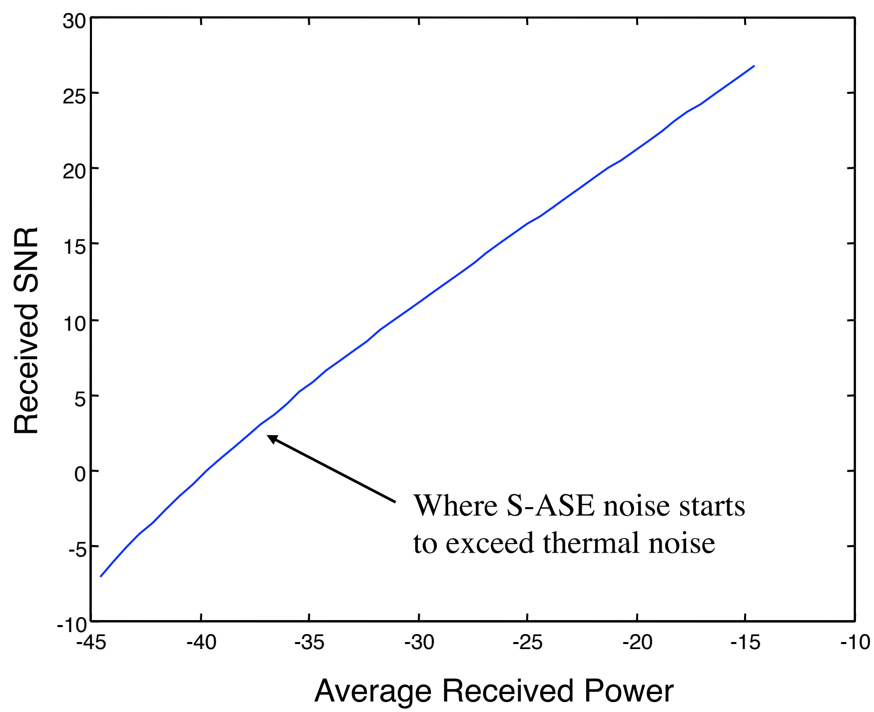
G_0 =15 dB

B=10Gbps

I_d =1nA

F_{ne} = 3dB

F_{nedfa} = 5dB

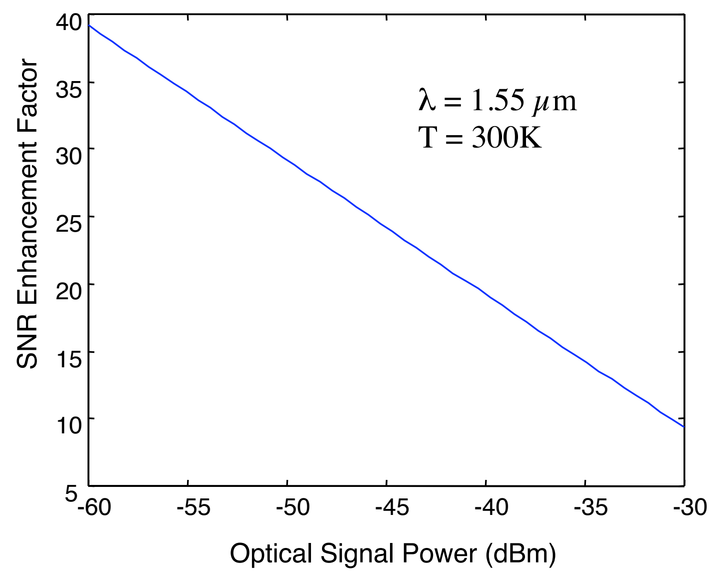


SNR Enhancement in OPRs

- The detected SNR can be enhanced using an optically pre-amplified receiver
- The OPR always degrades the SNR by a minimum of 3dB (i.e. F_n = 3dB)
- Yet there will be an improvement in the electrical SNR if
  - Thermal noise is present in the receiver and
  - The optical signal level is relatively high compared to the ASE noise power

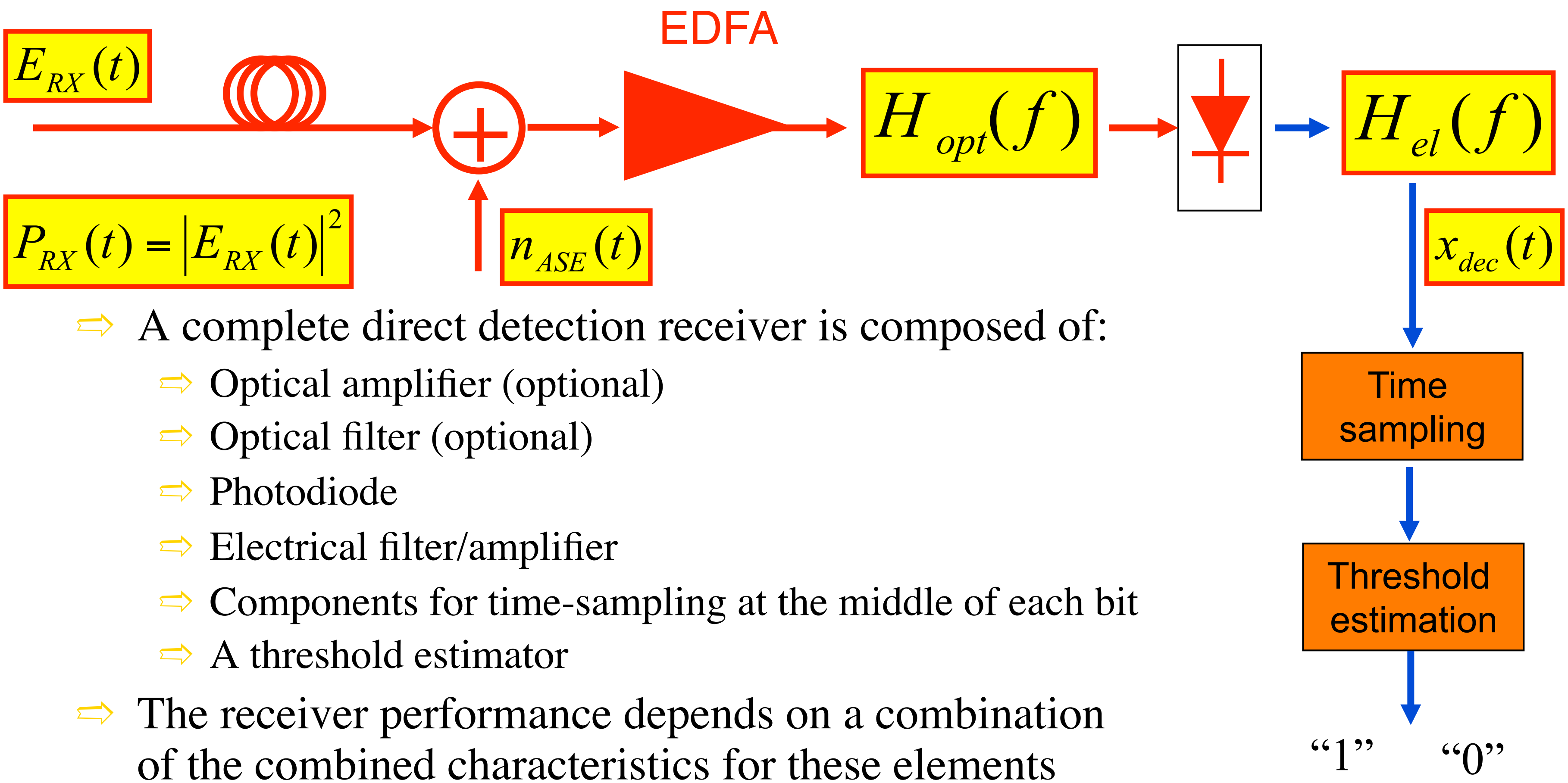
The SNR enhancement factor is given be the ratio of the SNR using an optical preamplifier (SNR_e^{opt}) to the SNR without optical preamplification (SNR_e) assuming large G and $P_s >> P_{ASE}$†

$$x = \frac{SNR_e^{OPR}}{SNR_e} = \frac{1}{2\eta n_{sp}}\left(1 + \frac{2k_B T h \nu_s}{R\eta q^2}\frac{1}{P_s}\right)$$



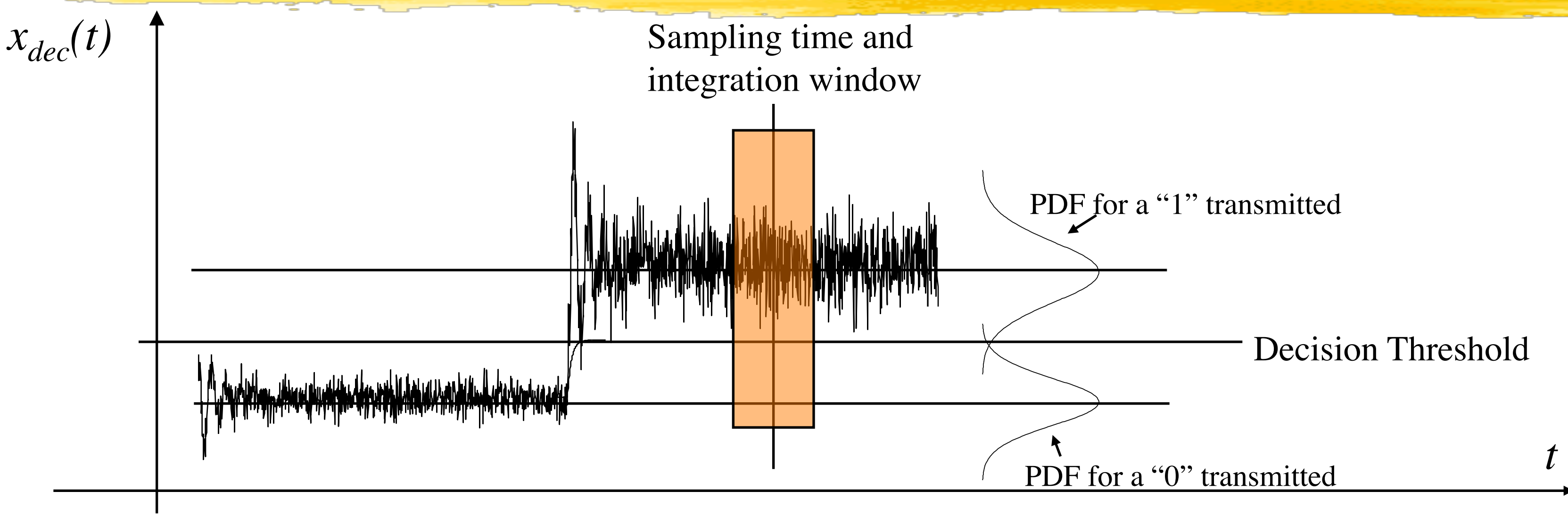
† Erbium Doped Fiber Amplifiers, E. Desurvire, Wiley-Interscience

Direct-Detection (DD) receivers



- A complete direct detection receiver is composed of:
  - Optical amplifier (optional)
  - Optical filter (optional)
  - Photodiode
  - Electrical filter/amplifier
  - Components for time-sampling at the middle of each bit
  - A threshold estimator
- The receiver performance depends on a combination of the combined characteristics for these elements

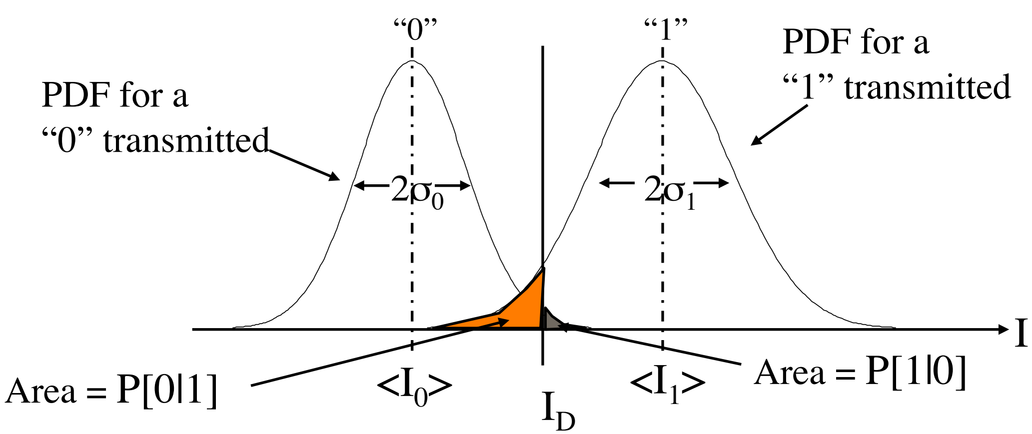
Data Recovery: DD Receivers



- The bit error rate (BER) of the system depends from the statistics of the resulting noise on the "1" and "0" levels

Bit Error Rate (BER)

- Probability of error = *P[0]P[1|0] + P[1]P[0|1]*
  - *P[0]* = Probability a "0" was transmitted
  - *P[1]* = Probability a "1" was transmitted
  - *P[1|0]* = Probability a "1" is received given that a "0" is transmitted
  - *P[0|1]* = Probability a "0" is received given that a "1" is transmitted



Under the gaussian assumption:

$$P[1\,|\,0] = \frac{1}{\sigma_0\sqrt{2\pi}}\int_{I_D}^{\infty} \exp\left\{\frac{\left(\langle I_0\rangle - I\right)^2}{2\sigma_0^2}\right\} dI$$

$$P[0\,|\,1] = \frac{1}{\sigma 1\sqrt{2\pi}}\int_{\infty}^{I_D} \exp\left\{\frac{\left(\langle I_1\rangle - I\right)^2}{2\sigma_1^2}\right\} dI$$

BER and Q-Factor

Substituting

$$Q_0 = \frac{I_D - \langle I_0 \rangle}{\sigma_0}$$

$$Q_1 = \frac{I_D - \langle I_1 \rangle}{\sigma_1}$$

$$P[1|0] = \frac{1}{\sqrt{2\pi}} \int_{Q_0}^{\infty} \exp\left\{-\frac{I^2}{2}\right\} dI$$

$$P[0|1] = \frac{1}{\sqrt{2\pi}} \int_{Q1}^{\infty} \exp\left\{-\frac{I^2}{2}\right\} dI$$

The "near" optimum decision threshold is $$I_D = \frac{\sigma_0 \langle I_1 \rangle + \sigma_1 \langle I_0 \rangle}{\sigma_0 + \sigma_1}$$

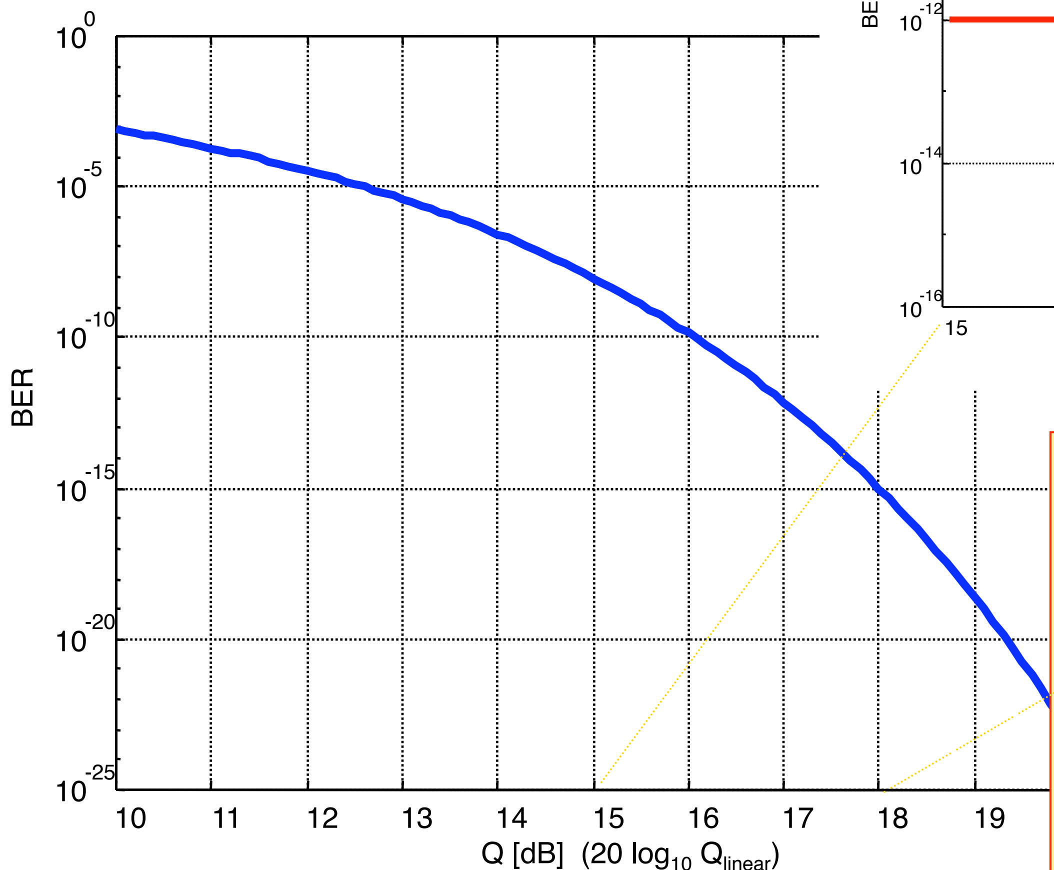
Defining the Q factor $$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}$$

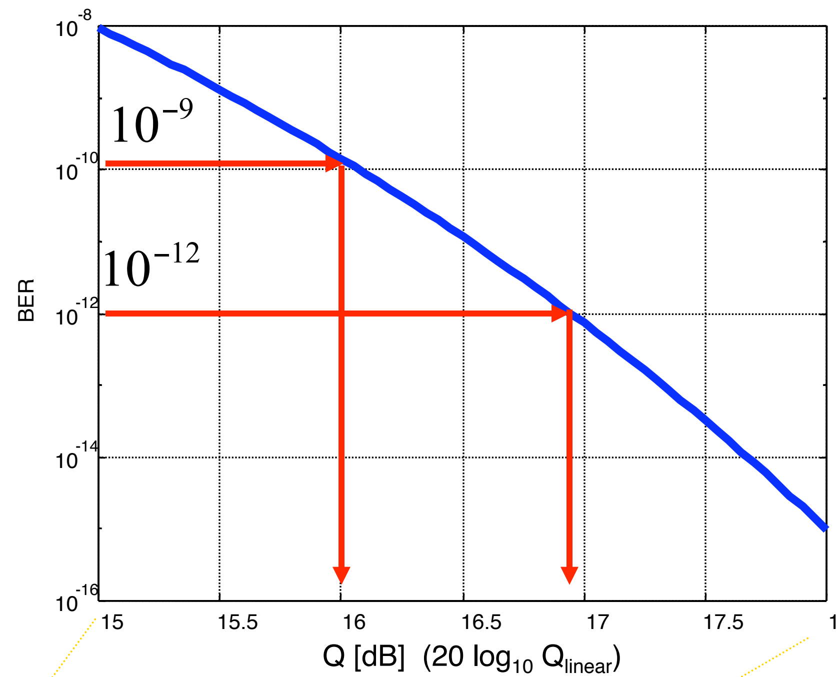
- BER is the most important performance indicator of a receiver
- Q-factor is a good indicator

The bit error rate (BER) assuming Gaussian noise can be written as

$$BER \cong \frac{1}{2} erfc\left(\frac{Q}{\sqrt{2}}\right) \approx \frac{\exp\left(-Q^2/2\right)}{Q\sqrt{2\pi}}$$

BER vs. Q-Factor

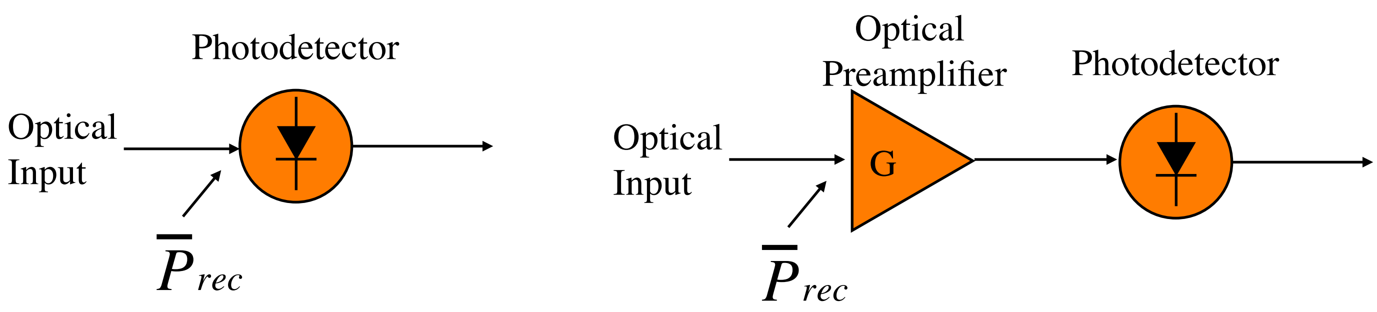




Important values:
- Q~16 dB -> BER=10^{-9}
- Q~17.0 dB-> BER=10^{-12}
- slope: 2 dpades per dB around BER=10^{-9} (slope increases when increasing reference BER)

Receiver Sensitivity

Define: Receiver Sensitivity is the minimum average power needed to achieve a certain BER at a given bit-rate. The receiver sensitivity is measure at the receiver input



The receiver sensitivity is expressed as an average received power

$$\overline{P}_{rec} = \frac{P_1 + P_0}{2}$$

Receiver Sensitivity

For a given Q *(BER)*, the minimum average received power can be found by solving for $\overline{P}_{rec}$ from σ_0 and σ_1 †

$$\overline{P}_{rec} = Q^2 h\nu_s B_e \left\{ F_0 + \frac{1}{Q}\sqrt{Mn_{sp}^2\left(2\frac{B_0}{B_e} - 1\right) + Mn_{sp}\frac{2}{\eta G}\frac{B_0}{B_e} + \frac{4k_BT}{Rq^2\eta^2G^2B_e}} \right\}$$

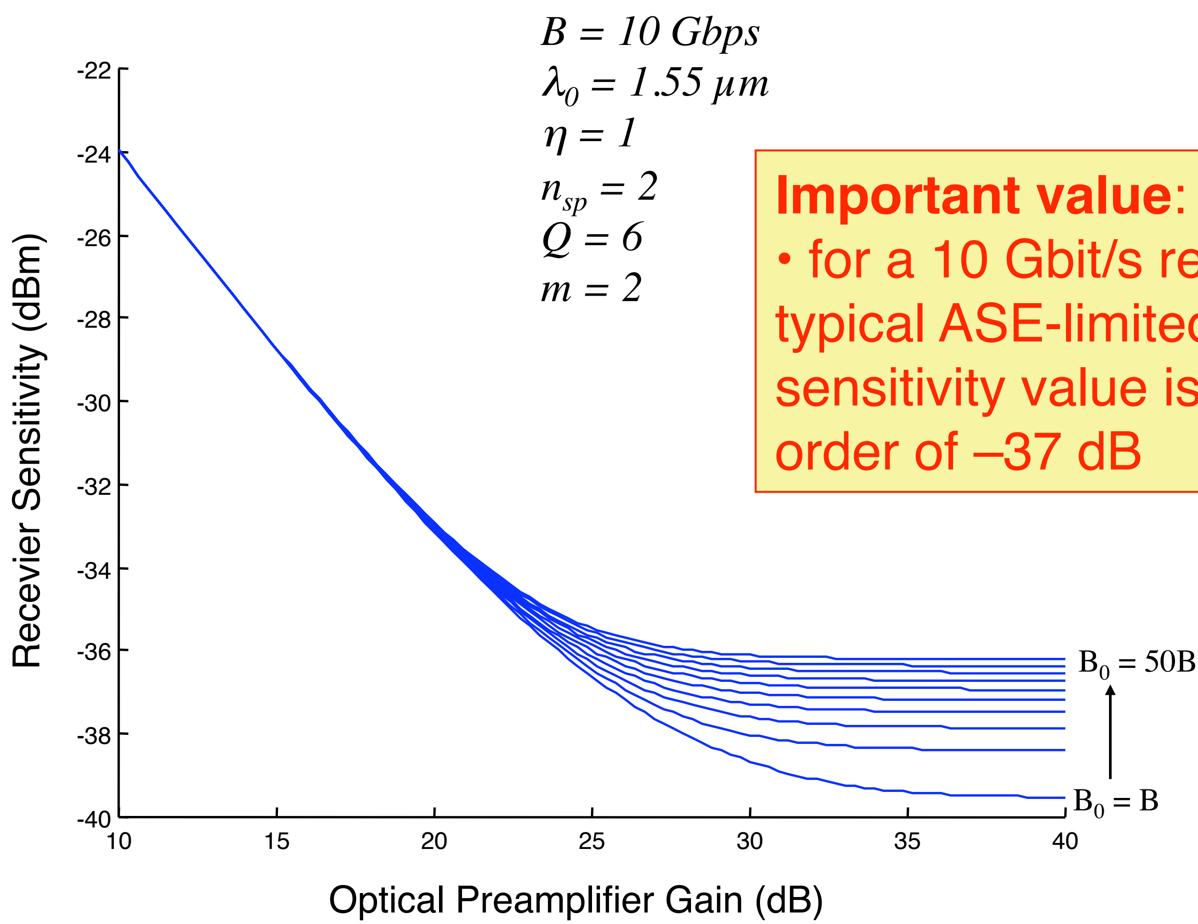
$$F_0 = \frac{1/\eta + 2n_{sp}G}{G}$$

For high G, $B_e = B_0/2$, and $Q = 6 = 16dB \rightarrow BER = 10^{-9}$ (M=2 for all polarization states)

$$\overline{P}_{rec} = 18 n_{sp} h\nu_s B_0 \left(2 + \sqrt{\frac{M}{12}}\right)$$

† Erbium Doped Fiber Amplifiers, E. Desurvire, Wiley-Interscience

Receiver Sensitivity

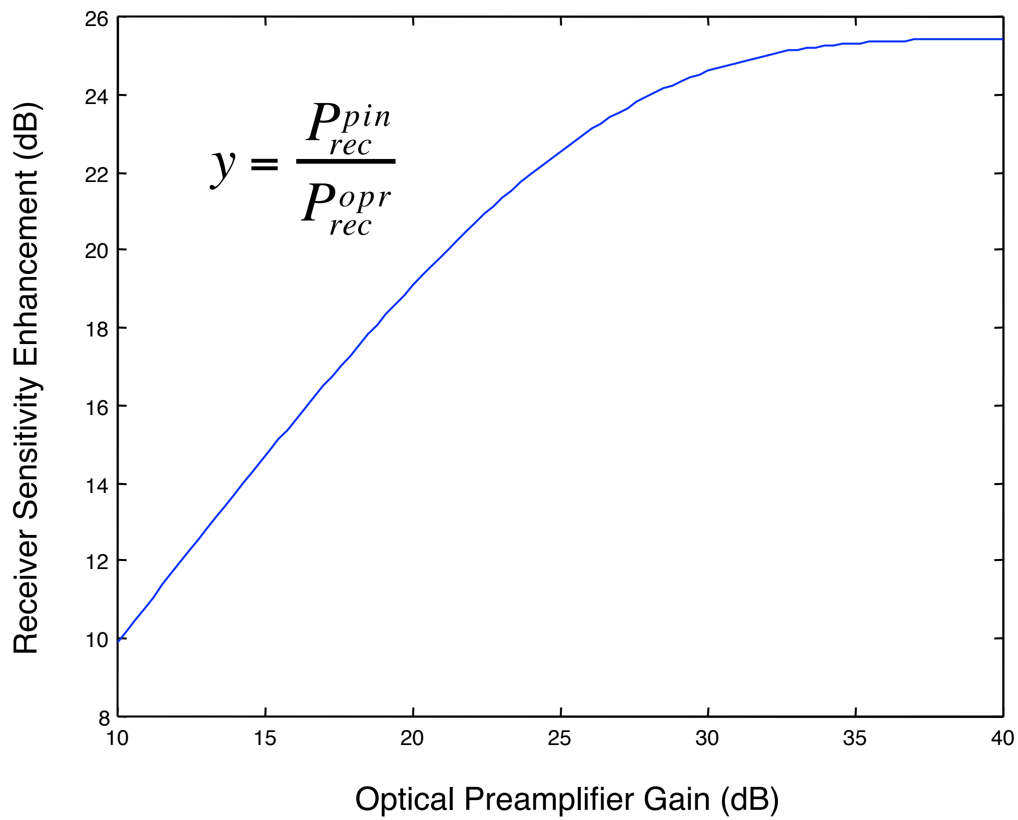


Important value:

- for a 10 Gbit/s receiver, the typical ASE-limited sensitivity value is of the order of –37 dB

Receiver Sensitivity Enhancement

Similar to SNR enhancement, we can define the improvement in Receiver Sensitivity of an optically preamplified receiver relative to a non-amplified pin receiver†



† Erbium Doped Fiber Amplifiers, E. Desurvire, Wiley-Interscience

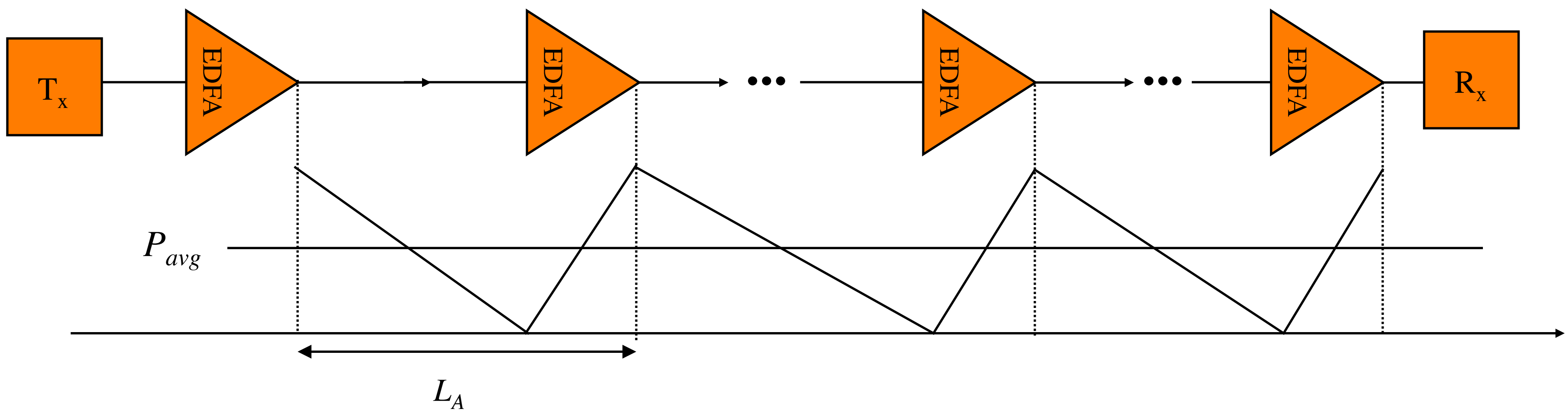
Summary on receiver sensitivity

The typical receiver sensitivities for a 10 Gbit/s system are:

- Theoretical quantum limit, direct detection, no optical amplification, shot noise limited
  - Sensitivity= - 45 dBm (approx, and never achieved in practice)
- Direct detection, no optical amplification, thermal noise limited
  - Sensitivity= -20 dBm (on the best available commercial receivers)
- Optically pre-amplified direct detection receiver
  - Sensitivity= -37 dBm (with the best available commercial receivers)

Long-haul optically amplified systems

To balance loss and gain: $e^{-\alpha L_A} = 1/G$



- Long haul optically amplified links are designed so that the EDFA gain exactly compensated the span loss
  - It is sometimes called the "transparency condition"
- In these systems, the only relevant noise effect is the accumulation of ASE noise introduced by each EDFA
  - Receiver electrical noise is usually negligible

OSNR after a chain of EDFA

- The OSNR at the output of these systems is approximately given by:

$$OSNR \cong P_{EDFA}^{out} - \alpha_{span} - 10\log_{10} N_{span} - F_{EDFA} + 58dB$$

- Where:
  - P_{out}^{EDFA} is EDFA the signal output power
  - α_{span} is the loss per span
  - N_{span} is the total number of spans
- Example: Trans-Pacific link, 8000 km
  - $P_{out}^{EDFA} = 0$ dBm (power per channel)
  - 50 km spans, α_{span} =12 dB
  - N_{span} =160
  - F_{EDFA}=5dB

$$\Rightarrow OSNR \cong 18dB$$

Important: the OSNR
- **increases with**
  - **the signal output power of the EDFA**
- **Decreases with**
  - **Span loss**
  - **Number of spans**