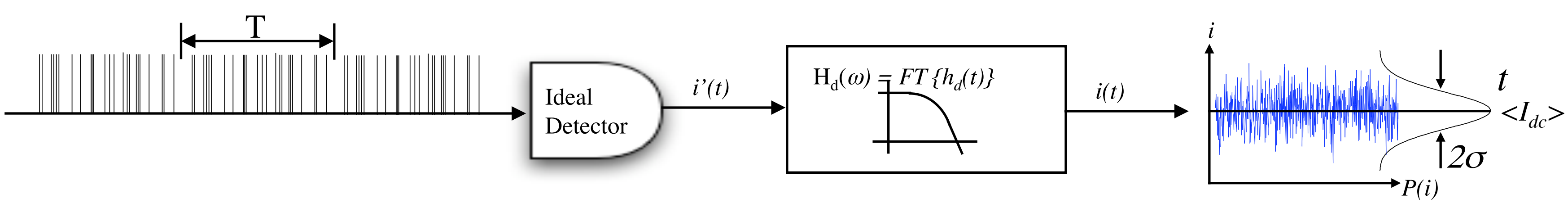
Lecture 2: Photodetection and Photodetectors

Photodetection (Continued)

Detecting Photons (3)

- As the average photon rate increases, the observed photo-current starts smoothing out, with a variance around the mean (average) count that is based on the statistics (which tends to Gaussian for large photon arrival rate)
- $P(i)$ is the probability function of measuring the current at a certain value at time t.



Detecting Photons (4)

- The detector output current *i(t)* can be modeled as a discrete "filtered Poisson" process

$$i(t) = \sum_{j=1}^{N} h_d(t - \tau_j)$$

  - Where $h_d(t)$ is PD impulse response, N is total number e-h pairs generated, τ_j is the random time the j^{th} photocarrier is generated.

- Define: Quantum Efficiency (QE), unitless, as

$$\eta = \frac{\text{number of photocarriers produced}}{\text{number of incident photons}}, 0 \leq \eta \leq 1$$

- Define: Time varying photon rate parameter ($\lambda(t)$) in units of photocarriers/second as

$$\lambda(t) = \frac{\eta}{h\upsilon} P_{recvd}(t)$$

Detecting Photons (5)

- The power incident on a photodetector of area A, in units of Watts, is

$$P(t) = \int_A I(\vec{p},t)\,dA$$

  - where the instantaneous optical intensity at an observation point $\vec{p}$ is given by

$$I(\vec{p},t) = \frac{1}{Z_0}\left|E(\vec{p},t)\right|^2$$

- The time varying photon rate parameter $\lambda(t)$ can then be written in terms of $P(t)$

$$\lambda(t) = \frac{\eta}{h\nu}\frac{\left|E(t)\right|^2}{Z_0}$$

Detecting Photons (6)

- If we consider an observation interval, over which we are going to average our photon count over
  - This can be due either to the inherent bandwidth of the detector or (as we will see later) on purpose to match the receiver bandwidth to the data bit rate
- Then the number of photocarriers generated over the interval T counted at the j^{th} observation interval

$$N_j = \int_0^T \lambda_j(\tau)d\tau$$

- Assuming a coherent source, the ***conditional inhomogeneous Poisson process*** describes this photon count during the j^{th} observation interval

$$P(N_j = N) = \frac{\left(\int_0^T \lambda_j(\tau)d\tau\right)^N}{N!} e^{\left(-\int_0^T \lambda_j(\tau)d\tau\right)}$$

Detecting Photons (7)

- If we assume a constant rate parameter over the time interval T (independent of *j*), then the photo-generated current can be written as

$$i(t) = \lambda(t) q$$

$$\lambda(t) = \frac{N}{T}$$

- Then the photocurrent produced by the photodetector can be written in Amperes, assuming the observation time is normalized to one second

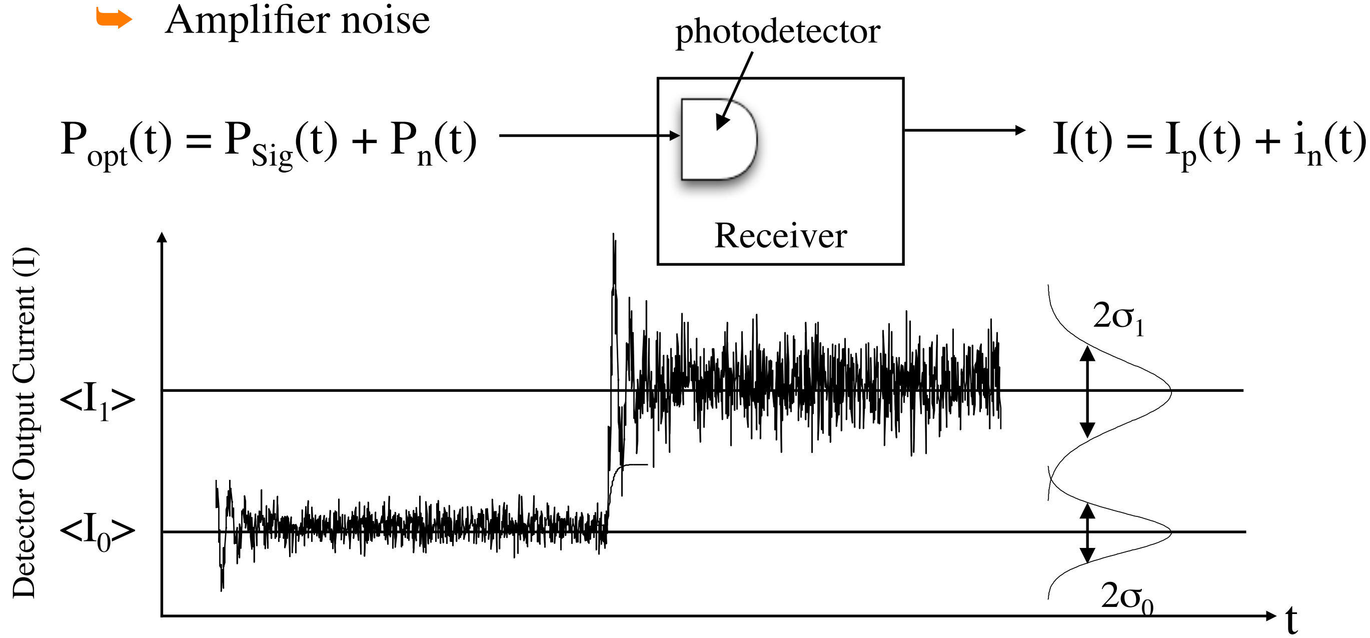
$$i(t) = \lambda(t) q = \frac{\eta q}{h\nu} P_{rcvd}(t)$$
$$= \Re P_{rcvd}(t)$$

  - Where we have defined the detector responsivity as

$$\Re = \frac{\eta q}{h\nu}$$

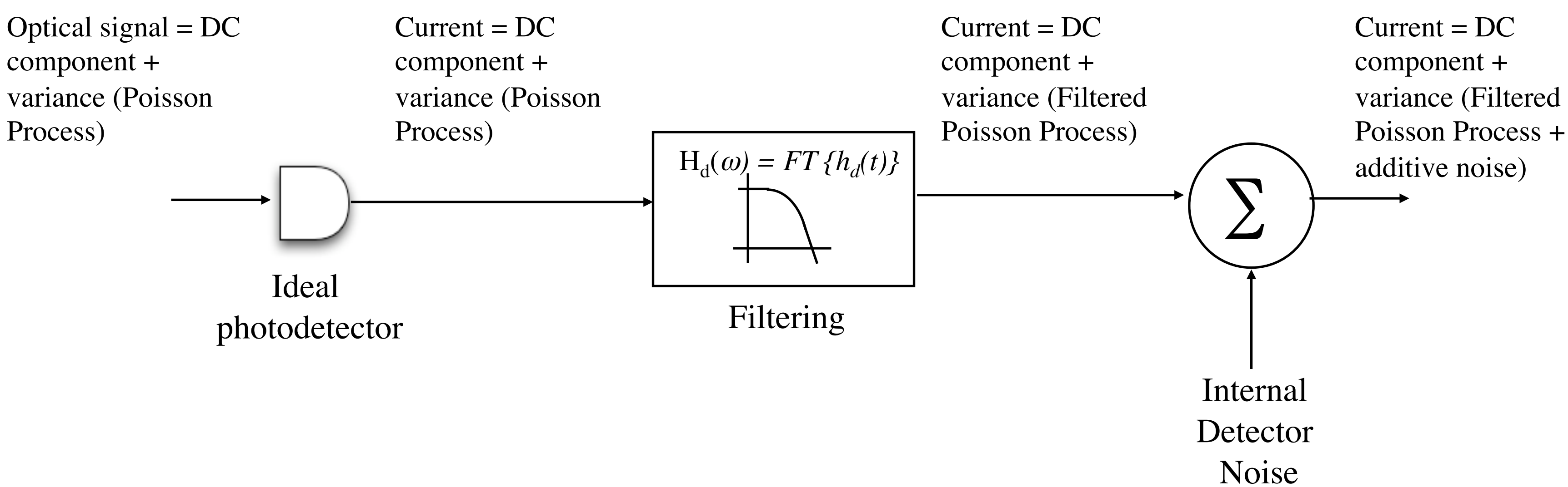
Electrical Signal-to-Noise Ratio (SNR)

- At the receiver, there is noise on the signal arriving at the input and and after detection added to that is noise that is injected at various stages of the receiver
  - The current output of the receiver $i_n(t)$ has current contributions from
    - Electrical shot noise
    - Thermal noise
    - APD detectors have additional multiplication noise
    - Amplifier noise



Modeling Detector SNR

- When observing the detector current output, it is difficult to tell which noise was present at the optical input and which noise was generated internal to the detector. So we tend to use several different models and combine them



Noise Current

- To quantify the statistical nature of noise, we can't determine random events ahead of time, but we can use their "spectral" characteristics to quantify statistical behavior
  - Define an ***Average*** (mean) value to quantify the amount of power (energy) in the non-time varying part of the signal
  - Define a ***Variance*** to quantify the amount of power (energy) in the noisy part of the signal
- Define the "noise" current as

$$i(t) = I_{DC} + i_{noise}(t)$$

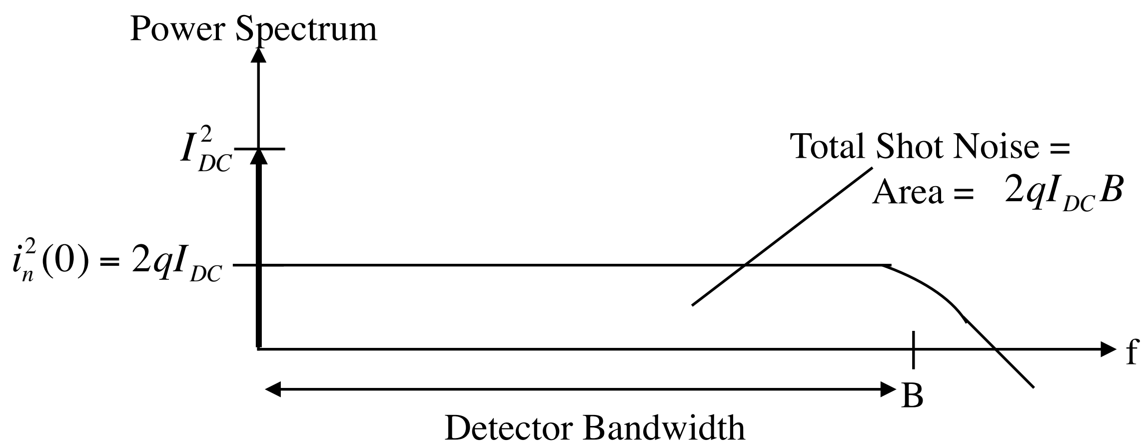
Shot Noise Mean and Variance

- For constant power illumination, the rate parameter is constant, and the signal is the mean
- The noise corresponds to the photocurrent variance
- For a filter, homogeneous Poisson process

Mean (Amps) $$i_s(t) = \overline{i(t)} = \frac{\eta q}{h\nu} P_{recvd} \int_0^t h_d(\tau)\, d\tau$$

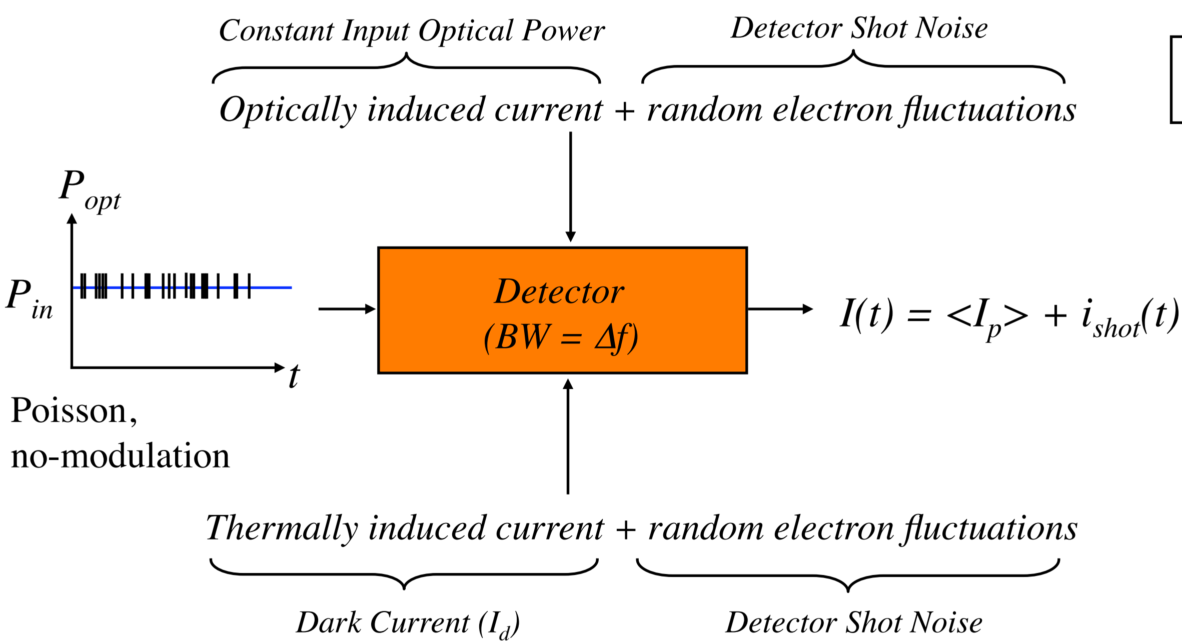
Variance (Amps²) $$i_n^2(t) = \mathrm{var}\{i(t)\} = \frac{\eta q}{h\nu} P_{recvd} \int_0^t h_d^2(\tau)\, d\tau$$

- Both mean and variance are linear with P_{revd}
- As P_{rcvd} is increased, both signal and noise increase

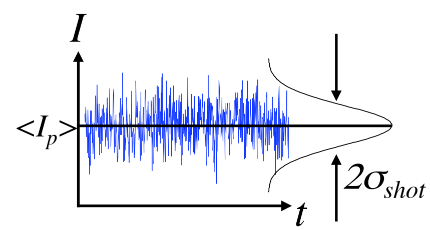


Photodetector Shot Noise

- The shot noise generated in the photodetection process is physically due to the "quantum granularity" of the received (and photo converted) optical signal
- Shot noise sets the ultimate limit of an optical receiver
- Shot noise is a Poisson noise, but it is usually approximated as a Gaussian noise
- Hallmark of shot noise is dependence on q, the electron charge



$$\sigma^2_{Shot} = 2q(I_p + I_d)\Delta f$$



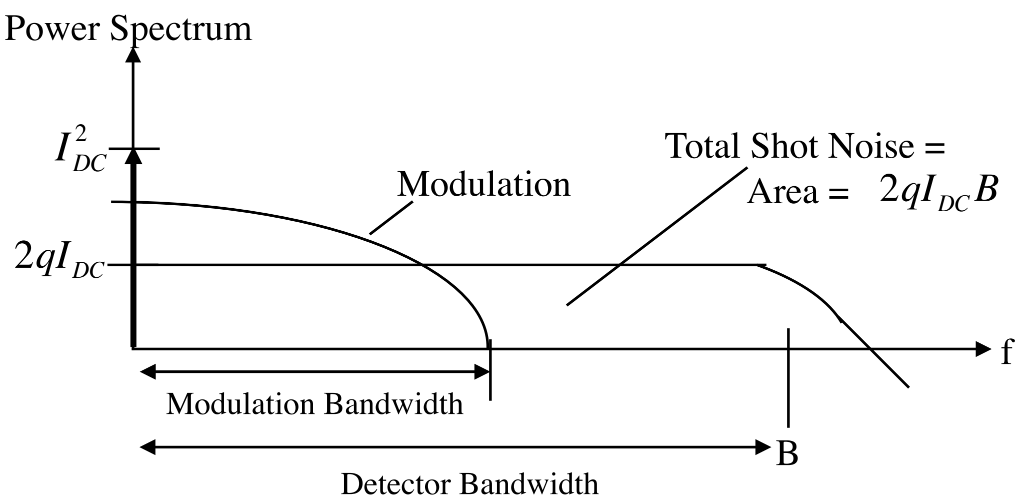
Shot Noise with Data Modulation

- Consider how the picture changes when we have information modulated on the optical carrier
- Let *m(t)* be the information transmitted
  - Then P_{rcvd} (t) and λ(t) are functions of *m(t)*
  - Assuming the photodetector filter impulse function can change in amplitude from time period to time period, let G_j be a time varying parameter

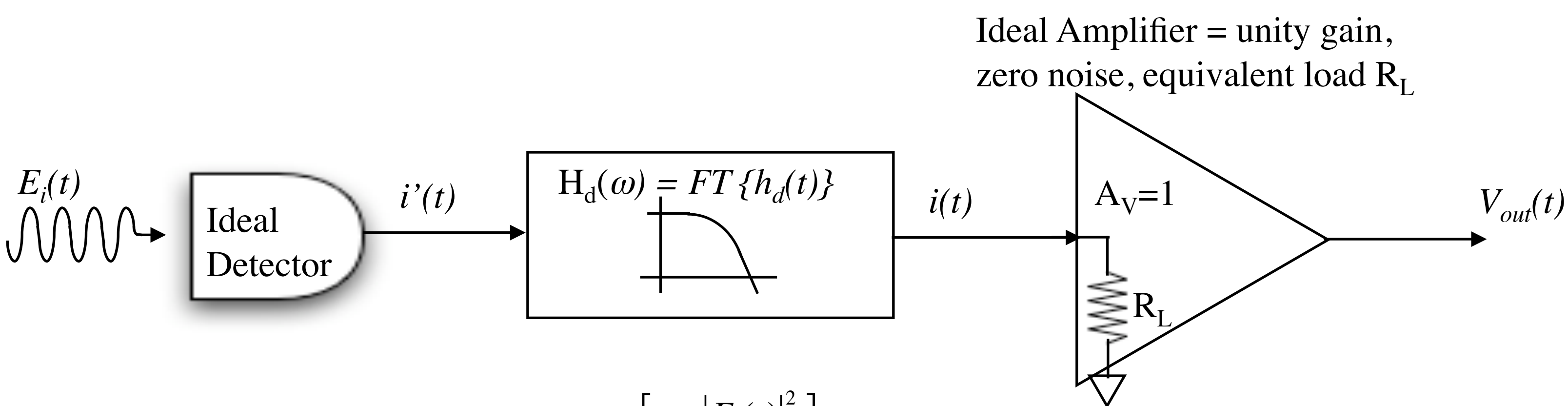
$$i(t) = \sum_{j=1}^{N} G_j h_d(t - \tau_j)$$

$$i_s(t) = \overline{i(t)} = \frac{\eta}{h\nu}\overline{G}\int_{-\infty}^{t} P_{rcvd}(\tau) h_d'(t-\tau)\,d\tau$$

$$i_n^2(t) = \mathrm{var}\{i(t)\} = \frac{\eta}{h\nu}\overline{G^2}\int_{-\infty}^{t} P_{rcvd}(\tau) h_d^2(t-\tau)\,d\tau$$



Ideal Direct Detection (1)



$$E_i(t) = \sqrt{2P_sZ_0}\cos(\omega_s t + \phi)$$

$$P_{avg} = \left(\frac{|E_i(t)|^2}{\sqrt{2}} \Big/ Z_0 \right) = P_s$$

$$i(t) = LPF\left[\frac{\eta q}{h\nu}\frac{|E_i(t)|^2}{Z_0}\right]$$

$$= LPF\left[\frac{\eta q}{h\nu}\frac{2P_sZ_0}{Z_0}Cos^2(\omega_s t + \phi)\right]$$

$$= LPF\left[\frac{\eta q}{h\nu}2P_s\frac{1}{2}\left[1 + \cos 2(\omega_s t + \phi)\right]\right]$$

$$= i_{dc} = \frac{\eta q}{h\nu}P_s$$

$$V_{out}(t) = i(t)R_L$$

$$= i_{dc}R_L = \frac{\eta q}{h\nu}P_sR_L$$

Ideal Direct Detection (2)

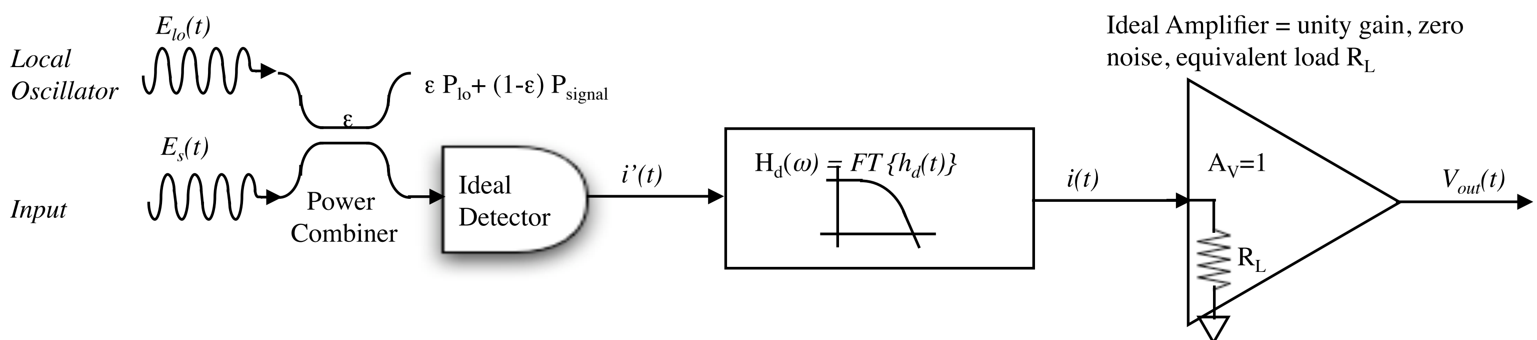
- Electrical SNR is found using the ratio between the signal power (DC) generated in the load resistor and the noise power (shot noise) generated in the load resistor

$$SNR_{dd} = \frac{P_{signal}}{P_{noise}} = \frac{i^2_{signal} R_L}{i^2_{noise} R_L} = \frac{\left(\frac{\eta q}{h\nu} P_{signal}\right)^2}{2qh\nu P_{signal} B} = \frac{1}{2} \frac{\eta P_{signal}}{h\nu B}$$

- This equation shows the fundamental, ***quantum shot noise limit***, where the SNR is limited only by the shot noise itself -> ***Shot Noise Limited Direct (Incoherent) Detection***
- SNR improves linearly with input signal strength
- We will discuss other noise contributions that exist that make it difficult to reach this limit

Ideal Coherent Detection (1)

- Consider the following ideal Heterodyne Coherent Receiver
  - Heterodyne implies that a non-zero intermediate frequency (ω_{IF}) is generated prior to data recovery



$$E_i(t) = \sqrt{\varepsilon} E_{lo}(t) + \sqrt{1-\varepsilon} E_s(t)$$
$$= \sqrt{\varepsilon}\sqrt{2P_{lo}Z_0}\cos(\omega_{lo}t + \phi)$$
$$+ \sqrt{1-\varepsilon}\sqrt{2P_{rcvd}Z_0}\cos(\omega_s t + \phi)$$

$$P_{avg} = \left(\frac{|E_i(t)|^2}{\sqrt{2}} \Big/ Z_0 \right) = P_s$$

$$i(t) = LPF\left[\frac{\eta q}{h\nu}\frac{1}{Z_0} \left| \sqrt{\varepsilon}\sqrt{2P_{lo}Z_0}\cos(\omega_{lo}t+\phi) + \sqrt{1-\varepsilon}\sqrt{2P_{rcvd}Z_0}\cos(\omega_s t+\phi) \right|^2 \right]$$

using $\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$

$$i(t) = LPF\left[\frac{\eta q}{h\nu}\left\{ P_{lo}\varepsilon + P_{rcvd}(1-\varepsilon) + 2\sqrt{P_{lo}P_{rcvd}\varepsilon(1-\varepsilon)}\cos\left[(\omega_s - \omega_{lo})t + \phi\right] \right\} \right]$$

Since typically $P_{rcvd} \ll P_{lo}$

$$I_{DC} \approx \frac{\eta q}{h\nu} P_{lo}\varepsilon$$

Assuming the intermmediate frequency ($\omega_{IF} = \omega_s - \omega_{lo}$) falls within the LPF bandwidth

$$i(t) = \frac{\eta q}{h\nu} 2\sqrt{P_{lo}P_{rcvd}\varepsilon(1-\varepsilon)}\cos\left[\omega_{IF}t + \phi\right]$$

Ideal Coherent Detection (2)

- Using the same approach as in direct detection to obtain the SNR

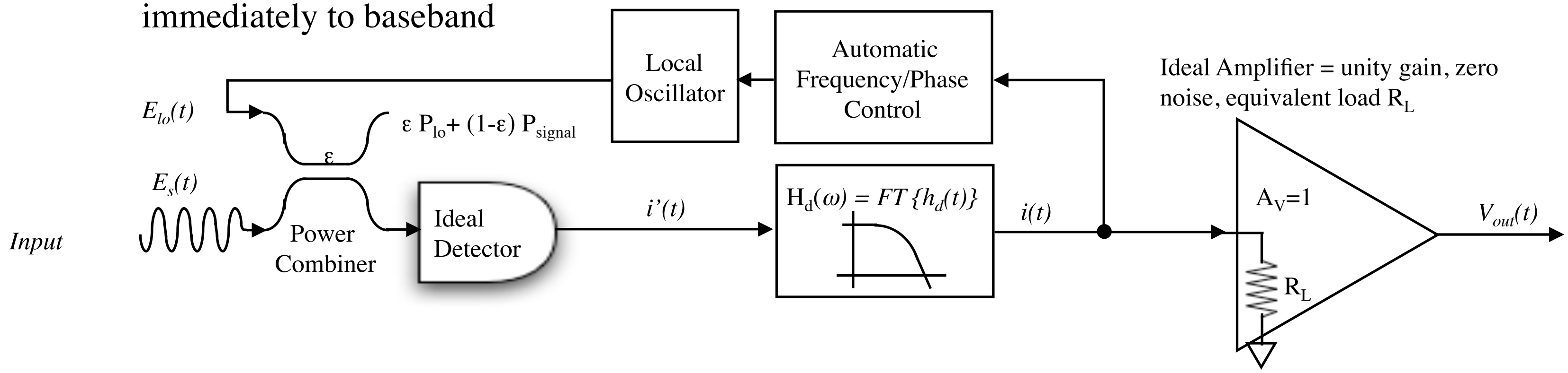
$$SNR_{het} = \frac{P_{signal}}{P_{noise}} = \frac{i_{signal}^2 R_L}{i_{noise}^2 R_L} = \frac{\left(i_{rms}\right)^2}{2qI_{DC}BR_L} = \frac{\left(\frac{i_{peak}}{\sqrt{2}}\right)^2}{2qI_{DC}BR_L}$$

$$= \frac{\left(\frac{\frac{\eta q}{h\nu} 2\sqrt{P_{lo}P_{rcvd}\varepsilon(1-\varepsilon)}}{\sqrt{2}}\right)^2}{2q\left(\frac{\eta q}{h\nu}P_{lo}\varepsilon\right)BR_L} = \frac{\eta(1-\varepsilon)P_{rcvd}}{h\nu B} \approx \left.\frac{\eta P_{rcvd}}{h\nu B}\right|_{\text{limit } P_{lo}\to\infty,\varepsilon\to 0}$$

- Note that ***shot noise limited heterodyne coherent detection***, in the limit where the local oscillator is much stronger than the received signal,
  - Is a factor of 2 (3dB) better than the ***shot noise limited incoherent detection***

Ideal Coherent Detection (3)

- The other coherent approach is ***Homodyne Coherent Detection***
  - The intermediate frequency (ω_{IF}) is driven to zero ($\omega_{IF}=0$) at phase is driven to $\phi=0$ bringing the data immediately to baseband



$$E_i(t) = \sqrt{\varepsilon}E_{lo}(t) + \sqrt{1-\varepsilon}E_s(t)$$
$$= \sqrt{\varepsilon}\sqrt{2P_{lo}Z_0}\cos(\omega_{lo}t+\phi)$$
$$+\sqrt{1-\varepsilon}\sqrt{2P_{rcvd}Z_0}\cos(\omega_s t+\phi)$$

$$P_{avg} = \left(\frac{\left|E_i(t)\right|^2}{\sqrt{2}} \Big/ Z_0 \right) = P_s$$

$$i(t) = LPF\left[\frac{\eta q}{h\nu}\frac{1}{Z_0}\left|\sqrt{\varepsilon}\sqrt{2P_{lo}Z_0}\cos(\omega_{lo}t+\phi) + \sqrt{1-\varepsilon}\sqrt{2P_{rcvd}Z_0}\cos(\omega_s t+\phi)\right|^2\right]$$

Since we are using an AFC/APC control to drive $\omega_{IF} = 0$

$$i(t) = LPF\left[\frac{\eta q}{h\nu}\left\{P_{lo}\varepsilon + P_{rcvd}(1-\varepsilon) + 2\sqrt{P_{lo}P_{rcvd}\varepsilon(1-\varepsilon)}\right\}\right]$$

Since typically $P_{rcvd} \ll P_{lo}$

$$I_{DC} \approx \frac{\eta q}{h\nu}P_{lo}\varepsilon$$

$$i(t) = \frac{\eta q}{h\nu}2\sqrt{P_{lo}P_{rcvd}\varepsilon(1-\varepsilon)}$$

Ideal Coherent Detection (4)

- Using the same approach as in direct detection to obtain the SNR

$$SNR_{het} = \frac{P_{signal}}{P_{noise}} = \frac{i_{signal}^2 R_L}{i_{noise}^2 R_L} = \frac{\left(i_{rms}\right)^2}{2qI_{DC}B}$$

$$= \frac{\left(\frac{\eta q}{h\nu} 2\sqrt{P_{lo}P_{rcvd}\varepsilon(1-\varepsilon)}\right)^2}{2q\left(\frac{\eta q}{h\nu}P_{lo}\varepsilon\right)B} = \frac{\eta 2(1-\varepsilon)P_{rcvd}}{h\nu B} \square\ 2\frac{\eta P_{rcvd}}{h\nu B}\bigg|_{\text{limit } P_{lo}\to\infty,\varepsilon\to 0}$$

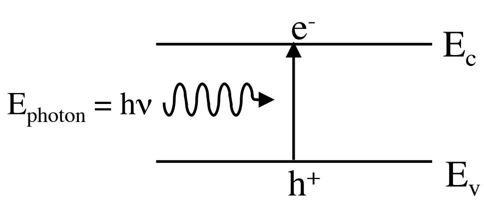
- Note that ***shot noise limited homodyne coherent detection***, in the limit where the local oscillator is much stronger than the received signal,
  - Is a factor of 2 (3dB) better than the ***shot noise limited heterodyne receiver*** and factor of 4 (6dB) better than the ***shot noise limited incoherent detection***

Photodetectors

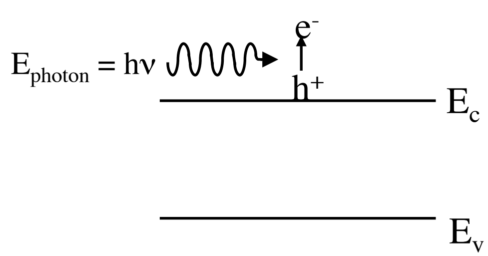
Photoconductors (1)

- Photon absorption in semiconductor materials.
- Three main absorption mechanisms: Intrinsic (band-to-band), Free-Carrier Absorption and Band-and-Impurity Absorption
- Intrinsic (band-to-band) is the dominant effect in most SC photoconductors

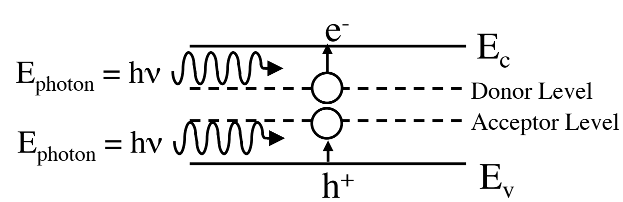
Intrinsic (band-to-band)



Free-Carrier Absorption



Band-and-Impurity Absorption



•Incident photon $E_{photon} = h\nu = E_c - E_v$

Photoconductors (2)

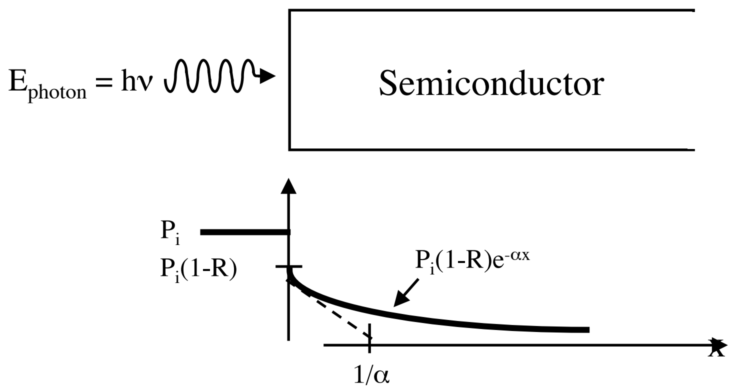
- For intrinsic absorption, photons can be absorbed if

$$\lambda(\mu m) > \frac{hc}{E_c - E_V} = \frac{1.24}{E_g(eV)}$$

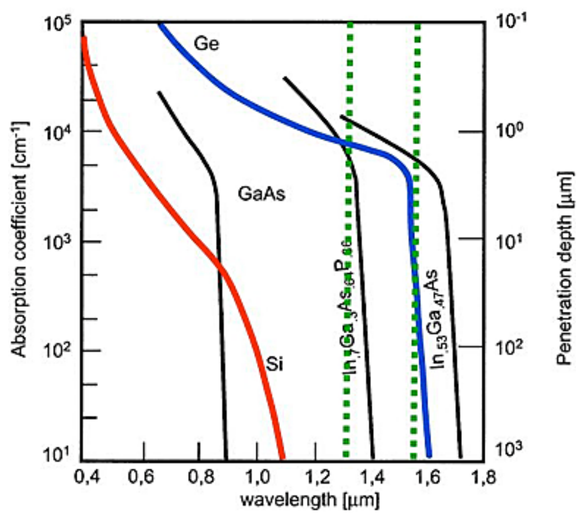
$$\lambda(nm) > \frac{1240}{E_g(eV)}$$

Material	Bandgap (eV)	Maximum λ (nm)	Typical Operating Range (nm)
Si	1.12	1110	500-900
Ge	0.67	1850	900-1300
GaAs	1.43	870	750-850
$In_xGa_{1-x}As_yP_{1-y}$	0.38-2.25	550-3260	1000-1600

Photoconductors (3)



- Define:
  - P_i = incident optical power
  - $R(\lambda)$ power reflectivity from input medium to semiconductor
  - $\alpha(\lambda)$ = 1/e absorption length
  - $1/\alpha(\lambda)$ = penetration depth



- Power absorbed by the semiconductor is

$$P_{abs}(x) = P_i(1-R)(1-e^{-\alpha(\lambda)x})$$
$$= \eta(\lambda,x)P_i$$

- defining the efficiency

$$\eta(\lambda,x) = \frac{\text{number of photocarriers produced}}{\text{number of incident photons}}$$
$$= (1-R)(1-e^{-\alpha(\lambda)x})$$
$$0 \le \eta(\lambda,x) \le 1$$

Photoconductive Photodetectors (1)

- Photogenerated current will have time and wavelength dependence

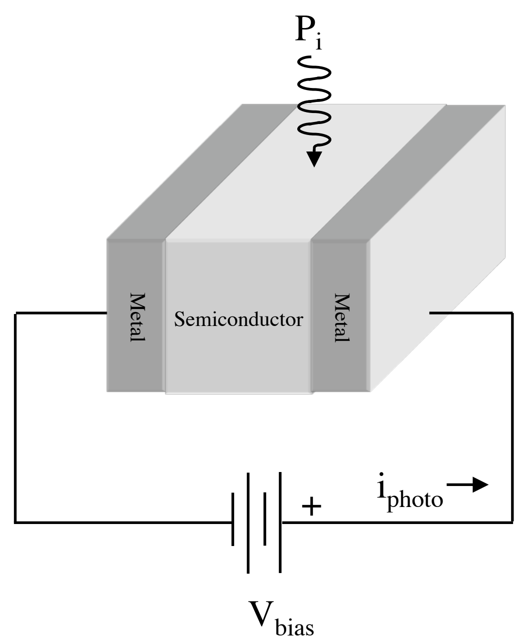
$$i_{photo}(t) = \frac{\eta q}{h\nu} G P_{rcvd}(t) + i_{dark}$$

$$\tau_{carrier} = \text{mean free carrier lifetime}$$

$$\tau_{transit} = \text{transit time between eletrical contacts}$$

$$G = \left(\frac{\tau_{carrier}}{\tau_{transit}}\right) = \text{photoconductive gain}$$

$$i_{dark} = \text{dark current}$$



- The transit time for electrons and holes can be different and in many SCs the eletron mobility is greater than that of the hole

$$\upsilon_e = \mu_e E > \mu_h E = \upsilon_h$$

- The SC must remain charge neutral, for every electron generated, multiple holes will get pulled in until the photogenerated electron reaches the other contact. The carrier and transit times are limited by the slower carrier and the photoconductive gain is given by the ratio of the transit times

$$\tau_{carrier} = \frac{L_a}{\upsilon_h}$$

$$\tau_{transit} = \frac{L_a}{\upsilon_e}$$

Photoconductive Photodetectors (2)

- The carrier velocity is a linear function of electric field strength up to a saturation velocity (which is the same for both electrons and holes)
  - Field strength of about 10^5 V/cm result in velocities in range of 6×10^6 to 10^7 cm/s
  - Some materials have an electron drift velocity that peaks at 2×10^7 cm/s at 10^4 V/cm
- When photoconductive gain is desirable, detector is operated at low voltages
- Carrier lifetime also impacts the frequency response of the photoconductive photodetector

$$i_{photo}(\omega) = \Re G \frac{P_{rcvd}(\omega)}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

$$\omega_c = \frac{1}{\tau_{carrier}} = \text{cutoff frequency}$$