

HW#1 Solutions

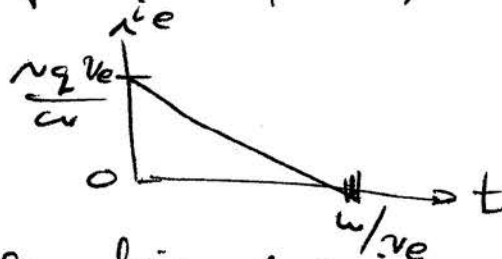
Problem 1 : 10 pts

Assuming photo carriers are generated uniformly throughout the sc, when electrons reach $x = \omega$ at holes reach $x = 0$, they stop contributing to photo current.

for N electrons generated uniformly at $t = 0$, the initial current is

$$i_e = \frac{Nq v_e}{\omega}$$

Assuming that at time $t = \frac{\omega - x}{v_e}$ all electrons generated at x have ceased contributing to current, independent of x , we must have



which is a line described by

$$i_e = -\frac{Nq v_e^2}{\omega^2} t + \frac{Nq v_e}{\omega}, \quad 0 \leq t \leq \omega/v_e$$

For holes, same situation

$$i_h = -\frac{Nq v_h^2}{\omega^2} t + \frac{Nq v_h}{\omega}, \quad 0 \leq t \leq \omega/v_h$$

And total current is sum

$$\begin{aligned} i_{\text{total}}(t) &= \frac{Nq}{\omega} \left[(v_h + v_e) - \frac{1}{\omega} (v_h^2 + v_e^2) t \right] \\ &\quad \text{for } 0 \leq t \leq \omega/v_e \\ &= \frac{Nq}{\omega} \left[v_h - \frac{v_h^2}{\omega} t \right] \quad \frac{\omega}{v_e} \leq t \leq \frac{\omega}{v_h} \end{aligned}$$

Problem 2

$$\eta = \frac{3 \times 10^{12}}{8 \times 10^{12}} \frac{\text{electrons}}{\text{photon}} = 37.5\%$$

$$R = \frac{\eta q}{h \nu} = \frac{(0.375)(1.6022 \times 10^{-19})}{(6.62 \times 10^{-34})(5.5)(2 \times 10^8 \text{ m/s})} \lambda (\mu\text{m}) \times 10^{-6}$$

$$R(0.83) = .251$$

$$R(1.3) = .394$$

$$R(1.55) = .469$$

At $1 \mu\text{m}$ thickness, $\text{In}_{.7}\text{Ga}_{.3}\text{As}_{.64}\text{P}_{.36}$ has
 $\alpha \approx 3000 \text{ cm}^{-1}$

$$\begin{aligned} \text{Now } \eta &= R (1 - e^{-\alpha \cdot 10^{-4}}) \\ &= R (.74) \end{aligned}$$

$$\eta(1.3) = (.394)(.74) = .292$$

Problem 3

(a) rms noise current due to shot noise must consider both photo generated + dark noise currents

$$\sigma_{\text{rms}} = \sqrt{2q(I_p + I_d) \Delta f}$$

$$\text{where } I_p = \frac{\eta q}{h \nu} P_{\text{in}} = 0.4 \text{ nA}$$

$$\begin{aligned} \sigma_{\text{rms}} &= \text{Sqrt} (2 \cdot 1.6022 \times 10^{-19} (0.4 \text{ nA} + 1 \text{ nA}) 20 \times 10^6) \\ &= 5.06 \times 10^{-8} \text{ A} \end{aligned}$$

problem 3b (contd)

(b) SNR due to shot noise

$$SNR = \frac{\bar{I}_p^2}{\sigma_{rms}^2} = \frac{(0.4 \text{ mA})^2}{(5.06 \times 10^{-8})^2} = 6.25 \times 10^7$$

$$= 77.95 \text{ dB}$$

(c) $SNR = 100 = \frac{\bar{I}_p^2}{2q(\bar{I}_p + I_d)\Delta f}$

$$\bar{I}_p^2 - 2q(100)\Delta f \bar{I}_p - 2q(100)\Delta f I_d$$

Solving for root

$$\bar{I}_p = 1.18 \text{ nA}$$

$$\therefore P_{in} = \frac{hc}{2q\lambda} \bar{I}_p$$

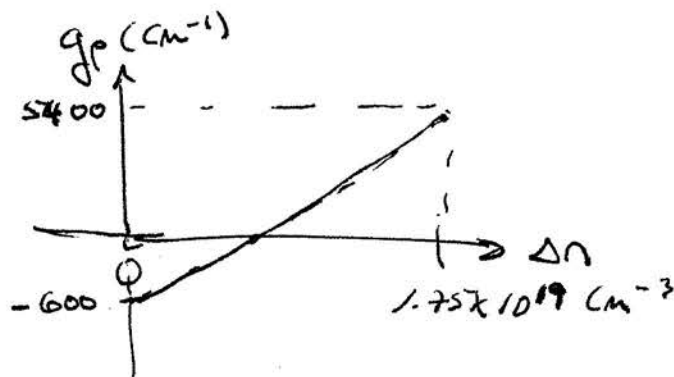
$$= \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{(0.65)(1.6022 \times 10^{-19})(0.8 \times 10^{-6})} \cdot 1.18 \text{ nA}$$

$$= 2.81 \text{ nW}$$

problem 4

(A)

$$g_p = \alpha \left(\frac{\Delta n}{\Delta n_T} - 1 \right)$$



(B) Mode Separation $\Delta d_m = -\frac{\lambda^2}{2nL} = -\frac{(1.31)^2 \mu\text{m}^2}{2 \cdot 4 \cdot 350 \mu\text{m}} = .613 \text{ nm}$

(C) # Modes oscillate = $\frac{5 \text{ nm}}{.613 \text{ nm}} \approx 8 \text{ Modes}$

(D) Mode peak is at $M=0$

Problem 5

400 μm long grating
 $n_{\text{eff}} = 3.5$

$$\Lambda = 22 \mu\text{m}$$

$$\therefore \lambda_B = (2)(22 \mu\text{m})(3.5) = 1.54 \mu\text{m}$$

From Fig assume nearest modes to $\Delta\beta = 0$,
or $\Delta\beta L = 0.5$

$$\Rightarrow \Delta\beta L \cong 2$$

Solving

$$\Delta\beta L = 2 = 400 \mu\text{m} \cdot 2\pi \cdot 3.5 \left(\frac{1}{\lambda} - \frac{1}{\lambda_B} \right)$$

$$\frac{1}{\lambda} = \frac{1}{1.54} \pm 7.1 \times 10^{-4}$$

OR two losing modes

$$\lambda = 1.538 \mu\text{m}, 1.542 \mu\text{m}$$

With $\pi/4$ phase shift in grating

$$\Delta\beta L = 0 \text{ and}$$

$$\lambda_{\text{losing}} = \lambda_B = 1.54 \mu\text{m}$$

Problem 6

Active Volume $V = 400\mu\text{m} \times 2\mu\text{m} \times 0.3\mu\text{m}$
 $= 2.4 \times 10^{-10} \text{ cm}^3$

Current vs. carrier concentration is obtained from rate eqn.

$$\frac{I_g}{qV} = \beta N^2 + \alpha N^3$$

$$I_g = (1.6022 \times 10^{-19}) (2.4 \times 10^{-10})$$

$$\cdot [10^{-10} \text{ N}^2 + 3 \times 10^{-29} \text{ N}^3]$$

Solving for N for I_g from 0mA to 100mA

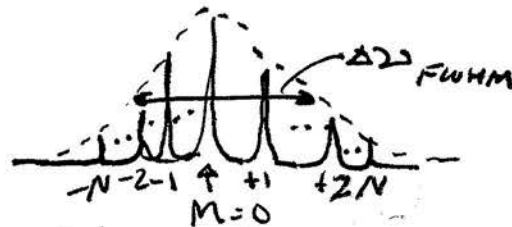
I_g (mA)	N	Δd (nm)
10	1.35×10^{18}	-1.7
20	1.83×10^{18}	-2.3
30	2.17×10^{18}	-2.8
40	2.45×10^{18}	-2.8
50	2.68×10^{18}	-3.1
60	2.89×10^{18}	-3.7

Using $\Delta d = \frac{\beta q d_g P_t N}{n_{eff}} = \frac{(-1.3 \times 10^{-20})(1300 \text{ nm})(0.3)N}{4}$

See next page

problem 7

Assume the gain curve and modes look something as follows



Total # modes
 $M = 2N + 1$

pulse width: The gain line shape has a FWHM of $\Delta\nu$, so

$$\tau_{\text{pulse}} \approx \frac{1}{\Delta\nu}$$

Number of modes is given by mode separation ν_m and linewidth

$$M = \frac{\Delta\nu}{\nu_m}$$

mean intensity

$$\bar{I} = M |A|^2 = M P$$

peak intensity

$$I_p = M \bar{I} = M^2 P$$