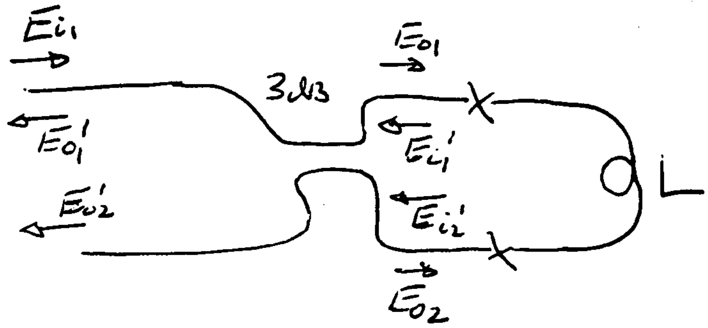
HW#2 Solutions
ECE 228B

Problem 3.1

Input →



For the 3dB Coupler with single input illuminated:

$$\begin{pmatrix} E_{o1} \\ E_{o2} \end{pmatrix} = e^{-i\beta l} \begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} E_{i1} \\ 0 \end{pmatrix}$$

After propagating through a fiber of length L

$$\begin{pmatrix} E_{i1}' \\ E_{i2}' \end{pmatrix} = e^{-i\beta L} \begin{pmatrix} E_{o2} \\ E_{o1} \end{pmatrix}$$

After passing through the 3dB Coupler for the second time

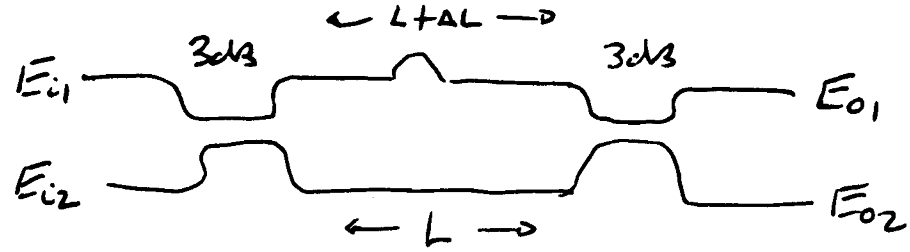
$$\begin{pmatrix} E_{o1}' \\ E_{o2}' \end{pmatrix} = e^{-i\beta l} \begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} E_{i1}' \\ E_{i2}' \end{pmatrix}$$

Solving

$$\begin{aligned} E_{o1}' &= e^{-i\beta l}\left[\tfrac{1}{\sqrt{2}} E_{i1}' + \tfrac{i}{\sqrt{2}} E_{i2}'\right] \\ &= e^{-i\beta l}\left\{\tfrac{1}{\sqrt{2}} e^{-i\beta L} E_{o2} + \tfrac{i}{\sqrt{2}} e^{-i\beta L} E_{o1}\right] \\ &= e^{-i\beta(2l+L)}\left[\tfrac{i}{2} E_{i1} + \tfrac{i}{2} E_{i1}\right] \\ &= e^{-i[\beta(2l+L)+\frac{\pi}{2}]} E_{i1} \end{aligned}$$

$P_{o1}' = |E_{o1}'|^2 = |E_{i1}|^2$ All power is reflected back to input ⇒ Loop mirror!

Problem 3.11



In general

$$\begin{pmatrix} E_{o1} \\ E_{o2} \end{pmatrix} = e^{-i\beta l}\begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} e^{-i\beta L}\begin{pmatrix} e^{i\beta\Delta L} & 0 \\ 0 & 1 \end{pmatrix} e^{-i\beta l}\begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}\begin{pmatrix} E_{i1} \\ E_{i2} \end{pmatrix}$$

$$= e^{-i\beta(2l+L+\Delta L/2)}\begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}\begin{pmatrix} e^{i\beta\Delta L/2} & 0 \\ 0 & e^{-i\beta\Delta L/2} \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}\begin{pmatrix} E_{i1} \\ E_{i2} \end{pmatrix}$$

$$= e^{-i\beta(2l+L+\Delta L/2)}\begin{pmatrix} \frac{e^{i\beta\Delta L/2}}{2} - \frac{e^{-i\beta\Delta L/2}}{2} & \frac{ie^{i\beta\Delta L/2}}{2} + \frac{ie^{-i\beta\Delta L/2}}{2} \\ \frac{ie^{i\beta\Delta L/2}}{2} + \frac{ie^{-i\beta\Delta L/2}}{2} & -\frac{e^{i\beta\Delta L/2}}{2} + \frac{e^{-i\beta\Delta L/2}}{2} \end{pmatrix}\begin{pmatrix} E_{i1} \\ E_{i2} \end{pmatrix}$$

$$= e^{-\beta(2l+L+\Delta L/2)}\begin{pmatrix} i\sin(\Delta\beta L/2) & i\cos(\Delta\beta L/2) \\ i\cos(\Delta\beta L/2) & -i\sin(\Delta\beta L/2) \end{pmatrix}\begin{pmatrix} E_{i1} \\ E_{i2} \end{pmatrix}$$

Assuming $E_{i2} = 0$

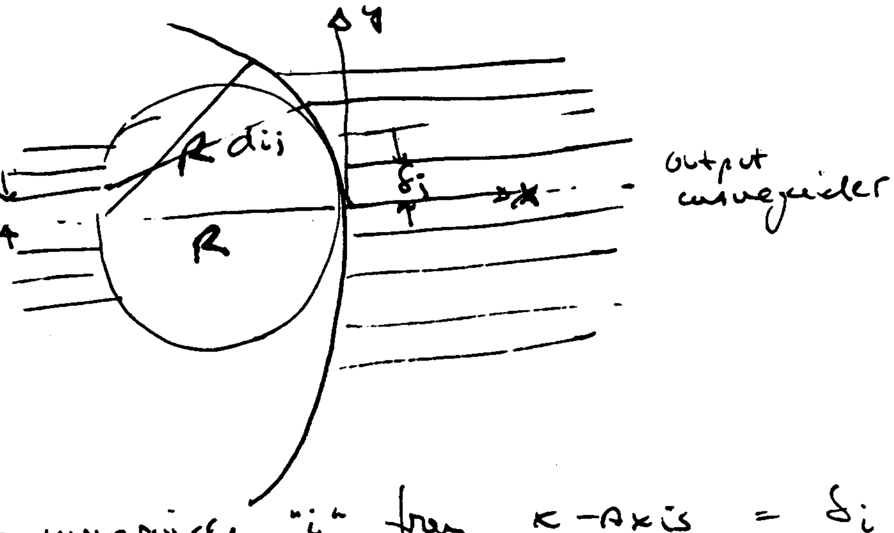
$$\begin{pmatrix} E_{o1} \\ E_{o2} \end{pmatrix} = ie^{-\beta(2l+L+\Delta L/2)}\begin{pmatrix} \sin(\Delta\beta L/2) \\ \cos(\Delta\beta L/2) \end{pmatrix} E_{i1}$$

and the power transfer function is given by the magnitude squared

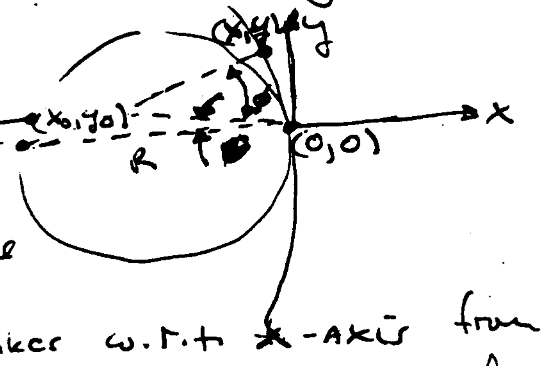
$$T = \begin{pmatrix} \sin^2(\Delta\beta L/2) \\ \cos^2 \Delta\beta L/2 \end{pmatrix}$$

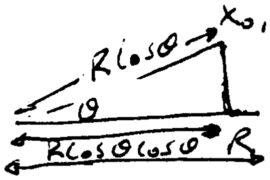
...blem
3.13...

The Rowland Circle Construction is given by



Define offset of input waveguide "i" from x-Axis $= \delta_i$
offset of output waveguide "j" from x-Axis $= \delta_j$
using the following coordinate system:





θ = Angle x_0, y_0 makes w.r.t x-Axis from $(0,0)$

ϕ = Angle x, y makes w.r.t x-Axis from $(-R, 0)$

then $x_0 = -R\cos\theta\cos\theta = -R\cos^2\theta$

$y_0 = R\cos\theta\sin\theta$

$x = -(R - R\cos\phi) = -R(1-\cos\phi) = -R\,2\sin^2\phi/2 \simeq -2R\left(\frac{\phi}{2}\right)^2$ small ϕ

$y = R\sin\phi \simeq R\phi$ for small ϕ

Distance from input "i" to output "j"

$d_{ij}^2 = (x-x_0)^2 + (y-y_0)^2 = x^2 + x_0^2 - 2xx_0 + y^2 + y_0^2 - 2yy_0$

$(x^2 \simeq 0$ for small $\phi) \Rightarrow \quad = (x_0^2 + y_0^2) + \cancel{x^2} + y^2 - 2xx_0 - 2yy_0$

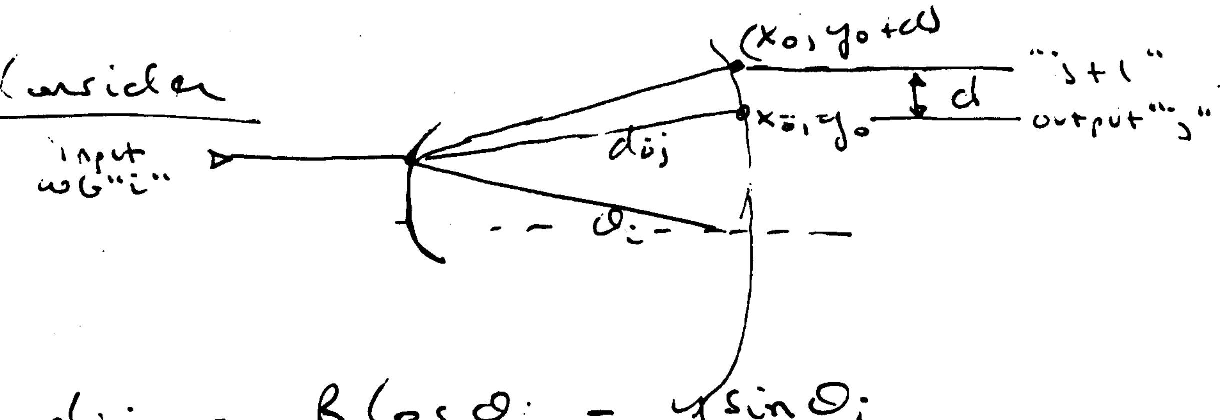
$= R^2\cos^2\theta + R^2\phi^2 - 4R^2\cos^2\theta\left(\frac{\phi}{2}\right)^2 - 2R^2\phi\cos\theta\sin\theta$

Back to input $\Rightarrow$ Laser mirror!

$$
\begin{aligned}
d_{ij}^2 &= R^2\cos^2\theta + R^2\phi^2 - 4R^2\cos^2\theta\,\frac{\phi^2}{4} - 2R^2\phi\cos \\
&= R^2\cos^2\theta + y^2 - \cancel{R^2}\cos^2\theta\,\frac{y^2}{\cancel{R^2}} - 2Ry\cos\theta \\
&= R^2\cos^2\theta + y^2(1-\cos^2\theta) - 2Ry\cos\theta \\
&= R^2\cos^2\theta + y^2\sin^2\theta - 2Ry\cos\theta\sin \\
&= (R\cos\theta - y\sin\theta)^2
\end{aligned}
$$

or $\quad d_{ij} = R\cos\theta - y\sin\theta$

Now consider



$$
\begin{aligned}
d_{ij} &= R\cos\theta_i - y\sin\theta_i \\
d_{ij+1} &= R\cos\theta_i - (y+d)\sin\theta_i
\end{aligned}
$$

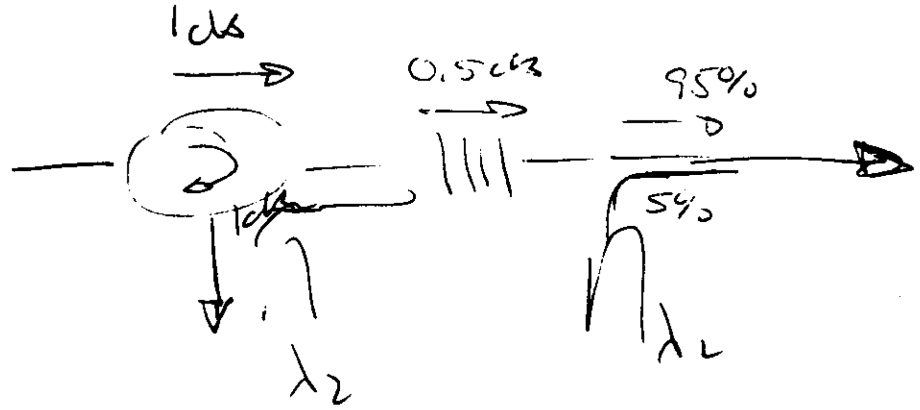
or $\quad |d_{ij+1} - d_{ij}| = (R\cos\theta_i - (y+d)\sin\theta_i)$

$$- R\cos\theta_i - y\sin\theta_i$$

$$= \underline{\underline{d\sin\theta_i = \delta_{ij}}}$$

3.24 10 pts

a)



Converting Tap loss to dB

$10 \log_{10}(0.95) = -0.223$ dB

$10 \log_{10}(0.05) = -13.01$ dB

$P_{in} - 1\text{ dB} - 0.5\text{ dB} - 0.223\text{ dB} = P_{passed}$

$P_{in} - 1\text{ dB} - 1\text{ dB} = P_{drop}$

$P_{add} - 13.01\text{ dB} = P_{add}$

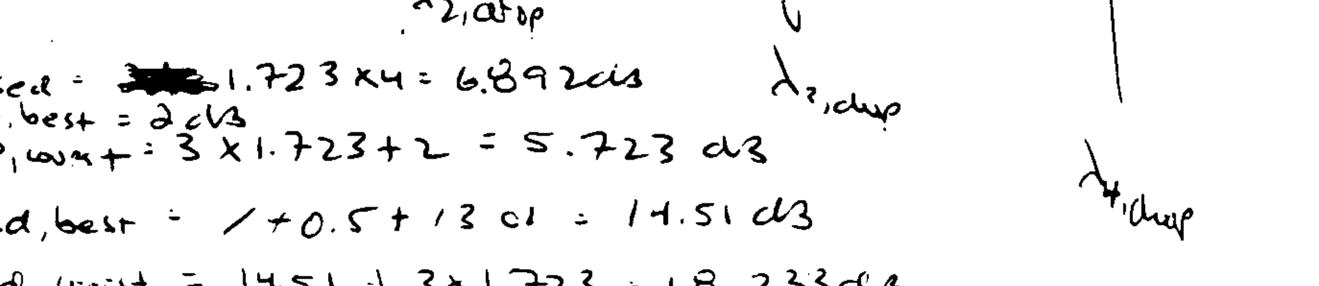
For $P_{in} = -15$ dBm $\Rightarrow$ $P_{passed} = -16.723$ dBm

$P_{drop} = -17$ dB

$P_{added} = -16.723 = P_{add} - 13.01$ dB

$P_{add} = -3.713$ dBm

b)



$\alpha_{passed} =$ [redacted] $1.723 \times 4 = 6.892$ dB

$\alpha_{drop,best} = 2$ dB

$\alpha_{drop,worst} = 3 \times 1.723 + 2 = 5.723$ dB

$\alpha_{add,best} = 1 + 0.5 + 13.01 = 14.51$ dB

$\alpha_{add,worst} = 14.51 + 3 \times 1.723 = 18.233$ dB