Homework #2 Solutions

Problem 8.2 (Ramaswami)

Part (a)

The following RWA will work for the four nodes, where each wavelength is one unit in the traffic matrix and we assume bi-directional links. Note that the B-D lightpath can be configured using both directions in the ring (indicated by the bolded wavelength), reducing the number of wavelengths by one over using one direction only. We can use 4 wavelengths.

Lightpath	Wavelengths
L_{AB}	λ_1 λ_2 λ_3
L_{BC}	λ_1 λ_2
L_{CD}	λ_1 λ_2
L_{AD}	λ_1 λ_2 λ_3
L_{BD}	λ_3 λ_4 $\boldsymbol{\lambda_4}$

Part (b)

The lower bound is the maximum node degree/2. Since node D has 8 ports connected, the lower bound is 4.

Part (c)

This is an interesting question. We can interpret the ADM as an all-optical (OADM) to provide bypass function, or any ADM that has optoelectronic termination. If we use the ADM definition, then there are more lightpaths then given in part (a). OADMs: Nodes A and C need to have optical bypass functionality to support the L_{AD} connections (where wavelengths are minimized), so (2) OADMs. If we were to add one more wavelength so that the L_{AD} connections passed in one direction only, then only node C needs an OADM, so there is a tradeoff between number of wavelengths and number of OADMs for this traffic matrix. The number of transponder ports required for this configuration (non-optical ADMs) is 23.

Part (d)

If we use only point to point (no optical bypass) then we don't need any OADMs, however, the number of lightpaths increases so the number of transponder ports increases to 4*8 = 32.

Problem 8.3 (Ramaswami)

Assuming there is *t* traffic between every node to all other nodes, and that communication is bi-directional around the ring

- The traffic between two adjacent nodes is *t/2* due to the source node transmitting in both directions around the ring.
- The traffic dropped at each node is *t/(N-1)* since each of the other *(N-1)* nodes are transmitting t units of traffic to each node.
- The traffic in one direction of the ring on each link has contributions from the nearest source node of (*t/2*) plus the traffic from the next neighboring node generating (*t/2*) with *t/(N-1)* traffic dropped at the neighboring node – hence (*t/2*)- *t/(N-1*). From the next node over, there is a contribution of (*t/2*)- *2t/(N-1)* and so on for *N/2* nodes (even) and *(N-1)/2* nodes odd.
- Therefore the total number of lightpaths is given by
    - Even number of nodes:

$$L = \frac{t}{2} + \left(\frac{t}{2} - \frac{t}{N-1}\right) + \left(\frac{t}{2} - \frac{2t}{N-1}\right) + \ldots + \left(\frac{t}{2} - \frac{\left(\frac{N}{2} - 1\right)t}{N-1}\right)$$

$$L = \sum_{i=1}^{N/2} \left[\frac{t}{2} - \frac{(i-1)t}{N-1}\right]$$

$$L = \left(\frac{t}{2}\right)\left(\frac{N}{2}\right) - \left(\frac{t}{N-1}\right)\sum_{i=1}^{N/2}[i-1]$$

$$L = \left(\frac{t}{2}\right)\left(\frac{N}{2}\right) - \left(\frac{t}{N-1}\right)\left(\sum_{i=1}^{N/2}[i] - \left(\frac{N}{2}\right)\right)$$

For M even $\sum_{i=1}^{M} i = \frac{M}{2}M + \frac{M}{2} = \frac{M}{2}[M+1]$

$$L = \left(\frac{t}{2}\right)\left(\frac{N}{2}\right) - \left(\frac{t}{N-1}\right)\left(\left[\frac{N}{4}\left(\frac{N}{2} + 1\right)\right] - \left(\frac{N}{2}\right)\right)$$

$$L = \left(\frac{t}{2}\right)\left(\frac{N}{2}\right) - \left(\frac{t}{N-1}\right)\left(\left[\left(\frac{N^2}{8} + \frac{N}{4}\right)\right] - \left(\frac{N}{2}\right)\right)$$

$$L = t\left(\frac{N}{4}\right) - \left(\frac{t}{N-1}\right)\left(\frac{N^2}{8} - \frac{N}{4}\right)$$

$$L = t\left(\frac{N}{4}\right) - \left(\frac{t}{N-1}\right)\left(\frac{N}{4}\right)\left(\frac{N}{2} - 1\right)$$

$$L = t\left(\frac{N}{4}\right) - \left(\frac{t}{N-1}\right)\left(\frac{N}{4}\right)\left(\frac{N-2}{2}\right)$$

$$L = t\left[\left(\frac{N}{4}\right) - \left(\frac{t}{N-1}\right)\left(\frac{N}{8}\right)(N-2)\right]$$

$$L = \frac{t}{8}\left[(2N) - N\left(\frac{N-2}{N-1}\right)\right]$$

$$L = \frac{t}{8}N\left[2 - \left(\frac{N-2}{N-1}\right)\right]$$

$$L = \frac{t}{8}N\left[\frac{2N-2}{N-1} - \left(\frac{N-2}{N-1}\right)\right]$$

$$L = \frac{t}{8}\left[\frac{N^2}{N-1}\right]$$

$$L = \frac{t}{8}\left[\frac{(N+1)(N-1)+1}{N-1}\right]$$

$$L = \frac{t}{8}\left[N + 1 + \frac{1}{N-1}\right]$$

- Odd number of nodes:

$$L = \sum_{i=1}^{[N-1]/2} \left[\frac{t}{2} - \frac{(i-1)t}{N-1}\right]$$

$$L = \left(\frac{t}{2}\right)\left(\frac{N}{2}\right) - \left(\frac{t}{N-1}\right)\sum_{i=1}^{[N-1]/2}[i-1]$$

$$L = \left(\frac{t}{2}\right)\left(\frac{N}{2}\right) - \left(\frac{t}{N-1}\right)\left(\sum_{i=1}^{[N-1]/2}[i] - \left(\frac{N-1}{2}\right)\right)$$

$$\sum_{i=1}^{M} i = \frac{M}{2}M + \frac{M}{2} = \frac{M}{2}[M+1]$$

$$L = \left(\frac{t}{2}\right)\left(\frac{N}{2}\right) - \left(\frac{t}{N-1}\right)\left(\frac{[N-1]/2}{2}\left(\frac{N-1}{2}+1\right) - \left(\frac{N-1}{2}\right)\right)$$

$$L = \left(\frac{t}{2}\right)\left(\frac{N}{2}\right) - \left(\frac{t}{N-1}\right)\left(\left(\frac{(N-1)^2}{8} + \frac{(N-1)}{4}\right) - \left(\frac{N-1}{2}\right)\right)$$

$$L = t\left(\frac{N}{4}\right) - \left(\frac{t}{N-1}\right)\left(\frac{N^2-2N+1}{8} + \frac{2N-2}{8} - \frac{4N-4}{8}\right)$$

$$L = t\left(\frac{N}{4}\right) - \left(\frac{t}{N-1}\right)\left(\frac{N^2-4N+3}{8}\right)$$

$$L = \frac{t}{8}\left[(2N) - \frac{(N^2-4N+3)}{N-1}\right]$$

$$L = \frac{t}{8}\left[\frac{(2N^2-2N)-(N^2-4N+3)}{N-1}\right]$$

$$L = \frac{t}{8}\left[\frac{N^2+2N-3}{N-1}\right]$$

$$L = \frac{t}{8}\left[\frac{(N-1)(N+3)}{N-1}\right]$$

$$L = \frac{t}{8}[(N+3)]$$

Problem 8.4 (Ramaswami)

Assuming the number of router ports for a given configuration is *Q = 2[t]* for a hubbed ring network, where *t* is the number of traffic units with one unit per wavelength and *N*, the number of nodes, then the upper bound on the number of wavelengths (*W*) required is found by:

Each node sends *t* units to the hub and receives *t* units from the hub. Since the hub can be configured with ½ the nodes on bi-directional ports connecting the other ½ nodes on bi-directional ports, wavelength reuse requires that at most wavelengths required are bound by

$$W = \frac{N}{2}\lceil t \rceil$$

Problem 8.5 (Ramaswami)

For an even number N of nodes, (N-2)/2 of the nodes will take from 1 to (N-1)/2 hops and the other (N-2)/2 of the nodes will also take (N-1)/2 hops (considering the destination node as a hop). One node will take N/2 hops (we can't count this one twice). So the average number of hops is given by the sum of all possible hops divided by the number of nodes that can communicate with the destination node.

$$\langle H \rangle = \frac{\sum_{i=1}^{N/2-1} i + \sum_{i=1}^{N/2-1} i + \frac{N}{2}}{N-1} = \frac{2\left[\frac{N/2-1}{2}\left(N/2-1+1\right)\right] + \frac{N}{2}}{N-1} = \frac{\left[\frac{N^2/2-N}{2}\right] + \frac{N}{2}}{N-1} = \frac{N^2}{4\left(N-1\right)}$$

$$= \frac{N^2+1-1}{4\left(N-1\right)} = \frac{(N+1)(N-1)+1}{4\left(N-1\right)} = \frac{(N+1)}{4} + \frac{1}{4\left(N-1\right)}$$

For an odd number of nodes, we subtract the node that takes N/2 hops

$$\langle H \rangle = \frac{\sum_{i=1}^{N/2-1} i + \sum_{i=1}^{N/2-1} i}{N-1} = \frac{2\left[\frac{N/2-1}{2}\left(N/2-1+1\right)\right]}{N-1} = \frac{\left[\frac{N^2/2-N}{2}\right]}{N-1} = \frac{N^2-2N}{4\left(N-1\right)}$$

$$= \frac{(N-1)(N-1)-1}{4(N-1)} = \frac{(N-1)}{4} - \frac{1}{4(N-1)}$$