## Lecture 10

## Congestion Parameter and Queuing Delay

$\Rightarrow$ Define congestion parameter $\lambda_{\text {max }}=\max _{\mathrm{ij}}\left\{\lambda_{\mathrm{ij}}\right\}$
$\Rightarrow$ We must determine variables $\lambda_{\mathrm{ij}}^{\text {sd }}, \lambda_{\text {max }}$ and $\lambda_{\mathrm{ij}}$
$\Rightarrow$ The process of determining $\lambda_{\mathrm{ij}}{ }^{\text {sd }}, \lambda_{\text {max }}$ and $\lambda_{\mathrm{ij}}$ is equivalent to the problem of finding a routing algorithm
$\Rightarrow$ Example:
$\Rightarrow$ Packet transmission times are exponentially distributed (Poisson Process) with mean time $1 / \mu$ seconds.
$\Rightarrow$ Model each link as an $M / M / 1$ queue, where it is assumed that traffic offered to a link in the network is independent of the traffic offered to other links.
$\Rightarrow$ The average queuing delay under these assumptions is

$$
d_{i j}=\frac{1}{\mu-\lambda_{i j}}
$$

## Throughput

$\leftrightharpoons$ Throughput is defined as minimum value of the offered load for which any link experiences infinite delay, which happens when

$$
\lambda_{\max }=\max _{i j} \lambda_{i j}=\mu
$$

$\Rightarrow$ Performance objective

$$
\operatorname{Minimize}\left(\lambda_{\max }\right)
$$

$\Rightarrow$ Subject to the following flow conservation at each node

$$
\sum_{j} \lambda_{i j}^{s d}-\sum_{j} \lambda_{j i}^{s d}=\left\{\begin{array}{l}
\lambda^{s d} \text { if } s=i, \\
-\lambda^{s d} \text { if } d=i, \\
0 \text { otherwise, }
\end{array} \quad \text { for all } s, d, i\right.
$$

$\Rightarrow$ Subject to the total flow on a logical link

$$
\begin{array}{ll}
\lambda_{i j}=\sum_{s, d} \lambda_{i j}^{s d}, & \text { for all } i, j, \\
\lambda_{i j} \leq \lambda_{\max }, & \text { for all } i, j, \\
\lambda_{i j}^{s d} \leq b_{i j} \lambda^{s d}, & \text { for all } i, j
\end{array}
$$

## Throughput

$\Rightarrow$ Subject to degree constraints

$$
\begin{array}{ll}
\sum_{i} b_{i j} \leq \Delta, & \text { for all } j, \\
\sum_{j} b_{i j} \leq \Delta, & \text { for all } i .
\end{array}
$$

$\Rightarrow$ Subject to bidirectional lightpath constraint

$$
b_{i j}=b_{j i}, \quad \text { for all } \mathrm{i}, j .
$$

$\Rightarrow$ Subject to Nonnegativity and integer constraints

$$
\begin{gathered}
\lambda_{i j}^{s d}, \lambda_{i j}, \lambda_{\max } \geq 0, \quad \text { for all } \mathrm{i}, j, s, d \\
b_{i j}
\end{gathered} \in\{0,1\}, \quad \text { for all } \mathrm{i}, j . ~ \$
$$

## Throughput

$\Rightarrow$ We associate packets routed between pair ( $\mathrm{s}, \mathrm{d}$ ) as the flow of a commodities.
$\Rightarrow$ The flow conservation constraint at node $i$ yields the net flow out of node $i$ for one commodity $(s, d)$.
$\Rightarrow$ The net flow is the difference between the outgoing and ingoing flows.
$\leftrightharpoons$ The flow conservation is 0 if a node is neither a source or destination of that particular commodity.
$\Rightarrow$ If node $i$ is the source of the flow $(i=s)$ the net flow $=\lambda^{\text {sd }}$
$\Rightarrow$ If node $i$ is the destination of the flow $(i=d)$ the net flow $=-\lambda^{\text {sd }}$
$\Rightarrow$ No more than $\Delta$ links into and out of each node.

## LTD Algorithms

$\Rightarrow$ The problem to solve sets a condition that the

$$
b_{i j} \in\{0,1\}, \quad \text { for all } \mathrm{i}, j .
$$

$\Rightarrow$ However, this type of problem can be very difficult and time consuming (numerically) to solve. If the solution is a linear funcion of the variables, then we call the program that can be used to solve type of problem a Linear Program (LP). If the variables are also restricted to integer values, we use an Integer Linear Program (ILP). If only some of the variables are restricted to integer values, we use a Mixed Integer Linear Program (MILP).
$\leftrightharpoons$ If we allow the variables $\mathrm{b}_{\mathrm{ij}}$ to take on any value between 0 and 1, we call this an LTD-LP problem, etc.

## Example LTD Algorithm

$\Rightarrow$ A rounding algorithm would allow us to solve for the $\mathrm{b}_{\mathrm{ij}}$ using an LP approach, then we can round the values close to 0 or close to 1 down to 0 or up to 1 respectfully.
$\Rightarrow$ Algorithm
$\Rightarrow$ Arrange $\mathrm{b}_{\mathrm{ij}}$ obtained using an LTD-LP into decreasing order
$\Rightarrow$ Set each $\mathrm{b}_{\mathrm{ij}}=1$ if the degree constraint $(\Delta)$ is not violated.
Set $\mathrm{b}_{\mathrm{ij}}=0$ otherwise.
$\Rightarrow$ Stop when all the degree constraints are satisfied or there are no $b_{i j} \mathrm{~s}$ remaining
$\Rightarrow$ LP algorithms are not a subject covered in this class.

## Routing and Wavelength Assignment

$\Rightarrow$ We now look in more detail at the tradeoff between equipment cost in the optical vs. the higher electronics layers.
$\Rightarrow$ RWA Problem
$\Rightarrow$ Given a network topology and a set of lightpath (end-toend) requests, determine a route and wavelength for each request using the minimum number of wavelengths (and/or some other wavelength related resource).
$\Rightarrow$ Constraints
$\Rightarrow$ Two lightpaths cannot share the same wavelength on the same link.
$\leftrightharpoons$ Without wavelength conversion, a lightpath must be assigned the same wavelength on all the links along its path

## Directed and Undirected Edges

$\Rightarrow$ Define an undirected edge
$\Rightarrow$ topology with a network that has a pair of unidirectional fiber links in opposite directions between nodes.
$\Rightarrow$ all lightpaths are bi-directional with the same route and wavelength chosen for both directions.
$\Rightarrow$ (a) is an example of an undirected edge and undirected lightpath.
$\Rightarrow$ Note three wavelengths are needed without $\lambda$-conversion
$\Rightarrow(\mathrm{b})$ is an example of directed edges and directed lightpaths
$\Rightarrow$ Only two wavelengths needed without $\lambda$-conversion
$\Rightarrow(\mathrm{c})$ and (d) are examples of undirected edges and directed lightpaths
$\Rightarrow$ (c) requires a fiber in each direction and two wavelengths per fiber
$\Rightarrow$ (d) requires bi-directional transmission over the same fiber, utilizing different wavelengths for each direction

(a)

(b)

(c)

(d)

## Wavelength Conversion

$\Rightarrow$ Wavelength conversion is the ability to translate data from one wavelenght to another without leaving the optical layer.
$\Rightarrow$ Full wavelength conversion in an optical crossconnect allows any wavelength on any input to be changed to any wavelength on any output as shown below.


## Optical Crossconnects (OXCs)

- Electronic switch core and/or DWDM with OEO interfaces
- Electronic signal regeneration at each input/output
- Bit-rate and format dependent
- Inherently non-blocking (OEO wavelength conversion)



## OXC Scalability Issues

- OEO interfaces required on DWDM and switch side or on just switch side

-Increased power dissipation due to
-faster OEO
-faster switch fabric



## Photonic Crossconnects (PXCs)

- Purely optical switch core, OEO on DWDM side only
- Bit-rate and format independent
- Optical loss can be compensated with optical amplification
- Non-blocking with OEO transponders or wavelength converters



## PXC Scalability



- Port count and footprint are independent of line rate
- Optical amplification can be used to compensate for increased loss with increased size
-Power dissipation increases with increased bit-rate only on DWDM

> line-side
-Power dissipation of switch fabric independent of DWDM line rate


## Wavelength Selective Crossconnects (WSXC)

- Scales to high port count using smaller NxN switches
- Partially blocking (cannot connect between $\lambda \mathrm{s}$ )



## Wavelength Switch/Router



## Wavelength Interchanger



