Lecture 11
Graph Coloring

\( \Rightarrow \) We can view the problem of wavelength assignment as a graph coloring problem

\( \Rightarrow \) Define the network as \( G \)

\( \Rightarrow \) Route for a lightpath corresponds to a path in \( G \)

\( \Rightarrow \) Set of routes that have been specified corresponds to a set of paths \( P \)

\( \Rightarrow \) Define the Path Graph \( P(G) \)

\( \Rightarrow \) Each path in \( P \) corresponds to a node in \( P(G) \)

\( \Rightarrow \) Two nodes in \( P(G) \) are connected by an edge if the corresponding paths in \( P \) share a common edge in \( G \)

\( \Rightarrow \) Solving the WA problem is then equivalent to solving the graph coloring problem in \( P(G) \)
Dimensioning W-R Networks

- The number and set of wavelengths must be determined in a network design, including which wavelengths on each link (Wavelength Dimensioning Problem).

**Statistical Dimensioning**
- First Passage Model: More likely to be used in today’s semi-static networks
  - Assume network starts with no lightpaths.
  - Lightpath requests and establishment arrive randomly according to some statistical model.
  - On average, over time, the number of lightpaths keeps increasing until a request has to be rejected.
  - Goal of model is to choose (dimension) the WDP such that there is a high probability that the first rejection will occur after a time T.
- Blocking Model: More likely to be used in future dynamic provisioned networks
  - Lightpaths requests are setup and torn down according to a statistical model, with total number on average staying constant (similar to telephone network modeling).
  - Most requests are honored, but some are blocked
  - Goal is to dimension so that blocking is low probability (e.g. 1%)
First-Passage Model

- Assume lightpath requests follow Poisson distribution and durations are exponentially distributed.
- Model the network using Markov chain where state of the chain is set of lightpaths in progress.
- Consider both cases with no wavelength conversion and with full wavelength conversion.
- Example is shown below for original ARPANET.
  - 20 node, 32 links, 190 possible routes.
  - Average lightpath lease was 1 year with exponential distribution.
  - Link capacities shown are determined such that probability any link needs to be upgraded within two years is less than 15%.
Blocking Model

- Define the *offered load*,
  - = Arrival rate of lightpath requests X the average lightpath duration
- Specify the maximum blocking probability (e.g. 1%)
- Determine the maximum offered load the network can support
- Define *reuse factor R*
  - Offered load per wavelength in the network that can be supported with a specified blocking probability
- R depends on
  - Network topology
  - Traffic distribution
  - RWA algorithm used
  - Number of wavelengths available
- Example algorithm

Interesting result:
- Reuse factor improves as the number of wavelengths increases!
- Called *trunking efficiency*

\[
P_b(\rho, W) = \frac{\rho^W}{W!} \sum_{i=0}^{W} \frac{\rho_i}{i!}
\]

\[
P_b(\alpha \rho, \alpha W) < P_b(\alpha \rho, \alpha W)
\]
Wavelength Reuse

- The impact of using wavelength reuse and wavelength conversion to alleviate blocking the network is an important issue to understand and be able to quantify.
- Without wavelength conversion, the number of hops that each lightpath takes and the overlap between lightpaths on link segments will heavily impact the number of wavelengths needed to support a network of a certain topology, size, and load.
- The effect that wavelength conversion can have on alleviating this problem is called wavelength conversion gain.
Wavelength Reuse

- Let's analyze (compare) a network without wavelength conversion to a network with wavelength conversion in terms of the probability that lightpath request will be blocked.
- Assume a statistical model of lightpath requests
- Assume that the route for each lightpath through the network is pre-assigned. This is important since if all lightpath routes are preassigned, then the state of the network is deterministic and non-blocking paths can be readily identified.
- Assume a network without wavelength conversion:
  - assigns an arbitrary, that is the same wavelength on every link of the route, when one wavelength is free (not assigned to other lightpaths) on every link on the path)
- When the network uses wavelength conversion:
  - assigns an arbitrary free wavelength on each link of the route
Let the $\pi$ be the probability that a wavelength is used on a link with $W$ wavelengths per link.

Assume that $\pi$ is independent of the probability that any other wavelength on that link or on other links is in use.

The probability that a wavelength on a certain link is free is given by $(1 - \pi)$ and the probability that a wavelength is free on all links over $H$ hops on a route is given by $(1 - \pi)^H$.

Therefore the probability that a given wavelength is not free on some link on the route is $(1 - (1 - \pi)^H)$.

And the probability, without wavelength conversion, that all $W$ wavelengths are not free on some link on a requested lightpath is given by

$$P_{b,nc} = (1 - (1 - \pi)^H)^W$$

If we introduce wavelength conversion, then the probability of blocking is given by the probability that any of the links on the route have exhausted their supply of wavelengths

$$P_{b,fc} = 1 - (1 - \pi^W)^H$$
We can now define the possible link utilization (that which can be achieved) for a given blocking probability with and without wavelength conversion.

\[ \pi_{nc} = 1 - (1 - P_{b,nc}^{1/W})^{1/H} \]

\[ \pi_{fc} = (1 - (1 - P_{b,fc}^{1/H})^{1/W}) \]

For small \( P_b \) and \( W \), we can approximate this as

\[ \pi_{nc} \approx \frac{P_{b,nc}^{1/W}}{H} \]

\[ \pi_{fc} \approx \left( \frac{P_{b,fc}}{H} \right)^{1/W} \]

And define the gain by the ratio, which shows the sensitivity of wavelength conversion on the achievable link utilization on the number of hops \( H \)

\[ \frac{\pi_{fc}}{\pi_{nc}} \approx H^{1 - \frac{1}{W}} \]
Conditional Dependence

⇒ What if we now remove the assumption that the probability of wavelength usage on each link is mutually independent.
⇒ For a network with no wavelength conversion, we define any lightpath that has already been established and uses one of the $H$ links that we want to use for a new lightpath, is an **interfering lightpath**.
⇒ Place the constraint that an interfering lightpath that uses link $i$ on one of the $H$ links, will not use the next link $i+1$ with probability $\pi_i$.
⇒ For any $\lambda$, we assume a new lightpath request that does not interfere on link $i-1$, will interfere on link $i$ on the route with probability $\pi_n$.
⇒ We have the following conditional probabilities:

\[
\text{Prob}(\lambda \text{ used on link } i \mid \lambda \text{ is not used on link } i-1) = \pi_n
\]

\[
\text{Prob}(\lambda \text{ used on link } i \mid \lambda \text{ used on link } i-1) = (1 - \pi_i) + \pi_i\pi_n
\]

⇒ And the probability of blocking with no wavelength conversion can be shown to be

\[
P_{b,nc} = (1 - (1 - \pi_n)^H)^W
\]