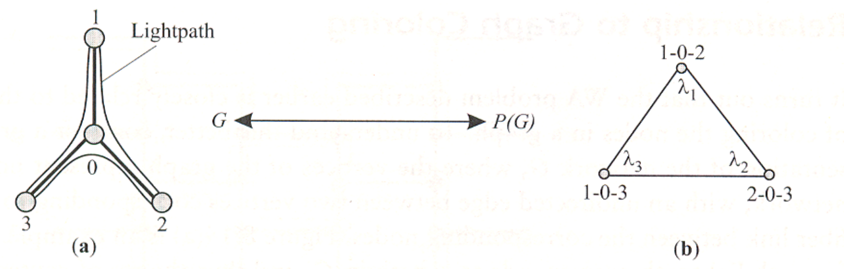
Lecture 11

Graph Coloring

- We can view the problem of wavelength assignment as a graph coloring problem
  - Define the network as G
  - Route for a lightpath corresponds to a path in G
  - Set of routes that have been specified corresponds to a set of paths P
  - Define the Path Graph P(G)
    - Each path in P corresponds to a node in P(G)
    - Two nodes in P(G) are connected by an edge if the corresponding paths in P share a common edge in G
  - Solving the WA problem is then equivalent to solving the graph coloring problem in P(G)

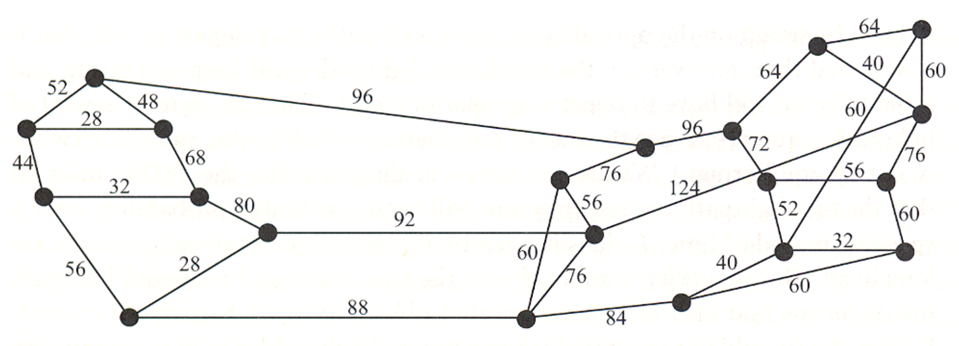


Dimensioning W-R Networks

- The number and set of wavelengths must be determined in a network design, including which wavelengths on each link > ***Wavelength Dimensioning Problem***
- Statistical Dimensioning
  - First Passage Model: More likely to be used in today's semi-static networks
    - Assume network starts with no lightpaths.
    - Lightpath requests and establishment arrive randomly according to some statistical model.
    - On average, over time, the number of lightpaths keeps increasing until a request has to be rejected.
    - Goal of model is to choose (dimension) the WDP such that there is a high probability that the first rejection will occur after a time T.
  - Blocking Model: More likely to be used in future dynamic provisioned networks
    - Lightpaths requests are setup and torn down according to a statistical model, with total number on average staying constant (similar to telephone network modeling).
    - Most requests are honored, but some are blocked
    - Goal is to dimension so that blocking is low probability (e.g. 1%)

First-Passage Model

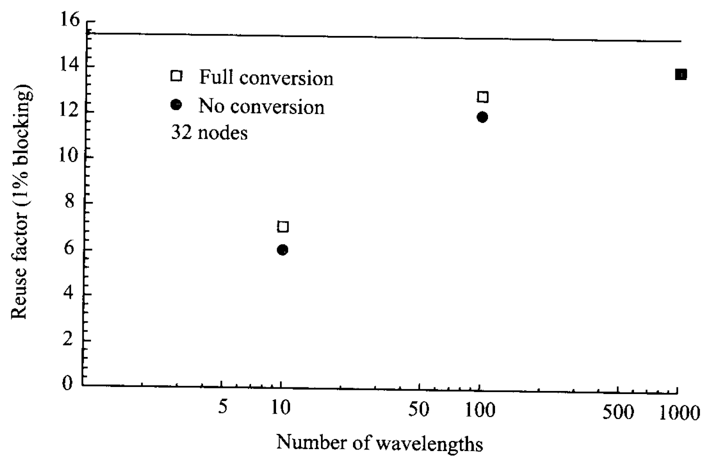
- ⇨ Assume lightpath requests follow Poisson distribution and durations are exponentially distributed
- ⇨ Model the network using Markov chain where state of the chain is set of lightpaths in progress.
- ⇨ Consider both cases with no wavelength conversion and with full wavelength conversion.
- ⇨ Example is shown below for original ARPANET.
  - ⇨ 20 node, 32 links, 190 possible routes.
  - ⇨ Average lightpath lease was 1 year with exponential distribution
  - ⇨ Link capacities shown are determined such that probability any link needs to be upgraded within two years is less than 15%.



Blocking Model

- Define the ***offered load***,
  - = Arrival rate of lightpath requests X the average lightpath duration
- Specify the maximum blocking probability (e.g. 1%)
- Determine the maximum offered load the network can support
- Define ***reuse factor R***
  - Offered load per wavelength in the network that can be supported with a specified blocking probability
- R depends on
  - Network topology
  - Traffic distribution
  - RWA algorithm used
  - Number of wavelengths available
- Example algorithm

- Interesting result:
  - Reuse factor improves as the number of wavelengths increases!
  - Called ***trunking efficiency***



$$P_b(\rho, W) = \frac{\frac{\rho^W}{W!}}{\sum_{i=0}^{W} \frac{\rho_i}{i!}}$$

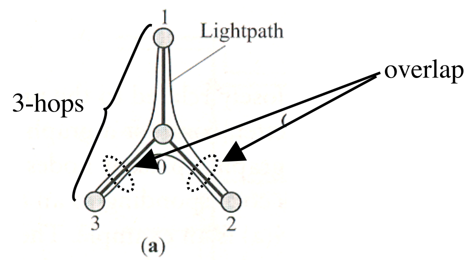
$$P_b(\alpha\rho, \alpha W) < P_b(\alpha\rho, \alpha W)$$

Wavelength Reuse

⇨ The impact of using wavelength reuse and wavelength conversion to alleviate blocking the network is an important issue to understand and be able to quantify

⇨ Without wavelength conversion, the number of hops that each lightpath takes and the overlap between lightpaths on link segments will heavily impact the number of wavelengths needed to support a network of a certain topology, size, and load.

⇨ The effect that wavelength conversion can have on alleviating this problem is called wavelength conversion gain.



Wavelength Reuse

- ⇨ Lets analyze (compare) a network without wavelength conversion to a network with wavelength conversion in terms of the probability that lightpath request will be blocked.
- ⇨ Assume a statistical model of lightpath requests
- ⇨ Assume that the route for each lightpath through the network is pre-assigned. This is important since if all lightpath routes are preassigned, then the state of the network is deterministic and non-blocking paths can be readily identified.
- ⇨ Assume a network without wavelength conversion:
  - ⇨ assigns an arbitrary, that is the same wavelength on every link of the route, when one wavelength is free (not assigned to other lightpaths) on every link on the path)
- ⇨ When the network uses wavelength conversion:
  - ⇨ assigns an arbitrary free wavelength on each link of the route

Wavelength Reuse Blocking Probability

⇨ Let the π be the probability that a wavelength is used on a link with W wavelengths per link.

⇨ Assume that π is independent of the probability that any other wavelength on that link or on other links is in use.

⇨ The probability that a wavelength on a certain link is free is given by $(1 - \pi)$ and the probability that a wavelength is free on all links over H hops on a route is given by $(1 - \pi)^H$.

⇨ Therefore the probability that a given wavelength is not free on some link on the route is $(1 - (1 - \pi)^H)$.

⇨ And the probability, without wavelength conversion, that all W wavelengths are not free on some link on a requested lightpath is given by

$$P_{b,nc} = (1 - (1 - \pi)^H)^W$$

⇨ If we introduce wavelength conversion, then the probability of blocking is given by the probability that any of the links on the route have exhausted their supply of wavelengths

$$P_{b,fc} = 1 - (1 - \pi^W)^H$$

Achievable Link Utilization

⇨ We can now define the possible link utilization (that which can be achieved) for a given blocking probability with and without wavelength conversion

$$\pi_{nc} = 1 - (1 - P_{b,nc}^{1/W})^{1/H}$$

$$\pi_{fc} = \left(1 - (1 - P_{b,fc})^{1/H}\right)^{1/W}$$

⇨ For small P_b and W, we can approximate this as

$$\pi_{nc} \cong \frac{P_{b,nc}^{1/W}}{H}$$

$$\pi_{fc} \cong \left(\frac{P_{b,fc}}{H}\right)^{1/W}$$

⇨ And define the gain by the ratio, which shows the sensitivity of wavelength conversion on the achievable link utilization on the number of hops H

$$\frac{\pi_{fc}}{\pi_{nc}} \cong H^{1-1/W}$$

Conditional Dependence

⇨ What if we now remove the assumption that the probability of wavelength usage on each link is mutually independent.

⇨ For a network with no wavelength conversion, we define any lightpath that has already been established and uses one of the H links that we want to use for a new lightpath, is an ***interfering lightpath***.

⇨ Place the constraint that an interfering lightpath that uses link i on one of the H links, will not use the next link $i+1$ with probability π_l.

⇨ For any λ, we assume a new lightpath request that does not interfere on link i-1, will interfere on link i on the route with probability π_n.

⇨ We have the following conditional probabilities:

$$\text{Prob}(\lambda \text{ used on link } i \,|\, \lambda \text{ is not used on link } i-1) = \pi_n$$

$$\text{Prob}(\lambda \text{ used on link } i \,|\, \lambda \text{ used on link } i-1) = (1-\pi_l) + \pi_l \pi_n$$

⇨ And the probability of blocking with no wavelength conversion can be shown to be

$$P_{b,nc} = (1-(1-\pi_n)^H)^W$$